

There follows Supplementary Information in three appendices

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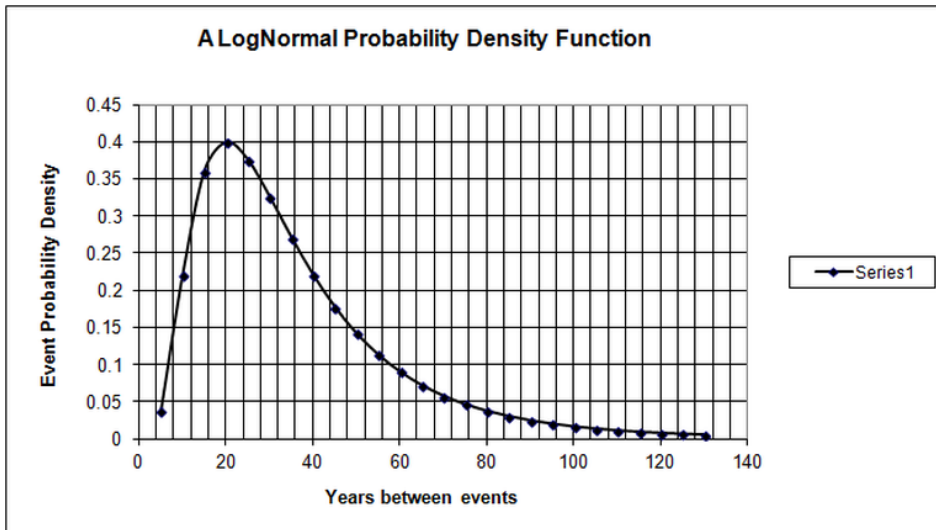
Appendix 1

The case for Log-Normal Likelihood Intervals. A non mathematical description.

Often where natural processes are concerned the Log-Normal curve is applicable. The uncertainty in the waiting time between death events can be shown to be adequately modeled by a Log-Normal distribution, particularly where the deaths are in a Select Group, for example within a mausoleum, or collections of family headstones and other such tomb scenarios. Suitable data are readily available for analysis, e.g. from lists of Monumental Inscriptions. Alternatively, multiple random simulations have shown that the result is quite general and may be applied without the need to test each case.

The Log-Normal distribution can be characterized by two parameters, the scale or location parameter and the shape parameter. Noting the number of times a gap of a particular length occurs (in years), between succeeding events, the resulting histogram of number of events for each interval may be tested for goodness of fit with a Log-Normal distribution. The distribution is skewed with a single mode and a long tail to the right with a lower probability for longer intervals between events. App Fig 1.

The location parameter moves the median of the curve left or right without changing its shape. Different select groups will change the location of the median. A Shape parameter of 'ln2' has been established empirically by simulation.



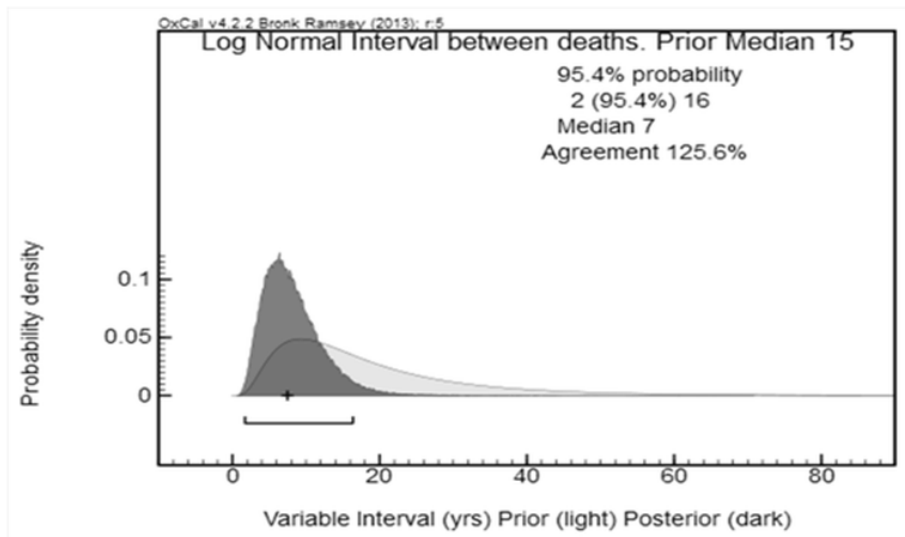
App1 Fig 1. An example of a Log-Normal PDF distribution

In the absence of stratification, this curve is valuable in defining a Likelihood distribution to describe the uncertainty in the *a priori* interval between deaths, for example in a Chambered Tomb. Bayesian analysis is predicated by the prior assumptions made within the model and the Likelihood data. The identification of a Log-Normal Likelihood Interval enables the Bayesian model to constrain the analysis to find a posterior interval between deaths that best fits the other model constraints and the radiocarbon data.

