Appendix to "Potentialism and S5"

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This is the appendix to the article "Potentialism and S5", which is published in the *Canadian Journal of Philosophy*. Doi: 10.1017/can.2023.2.

In this appendix I formalise **Strict Argument** (see §2) in a propositional modal logic and I provide a semantic tableau for the inference from (ii) and (V) to (vi). I will let 'A' stand for the proposition that Vetter exists (existed or will exist), use 'xx' as a plural objectual variable, and use '$\blacklozenge$' as a binary connective that takes a plural objectual variable as its first argument (notated as its subscript) and a sentence as its second argument such that '$\blacklozenge_{xx}p$' is interpreted as 'the xx have (had or will have) an iterated or non-iterated potentiality that p'.

(i) A (Assumption)

(ii) $\Diamond\neg A$ (Assumption)

(III) $\Box(\neg A \to \neg\exists xx \blacklozenge_{xx} A)$ (from Dependence)

(IV) $\Box(\neg\exists xx \blacklozenge_{xx} A \to \neg\Diamond A)$ (From Potentialism)

(V) $\Box(\neg A \to \neg\Diamond A)$ (From (III) and (IV))

(vi) $\neg\Box\Diamond A$ (From (ii) and (V))

Conclusion $A \wedge \neg\Box\Diamond A$ (From (i) and (vi))

Please note that although interpreting '$\exists xx \blacklozenge_{xx} A$' requires a semantic way to interpret quantification over individuals and into the subscript of our connective, the details of how this can be achieved need not bother us in the present context. Rather than using '$\exists xx \blacklozenge_{xx} A$', one could simply introduce 'B' as a propositional constant that abbreviates '$\exists xx \blacklozenge_{xx} A$'. Then the argument would be completely cast in an ordinary propositional modal logic without any additional devices.

The conclusion directly follows from (i) and (vi) by conjunction introduction. It is equivalent to $\neg(A \to \Box\Diamond A)$.

The following tree-proof shows that (ii) and (V) entail (vi) in every normal modal logic:

$\Diamond\neg A, \Box(\neg A \rightarrow \neg\Diamond A) \vdash \neg\Box\Diamond A$

1.	at w : $\Diamond\neg A$		Assumption
2.	at w : $\Box(\neg A \rightarrow \neg\Diamond A)$		Assumption
3.	at w : $\Box\Diamond A$		Negated Conclusion
4.	w has access to v		(1)
5.	at v : $\neg A$		(1)
6.	at v : $\Diamond A$		(3+4)
7.	v has access to u		(6)
8.	at u : A		(6)
9.	at v : $\neg A \rightarrow \neg\Diamond A$		(2+4)
10.	at v : $\neg\neg A$	at v : $\neg\Diamond A$	(9)
11.	at v : A		(10)
12.	$\otimes$ 5	at u : $\neg A$	(10+7)
		$\otimes$ 8	

The only other step in the argument that has not yet been discussed is the inference from (III) and (IV) to (V). I take the result that the strict conditional is transitive in every normal modal logic for granted (i.e. the result that $\Box(p \rightarrow q), \Box(q \rightarrow r) \vdash \Box(p \rightarrow r)$ holds in every normal modal logic). This result immediately guarantees the inference from (III) and (IV) to (V).