

# SUPPLEMENTARY MATERIAL

## External Resources and Indiscriminate Violence

### Evidence from German-Occupied Belarus

*World Politics*, Vol. 69, no. 1, January 2017

doi:10.1017/S0043887116000149

Yuri M. Zhukov

Department of Political Science, University of Michigan

June 4, 2016

This supplemental material formalizes the theoretical narrative described qualitatively in Section 2 of the main text. I develop a dynamic model that describes the effect of key strategic choices and exogenous parameters on the outcome of an asymmetric irregular war. In particular, I examine the role of coercion, civilian cooperation, and information asymmetry on combatants' ability to establish a monopoly on the use of force within a conflict zone.

I begin with a benchmark case where combatants have access to only local resources, attained through the cooperation of a security-seeking civilian population. I show that, in the absence of external resources, the government can establish a monopoly only if it can outproduce the rebels in selective violence. I then consider an extension of the model, in which one or both combatants have access to external resources. I show that such resources make it possible for the government to achieve victory despite a disadvantage in selective violence. By interdicting this external support, rebels can prevent the government from establishing a monopoly, and ensure a long-term decline in government violence.

## 1 Local support

Imagine a conflict zone inhabited by three groups of actors: rebels ( $R$ ), government ( $G$ ) and civilians ( $C$ ).  $G$  and  $R$  each seeks a monopoly on the use of force, which requires cooperation from  $C$  – in the form of taxes, manpower, intelligence and other forms of support. At a minimum,  $G$  and  $R$  want to ensure that  $C$  does not cooperate with their opponent, and does not actively resist their claims to sovereignty.  $C$  is interested in security above all else, and will cooperate with one of the two sides or remain neutral, depending on the relative costs of the three options.

Let  $G_t$  and  $R_t$  denote the sizes of the groups cooperating with government and rebel forces at time  $t$ . Let  $C_t$  denote the size of the neutral civilian population at time  $t$ . Let  $\pi_G(\mathbf{s}) = \frac{G_{eq}}{G_{eq} + R_{eq}} \in [0, 1]$  denote the government's payoff from strategy set  $\mathbf{s} = \{s_G, s_R, s_C\}$ , or the government's share of public support at equilibrium. Similarly, let  $\pi_R(\mathbf{s}) = \frac{R_{eq}}{G_{eq} + R_{eq}} \in [0, 1]$  denote the rebels' payoff. An equilibrium outcome with  $\pi_G = 1, \pi_R = 0$  is a *government monopoly*, in which the rebel population

converges to zero and the government establishes a monopoly on the use of force. An outcome with  $\pi_G = 0, \pi_R = 1$  is a *rebel monopoly*, similarly defined.

The combatants  $i \in \{G, R\}$  maximize their equilibrium shares of popular support by increasing the costs of cooperation with their opponents. Let  $s_R: \rho_R > 0$  be the intensity of rebel coercion against government forces and  $s_G: \rho_G > 0$  be the intensity of government coercion against the rebels. Let  $\theta_i \in (0, 1)$  be selectivity, or the ability of combatant  $i$  to accurately identify her opponents, such that some fraction  $\rho_i \theta_i$  of the combatant's violence will reach the intended targets, but the remainder  $\rho_i(1 - \theta_i)$  will be indiscriminately inflicted on neutral civilians.

Let  $\pi_C\{\mathbf{s}\} = -\kappa \in (-\infty, 0]$  be the costs inflicted on civilians by fighting between the combatants. If civilians join  $G$  or  $R$ , they will pay costs proportional to levels of selective violence inflicted against that group. If civilians stay neutral, they will pay costs in proportion to overall indiscriminate violence directed at civilians.

