Jordi Martí-Henneberg, "The radiality of the railway network in Spain during its early stages (1830-1867). An assessment of its territorial coherence."

Supplementary Appendix

We mathematically formulated the passing place indicator. Let *P* represent all of the judicial districts in Spain. Then, let $a_{i \to j}(k): P \to \{0,1\}$ be a function that, for a given connection $i \to j$, determines whether *k* is an acceptable stopping point. In particular:

$$a_{i \rightarrow j}(k) = \{0 \quad if \ k \ is \ not \ a \ good \ place \ to \ pass \ 1 \qquad if \ k \ is \ a \ good \ place \ to \ pass \ Y \ k \ \epsilon \ P$$

To determine the value of $a_{i \rightarrow j}(k)$ we must make:

$$a_{i \to j}(k) = 1 \iff \frac{d(i,k) + d(k,j)}{d(i,j)} \le detour$$

In other words, $a_{i \rightarrow j}(k) = 1$ if we do not have to deviate from the route by more than the maximum permitted *detour* in order to pass through judicial district *k*. In general, it is acceptable to deviate from the route to pass through what could be considered an important intermediary point (based on its GDP) but not to stop at an insignificant one. We can therefore suggest creating a detour dependence based on the GDP of k. We propose:

$$detour(k) = \frac{PIB_k}{PIB_{max}} + 1$$

where PIB_{max} is the greatest GDP value in a given Spanish judicial district. In the case that concerns us here: $PIB_{max} = PIB_{Madrid}$. It is therefore clear that $detour(k) \in [1,2]$ and, as a result, with a linear relationship in the GDP, it is possible to have detours between 1 –in other words no detour– and 2 –of double the straight-line distance–. With this, the previous expression becomes:

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$$\begin{aligned} a_{i \to j}(k) &= \{0 \quad if \quad \frac{d(i,k) + d(k,j)}{d(i,j)} > \frac{PIB_k}{PIB_{max}} + 1 1 \quad if \quad \frac{d(i,k) + d(k,j)}{d(i,j)} \\ &\leq \frac{PIB_k}{PIB_{max}} + 1 \end{aligned}$$

We can now create the indicator p(k) to quantify how many times a judicial district is a good place to pass for the 104653¹ possible connections between all of the other Spanish judicial districts:

$$p(k) = \sum_{\forall i \in P \setminus k} \sum_{\forall j \in P \setminus k \text{ s.t. } j < i} a_{i \to j}(k)$$

¹ There are 459 judicial districts on the Spanish Peninsula. The number of all the connections between 458 of them –459 minus judicial district k, for which we want to calculate the indicator p(k) – is 458 2 =104653. This is precisely the value that comes from calculating: i=1458j=1i-11