The function used to define the *Likelihood* uncertainty waiting time between events is: **Interval (LnN(ln(a),ln(b)))** where 'a' and 'b' are associated with the location and shape parameters of a Log-Normal distribution.

It is necessary to establish a rough estimate for 'a' to give a location parameter, as it will vary for different select groups. An appropriate value of 'b' for the shape parameter is empirically set at 2 (i.e. ln2) from consideration of existing Monumental Inscription data and averaging multiple runs of simulated data.

A default value of 10 is suggested for 'a'. Alternatively a useful approximation given by the difference in years between the earliest and latest C-14 years BP, divided by the number of dated events, gives the location parameter 'a'. A multiplication factor of (3) is introduced, found empirically from the simulation runs. In any event the Bayesian modeling modifies the distribution to provide a best fit as in App 1 Fig 2.



App1 Fig 2. One of the Intervals between dates from a tomb. From a Likelihood uncertainty Interval Median of 15 years, (light) the Bayesian Model Posterior uncertainty Interval Median is 7 years (dark).

The method is useful where stratified event information is sparse or unavailable and where the death population may be described as a Select Group. The model assumes that the deaths are sequential. This is a reasonable assumption in the absence of evidence to the contrary.

Although not required for the OxCal examples in Appendix 3, nor in general, a more precise method of deriving the Log-Normal mean and standard deviation parameters for the intervals employs the following R program, where the 'sg.csv' is the list of the ordered dates from appendix 2.

```

setwd("") ## change directory as necessary
require(MASS)
distributions = c("log-normal","Poisson")
params = c(2,1)
fit = list()
for (filename in c("sg")) {
  cat(paste0("\n",filename,"\n=====\n"))
  yeardat = read.csv(paste0("sg",".csv"))
  yeardata = yeardat[,1] # change the digit for each group column if multiple Select Groups
  print("yeardata")
  print(yeardata) # prints the yeardata
  GapYears=yeardata[1:21]-yeardata[2:22]
  print("GapYears")
  print(GapYears) # prints the GapYears
  h = hist(GapYears,breaks=-0.25:81.25,plot=F)
  plot(h,freq=F,main=paste0("Fits to gaps in ",filename," (by one group)"),xlim=c(-1,82),ylim=range(pretty(h$density)))
  logliks = c()
  for (dist in distributions){
    if (dist=="Poisson") fityears = floor(GapYears) else fityears = GapYears
    fit[[dist]] = fitdistr(fityears,dist)
    logliks=c(logliks,fit[[dist]]$loglik)
    cat(paste0(dist,"\n"))
    print(fit[[dist]]$estimate)
  }
  lines(dlnorm(0.1:max(GapYears),fit[["log-normal"]]$estimate[1],fit[["log-normal"]]$estimate[2]),lwd=3)
}

```

```

lines(dpois(0:floor(max(GapYears)),fit[["Poisson"]])$estimate[1]),lwd=3,lty=2)
legend("topright",legend=distributions,lty=1:2,col=1,lwd=3)
names(logliks) = distributions
AIC = 2*params - 2*logliks
cat("Akaike information criterion\n")
print(AIC)
}

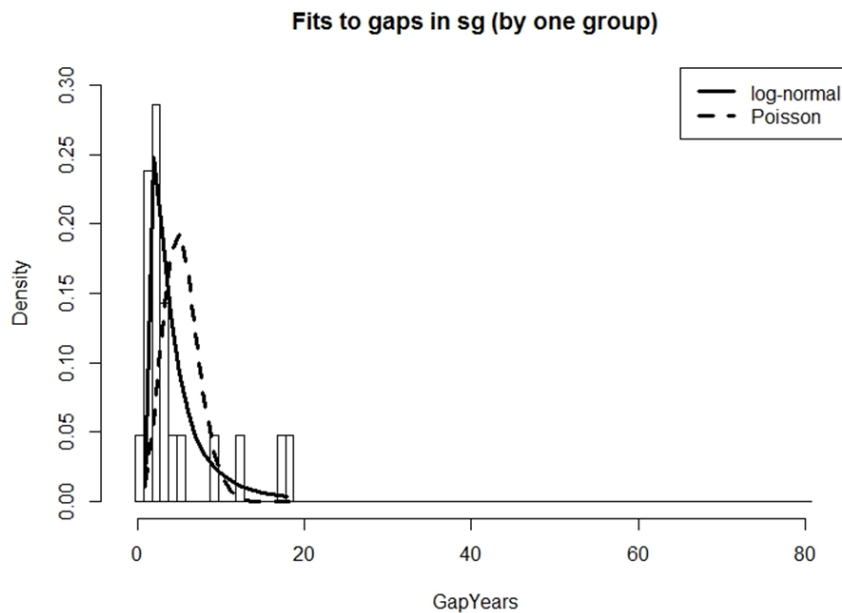
```