**Lemma 1.** *It is always more costly to remain neutral than to cooperate with one of the combatants.*

*Proof.* Let  $\kappa(i)$  denote the expected costs associated with membership in group  $i \in \{G, R, C\}$ , with  $\kappa(G) = \rho_R \theta_R$ ,  $\kappa(R) = \rho_G \theta_G$ , and  $\kappa(C) = \rho_R(1 - \theta_R) + \rho_G(1 - \theta_G)$ . The statement  $[\kappa(C) < \kappa(G)] \wedge [\kappa(C) < \kappa(R)]$  ("staying neutral is less costly than joining either combatant") is never true for any  $\rho_G \in (0, \infty), \rho_R \in (0, \infty), \theta_G \in [0, 1], \theta_R \in [0, 1]$  and  $\theta_G + \theta_R = 1$ . The statement  $[\kappa(C) < \kappa(G)] \wedge [\kappa(C) > \kappa(R)]$  ("staying neutral is less costly than joining  $G$  but more costly than joining  $R$ ") is true if and only if  $[\rho_G < \rho_R] \wedge [0 \leq \theta_G < \frac{\rho_R - \rho_G}{2\rho_R - \rho_G}]$ , and  $[\kappa(C) > \kappa(G)] \wedge [\kappa(C) < \kappa(R)]$  ("staying neutral is more costly than joining  $G$  but less costly than joining  $R$ ") is true if and only if  $[\rho_G > \rho_R] \wedge [\frac{\rho_G}{2\rho_G - \rho_R} < \theta_G \leq 1]$ . The statement  $[\kappa(C) > \kappa(G)] \wedge [\kappa(C) > \kappa(R)]$  ("staying neutral is more costly than joining  $G$  or  $R$ ") is true in all other cases: (1)  $[\rho_G > \rho_R] \wedge [0 \leq \theta_G < \frac{\rho_G}{2\rho_G - \rho_R}]$ , (2)  $[\rho_G < \rho_R] \wedge [\frac{\rho_R - \rho_G}{2\rho_R - \rho_G} < \theta_G \leq 1]$ .  $\square$

Lemma 1 shows that indiscriminate violence partially solves the combatants' collective action problem. If the damage jointly inflicted by indiscriminate government and rebel violence is greater than the selective damage inflicted by one side, neutrality will always be costlier than cooperation.

Let  $s_C: \mu_i = 1 - \frac{\rho_i \theta_i}{\rho_i + \rho_j}$  be the rate of civilian cooperation with group  $i$ . Intuitively, if  $G$  can inflict more selective violence against  $R$  than  $R$  can against  $G$  ( $\rho_G \theta_G > \rho_R \theta_R$ ), then  $C$  will cooperate with  $G$  at a higher rate than with  $R$  ( $\mu_G > \mu_R$ ).

These dynamics comprise a system of ordinary differential equations

$$\frac{\delta C}{\delta t} = k - (\mu_R R_t + \mu_G G_t - \rho_R(1 - \theta_R) - \rho_G(1 - \theta_G) - u) C_t \quad (1)$$

$$\frac{\delta G}{\delta t} = (\mu_G C_t - \rho_R \theta_R - u) G_t \quad (2)$$

$$\frac{\delta R}{\delta t} = (\mu_R C_t - \rho_G \theta_G - u) R_t \quad (3)$$

where  $\frac{\delta i}{\delta t}$  is the rate of change in the size of group  $i$  over time,  $k$  is an immigration parameter that

ensures a stable, non-negative population, and  $u$  is a natural death rate constant across all groups.

**Proposition 1.** *Without external support, a government victory equilibrium is stable if and only if the government's rate of selective violence is greater than that of the rebels.*

The proof of Proposition 1 depends on the following Lemma:

**Lemma 2.** *There exist three equilibrium solutions to (1-3) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

*Proof.* Define a *government victory equilibrium* of (1-3) as a fixed point satisfying  $\frac{\delta C}{\delta t} = 0$ ,  $\frac{\delta G}{\delta t} = 0$ ,  $\frac{\delta R}{\delta t} = 0$ ,  $C_{eq} \in [0, \infty)$ ,  $G_{eq} \in [0, \infty)$ ,  $R_{eq} \in [0, \infty)$  and  $\pi_G(\mathbf{s}) = 1$ ,  $\pi_R(\mathbf{s}) = 0$ . These conditions are satisfied at

$$C_{eq} = \frac{\rho_R \theta_R + u}{\mu_G} \quad (4)$$

$$G_{eq} = \frac{k}{\rho_R \theta_R + u} - \frac{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u}{\mu_G} \quad (5)$$

$$R_{eq} = 0 \quad (6)$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $k \in (0, \infty)$ ,  $u \in (0, \infty)$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ .

Define a *rebel victory equilibrium* of (1-3) as a fixed point satisfying  $\frac{\delta C}{\delta t} = 0$ ,  $\frac{\delta G}{\delta t} = 0$ ,  $\frac{\delta R}{\delta t} = 0$ ,  $C_{eq} \in [0, \infty)$ ,  $G_{eq} \in [0, \infty)$ ,  $R_{eq} \in [0, \infty)$  and  $\pi_G(\mathbf{s}) = 0$ ,  $\pi_R(\mathbf{s}) = 1$ . These conditions are satisfied at

$$C_{eq} = \frac{u + \rho_G \theta_G}{\mu_R} \quad (7)$$

$$G_{eq} = 0 \quad (8)$$

$$R_{eq} = \frac{k}{\rho_G \theta_G + u} - \frac{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u}{\mu_R} \quad (9)$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $k \in (0, \infty)$ ,  $u \in (0, \infty)$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ .

Define a *mutual destruction equilibrium* of (1-3) as a fixed point satisfying  $\frac{\delta C}{\delta t} = 0$ ,  $\frac{\delta G}{\delta t} = 0$ ,  $\frac{\delta R}{\delta t} = 0$ ,  $C_{eq} \in [0, \infty)$ ,  $G_{eq} \in [0, \infty)$ ,  $R_{eq} \in [0, \infty)$  and  $\pi_G(\mathbf{s}) = 0$ ,  $\pi_R(\mathbf{s}) = 0$ . These conditions are satisfied at

$$C_{eq} = \frac{k}{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u} \quad (10)$$

$$G_{eq} = 0 \quad (11)$$

$$R_{eq} = 0 \quad (12)$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $k \in (0, \infty)$ ,  $u \in (0, \infty)$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ .  $\square$

Now we can proceed to prove Proposition 1.

*Proof.* The stability of the equilibrium in (4-6) can be shown through linearization. Assume  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ . To ensure non-negative population values in equilibrium,

we impose a lower bound on immigration parameter  $k > \frac{(\rho_R \theta_R + u)(\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u)}{\mu_G}$ .

Let  $\mathbf{J}$  be the Jacobian of the system in (1-3), evaluated at fixed point (4-6).

$$\mathbf{J} = \begin{pmatrix} -\frac{k\mu_G}{\rho_R\theta_R+u} & -\rho_R\theta_R - u & -\frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} \\ 0 & 0 & \frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} - \rho_G\theta_G - u \\ \frac{k\mu_G - (\rho_R\theta_R+u)(\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u)}{\rho_R\theta_R+u} & 0 & 0 \end{pmatrix} \quad (13)$$

The determinant and trace of  $\mathbf{J}$  are

$$\det(\mathbf{J}) = \frac{(-\rho_R\theta_R - u) \left( \frac{\mu_R(\rho_R\theta_R+u)}{\mu_G} - \rho_G\theta_G - u \right) (k\mu_G - (\rho_R\theta_R+u)(\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u))}{\rho_R\theta_R+u} \quad (14)$$

$$\text{tr}(\mathbf{J}) = -\frac{k\mu_G}{\rho_R\theta_R+u} \quad (15)$$

The equilibrium point (4-6) is stable if all the eigenvalues of  $\mathbf{J}$  have negative real parts, or  $\det(\mathbf{J}) > 0$ ,  $\text{tr}(\mathbf{J}) < 0$ . These conditions hold if and only if  $\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$ .  $\square$

In the absence of external support, government victory requires that cooperation with rebels be more costly than cooperation with the government (Proposition 1). The stability of the government monopoly equilibrium depends on the selective violence ratio  $\frac{\rho_G\theta_G}{\rho_R\theta_R}$ . When  $\frac{\rho_G\theta_G}{\rho_R\theta_R} > 1$ , government forces have a *selective violence advantage* and are able to inflict costs on the rebels at a higher rate than the rebels can against them. If this happens, civilians cooperate in greater numbers with the government, and the system converges to a government monopoly ( $\pi_G(\cdot) = 1, \pi_R(\cdot) = 0$ ). When  $\frac{\rho_G\theta_G}{\rho_R\theta_R} < 1$ , a government victory becomes unsustainable and the system converges to a rebel monopoly ( $\pi_G(\cdot) = 0, \pi_R(\cdot) = 1$ ). The side better able to inflict selective violence will win the war.