This R-code provides the desired parameters and a histogram of the Gap Years with a Log-Normal and Poisson plot overlaid. The Akaike Information Criterion (AIC) values show Log-Normal as lower than Poisson, hence is the better fit. The log-normal mean log and sdlog values could be used directly as the Interval parameters although that detracts from the simplicity of the modeling method suggested in Appendix 3.

```

sg
=====
[1] "yeardata"
[1] 1891.0 1889.0 1887.0 1884.0 1876.0 1863.0 1862.0 1858.0 1853.0 1846.0 1842.0 1836.0 1832.
0 1826.5
[15] 1826.0 1824.0 1821.0 1820.0 1819.0 1815.0 1813.0 1808.0
[1] "GapYears"
[1] 2.0 2.0 3.0 8.0 13.0 1.0 4.0 5.0 7.0 4.0 6.0 4.0 5.5 0.5 2.0 3.0 1.0 1.0
4.0
[20] 2.0 5.0
log-normal
  meanlog      sdlog
1.1013109 0.7873878
Poisson
  lambda
3.904762
Akaike information criterion
log-normal      Poisson
 99.81103    105.98368

```



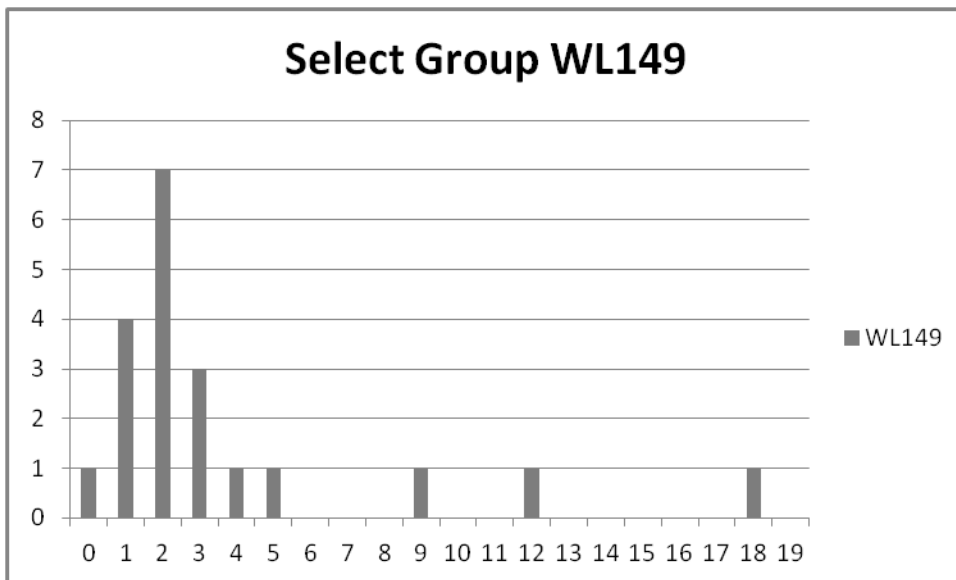
App1 Fig 3. Histogram of the gap year frequencies of SG WL149 with a Log-Normal and Poisson density plot overlaid.

Appendix 2.

The calendar year AD data from Monumental Inscription Lists

App2 Fig1. Select Group data with histogram chart of WL149 Gaps as an example. All are Log-Normal distributions.

SWM25,	SWM7,	WL31,	WL14,	WL113,	WL120,	WL149,	WL156,	WL174
1891,	1969,	1927,	1901.5,	1896,	1891,	1900,	1917,	1880.5
1889,	1941,	1917,	1901,	1891,	1884,	1888,	1894,	1880
1887,	1934,	1900,	1899.5,	1888,	1881.5,	1885,	1890,	1871
1884,	1901,	1895,	1899,	1885,	1881,	1884,	1886,	1862
1876,	1897,	1893.5,	1897,	1882,	1879,	1881,	1880,	1860
1863,	1888.5,	1893,	1896,	1880,	1876.5,	1879,	1866,	1853
1862,	1888,	1886,	1895,	1878,	1876,	1877.5,	1863,	1848
1858,	1886,	1880,	1893,	1871,	1870,	1877,	1862,	1845
1853,	1885,	1878,	1892,	1869,	1869,	1876,	1852,	1841
1846,	1883,	1875,	1888.5,	1856,	1855,	1875,	1849,	1840
1842,	1882.5,	1861,	1888,	1855,	1853,	1873,	1848.5,	1837
1836,	1882,	1857,	1884,	1848,	1850,	1871,	1848,	1834
1832,	1881,	1851,	1880,	1847,	1844,	1869,	1847.5,	1830
1826.5,	1866,	1850,	1878,	1843,	1842,	1865,	1847,	1789
1826,	1861,	1849,	1876,	1841,	1841,	1862,	1842,	1788
1824,	1858,	1848,	1869,	1840,	1837,	1853,	1835,	1770
1821,	1854,	1835,	1862.5,	1838,	1831,	1851,	1833,	1765
1820,	1850,	1830,	1862,	1837.5,	1814,	1849,	1831,	1762
1819,	1849,	1822,	1861,	1837,	1808,	1844,	1830,	1748
1815,	1842,	1819,	1857,	1826,	1807,	1843,	1814,	1730
1813,	1835,	1805,	1825,	1823,	1795,	1825,	1806,	1728
1808,	1815,	1725,	1818,	1820,	1769,	1808,	1782,	1724