Two empirical implications follow from the benchmark model: government coercion should be most extreme where (a) rebel coercion is high, and (b) government selectivity is poor. Formally, a stable victory requires the government to “outbid” its opponent’s use of coercion by matching the rebels’ level of violence, scaled by the initial balance of selectivity between them:  $\rho_G^* > \rho_R \frac{\theta_R}{\theta_G}$ , with  $\rho_G^*$  increasing in  $\rho_R$ , but decreasing in  $\theta_G$ . In other words, the kinds of areas where the government lacks the information for selective violence – and depends on indiscriminate force – are the same areas where incentives for escalation are greatest. An increase in rebel coercion will only provoke the government to escalate further.

## 2 External support

How does the availability of external resources change the dynamics of the conflict? Let  $\alpha_i \in [0, \infty)$  be the rate at which combatant  $i$  is able to draw on sources of support external to the conflict zone. For the government,  $\alpha_G$  may represent the ability to mobilize reserves, deploy reinforcements, send supplies, and draw on other sources of revenue and manpower that do not depend directly on local civilian cooperation. For rebels,  $\alpha_R$  may represent the ability to mobilize fighters and units from sanctuary areas of neighboring states, or attract capital and labor from governments, charities, and diasporas located outside the contested area. Let  $d \in [0, 1]$  be the proportion of  $G$ ’s external resources that  $R$  is able to interdict.<sup>1</sup>

<sup>1</sup>Although both sides can in principle interdict each other’s resources, for simplicity I limit the current discussion to rebel interdiction of the government’s external support. Because the model is symmetric, the same general results apply to the reverse case.

To permit this diversification of combatants' sources of support, we modify the system of equations in (1-3) in the following manner:

$$\frac{\delta C}{\delta t} = k - (\mu_R R_t + \mu_G G_t - \rho_R(1 - \theta_R) - \rho_G(1 - \theta_G) - u) C_t \quad (16)$$

$$\frac{\delta G}{\delta t} = (\mu_G C_t + (1 - d)\alpha_G - \rho_R\theta_R - u)G_t \quad (17)$$

$$\frac{\delta R}{\delta t} = (\mu_R C_t + \alpha_R - \rho_G\theta_G - u)R_t \quad (18)$$

Note that while local support requires interaction with the population ( $\mu_i C_t$ ), external support ( $\alpha_i$ ) does not depend on contact with civilians.

**Proposition 3.** *If external sources of support are available to the combatants, a selective violence advantage is neither necessary, nor sufficient for victory.*

Proposition 3 depends on the following Lemma:

**Lemma 3.** *There exist three equilibrium solutions to (16-18) in which the outcome of the fighting does not depend on the initial balance of forces: government victory, rebel victory and mutual destruction.*

*Proof.* Define a *government victory equilibrium* of (16-18) as a fixed point satisfying  $\frac{\delta C}{\delta t} = 0$ ,  $\frac{\delta G}{\delta t} = 0$ ,  $\frac{\delta R}{\delta t} = 0$ ,  $C_{eq} \in [0, \infty)$ ,  $G_{eq} \in [0, \infty)$ ,  $R_{eq} \in [0, \infty)$  and  $\pi_G(\mathbf{s}) = 1$ ,  $\pi_R(\mathbf{s}) = 0$ . These conditions are satisfied at

$$C_{eq} = \frac{\rho_R\theta_R + u - (1 - d)\alpha_G}{\mu_G} \quad (19)$$

$$G_{eq} = \frac{k}{\rho_R\theta_R + u - (1 - d)\alpha_G} - \frac{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u}{\mu_G} \quad (20)$$

$$R_{eq} = 0 \quad (21)$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $\alpha_G \in [0, \infty)$ ,  $\alpha_R \in [0, \infty)$ ,  $k \in (0, \infty)$ ,  $u \in (0, \infty)$ ,  $d \in [0, 1]$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ .

Define a *rebel victory equilibrium* of (16-18) as a fixed point satisfying  $\frac{\delta C}{\delta t} = 0$ ,  $\frac{\delta G}{\delta t} = 0$ ,  $\frac{\delta R}{\delta t} = 0$ ,  $C_{eq} \in [0, \infty)$ ,  $G_{eq} \in [0, \infty)$ ,  $R_{eq} \in [0, \infty)$  and  $\pi_G(\mathbf{s}) = 0$ ,  $\pi_R(\mathbf{s}) = 1$ . These conditions are satisfied at

$$C_{eq} = \frac{u + \rho_G\theta_G - \alpha_R}{\mu_R} \quad (22)$$

$$G_{eq} = 0 \quad (23)$$

$$R_{eq} = \frac{k}{\rho_G\theta_G + u - \alpha_R} - \frac{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u}{\mu_R} \quad (24)$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $\alpha_G \in [0, \infty)$ ,  $\alpha_R \in [0, \infty)$ ,  $k \in (0, \infty)$ ,  $u \in (0, \infty)$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ .

Define a *mutual destruction equilibrium* of (16-18) as a fixed point satisfying  $\frac{\delta C}{\delta t} = 0$ ,  $\frac{\delta G}{\delta t} = 0$ ,  $\frac{\delta R}{\delta t} = 0$ ,

Table 1: STABILITY CONDITIONS FOR GOVERNMENT MONOPOLY EQUILIBRIUM.

SELECTIVE VIOLENCE	EXTERNAL SUPPORT	
	$G$ advantage ( $\alpha_G > \alpha_R$ )	$R$ advantage ( $\alpha_G < \alpha_R$ )
$G$ advantage ( $\frac{\rho_G \theta_G}{\rho_R \theta_R} > 1$ )	Stable	Stable if $d < \bar{d}$
$R$ advantage ( $\frac{\rho_G \theta_G}{\rho_R \theta_R} < 1$ )	Stable if $d < \bar{d}$	Unstable

$C_{eq} \in [0, \infty)$ ,  $G_{eq} \in [0, \infty)$ ,  $R_{eq} \in [0, \infty)$  and  $\pi_G(\mathbf{s}) = 0$ ,  $\pi_R(\mathbf{s}) = 0$ . These conditions are satisfied at

$$C_{eq} = \frac{k}{\rho_G(1 - \theta_G) + \rho_R(1 - \theta_R) + u} \quad (25)$$

$$G_{eq} = 0 \quad (26)$$

$$R_{eq} = 0 \quad (27)$$

This equilibrium exists (i.e. yields non-negative equilibrium group sizes) for all  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $\alpha_G \in [0, \infty)$ ,  $\alpha_R \in [0, \infty)$ ,  $k \in (0, \infty)$ ,  $u \in (0, \infty)$ , with  $\mu_i = 1 - \frac{\rho_{-i}\theta_{-i}}{\rho_{-i} + \rho_i}$ .  $\square$

We can now proceed to prove Proposition 3.

*Proof.* Assume  $\rho_G \in (0, \infty)$ ,  $\rho_R \in (0, \infty)$ ,  $\theta_G \in [0, 1]$ ,  $\theta_R \in [0, 1]$ ,  $\alpha_G \in [0, \infty)$ ,  $\alpha_R \in [0, \infty)$ ,  $d \in [0, 1]$ . To ensure nonnegative population values in equilibrium, we impose a lower bound on the immigration parameter  $k > \frac{(\rho_R \theta_R + u - (1-d)\alpha_G)(\rho_G(1-\theta_G) + \rho_R(1-\theta_R) + u)}{\mu_G}$ , with  $\mu_G = 1 - \frac{\rho_R \theta_R}{\rho_R + \rho_G}$ . By linearization, the government victory equilibrium is stable if all the eigenvalues of the Jacobian matrix of the system in (16-18), evaluated at fixed point (19-21), have negative real parts, or  $\det(\mathbf{J}) > 0$ ,  $\text{tr}(\mathbf{J}) < 0$ . These conditions hold if either (a)  $\frac{\rho_G \theta_G}{\rho_R \theta_R} > 1$  and  $\alpha_R < \bar{\alpha}_R$ , where  $\bar{\alpha}_R = \frac{(1-d)\alpha_G(\rho_R + \rho_G(1-\theta_G)) + (\rho_G + \rho_R + u)(\theta_G \rho_G - \theta_R \rho_R)}{\rho_G + \rho_R(1-\theta_R)}$ , or (b)  $\frac{\rho_G \theta_G}{\rho_R \theta_R} < 1$ ,  $\alpha_R < \bar{\alpha}_R$ , and  $\alpha_G > \underline{\alpha}_G$ , where  $\underline{\alpha}_G = \frac{(\rho_G + \rho_R + u)(\theta_R \rho_R - \theta_G \rho_G)}{(1-d)(\rho_R + (1-\theta_G)\rho_G)}$ . The critical values  $\bar{\alpha}_R$  and  $\underline{\alpha}_G$  can be simplified to a single upper bound for  $d$ :  $\bar{d} = 1 - \frac{(\rho_G + \rho_R + u)(\theta_R \rho_R - \theta_G \rho_G) + \alpha_R(\rho_G + (1-\theta_R)\rho_R)}{\alpha_G(\rho_R + (1-\theta_G)\rho_G)}$ .  $\square$

As Proposition 3 states, external support creates new conditions for government victory. Crucially, a selective violence advantage ( $\frac{\rho_G \theta_G}{\rho_R \theta_R} > 1$ ) is no longer necessary for a government monopoly, as long as rebel interdiction falls below a critical value,

$$\bar{d} = 1 - \frac{(\rho_G + \rho_R + u)(\theta_R \rho_R - \theta_G \rho_G) + \alpha_R(\rho_G + (1-\theta_R)\rho_R)}{\alpha_G(\rho_R + (1-\theta_G)\rho_G)} \quad (28)$$

To evaluate the role of external support more intuitively, consider four scenarios, summarized in Table 1. In the government's 'best-case scenario', where it has an advantage in both selective violence and external support (upper left), a government monopoly is always stable. In the 'worst-case scenario' (lower right), where the government has advantages in neither selective violence nor external support, its monopoly is never stable.

The more intriguing scenarios appear in the off-diagonal elements of Table 1 – where the government has

an advantage in selective violence, but not external support (upper right), and where the government has a disadvantage in selective violence, but an advantage in external support (lower left). In each of these cases, the government can sustain victory as long as the rate of rebel interdiction falls below the critical value  $\bar{d}$ . If rebels interdict at a level above  $\bar{d}$ , the size of the government group – along with its capacity to produce violence – will diminish over time, and a rebel monopoly will emerge.

What determines the threshold level of interdiction needed to prevent a government victory? As the expression in (28) shows,  $\bar{d}$  is increasing in government external support ( $\frac{\partial \bar{d}}{\partial \alpha_G} > 0$ ). Where government access to such resources is abundant for logistical reasons, more interdiction is needed. Meanwhile,  $\bar{d}$  is decreasing in rebel external support ( $\frac{\partial \bar{d}}{\partial \alpha_R} < 0$ ). In the extreme case where rebels are completely isolated from external resources ( $\alpha_R = 0$ ),  $\bar{d}$  rises to  $1 - \frac{(\rho_G + \rho_R + u)(\theta_R \rho_R - \theta_G \rho_G)}{\alpha_G(\rho_R + (1 - \theta_G)\rho_G)}$ . Finally, this upper bound is decreasing in rebel selectivity and increasing in government selectivity ( $\frac{\partial \bar{d}}{\partial \theta_R} < 0, \frac{\partial \bar{d}}{\partial \theta_G} > 0$ ). Rebels do not need to interdict as many supplies where the government already lacks coercive leverage due to a poor informational endowment (e.g. in areas under rebel territorial control).

These results have two central implications. First, governments with access to sufficient external support are less reliant on the local population, and can achieve victory despite a disadvantage in selective violence. External resources can compensate for a lack of intelligence, a lack of coercive leverage, and can perpetuate a reliance on indiscriminate tactics. Second, and more optimistically, external resources create new liabilities for the government, and rebels are in a position to exploit them. If rebels interdict sufficient government resources ( $d > \bar{d}$ ), they can offset government advantages in both selective violence and external support, and prevent a government monopoly from taking hold. While rebel coercion provokes more government coercion, rebel interdiction suppresses government violence in the long run.