App2 Fig1. Select Group data with histogram chart of WL149 Gaps as an example. All fit Log-Normal distributions.

Appendix 3.

Example OxCal program of Sorted Sequence (SORT) model SGWL174 Simulated to a Plateau

```
//A reasonable Interval uncertainty default is 10 and 2 for a and b
//or calculate a and b for the LnN(ln(a),ln(b)) term.
//Location parameter a=3*(max-min R_Date)/(No of dates)
//and Shape parameter b= 2 as default unless there is sufficient
//reason to change it from LnN(ln(a),ln(b))
//Set the variable values b, n, d, a, for each set of burials
Plot()
{
  var(a);
  // calculated from d and n below
  var(b);
  // shape nominally '2' but may be changed
  var(d);
  // difference between Max and Min R_Date in the Sequence
  var(n);
  // n is the number of R_Dates
  Sequence("SGWL174_PGapSort")
  {
    Boundary("StartPGapSort");
    b=2;
    n=22;
    d=4540-4464;
    a=3*d/n;
    Sequence()
    {
      KDE_Plot("KDEA_PGapSort", )
      {
        };
        R_Date("A14", BP(calBP(5161))+26, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A17", BP(calBP(5185))+34, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A10", BP(calBP(5110))+25, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A1", BP(calBP(5070))+20, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A4", BP(calBP(5088))+3, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A2", BP(calBP(5070))+13, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A9", BP(calBP(5109))+13, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A6", BP(calBP(5097))+18, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A13", BP(calBP(5120))+13, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A5", BP(calBP(5090))+20, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A12", BP(calBP(5116))+9, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A8", BP(calBP(5105))+15, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A21", BP(calBP(5222))+4, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A7", BP(calBP(5102))+35, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A18", BP(calBP(5188))+4, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A16", BP(calBP(5180))+18, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A22", BP(calBP(5226))+4, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A19", BP(calBP(5202))+13, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A15", BP(calBP(5162))+37, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A11", BP(calBP(5113))+17, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A20", BP(calBP(5220))+21, 25);
        Interval (LnN(ln(a),ln(b)));
        R_Date("A3", BP(calBP(5079))+46, 25);
```

```

};
Span("WL174PGapSort");
Boundary("EndWL174PGapSort");
};
};

```

Example OxCal program of Sequence (SIM) model SGWL174 Simulated to a Plateau

```

//A reasonable Interval uncertainty default is 10 and 2 for a and b
//or calculate a and b for the LnN(ln(a),ln(b)) term.
//Location parameter a=3*(max-min R_Date)/(No of dates)
//and Shape parameter b= 2 as default unless there is sufficient
//reason to change it from LnN(ln(a),ln(b))
//Set the variable values b, n, d, a, for each set of burials
Plot()
{
var(a);
// calculated from d and n below
var(b);
// shape nominally '2' but may be changed
var(d);
// difference between Max and Min R_Date in the Sequence
var(n);
// n is the number of R_Dates
Sequence("SGWL174_PGapSim")
{
Boundary("StartPGapsSim");
b=2;
n=22;
d=4540-4464;
a=3*d/n;
Sequence()
{
KDE_Plot("KDEA_PGapSim", )
{
};
R_Date("A22", BP(calBP(5226))+4, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A21", BP(calBP(5222))+4, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A20", BP(calBP(5220))+21, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A19", BP(calBP(5202))+13, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A18", BP(calBP(5188))+4, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A17", BP(calBP(5185))+34, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A16", BP(calBP(5180))+18, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A15", BP(calBP(5162))+37, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A14", BP(calBP(5161))+26, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A13", BP(calBP(5120))+13, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A12", BP(calBP(5116))+9, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A11", BP(calBP(5113))+17, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A10", BP(calBP(5110))+25, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A9", BP(calBP(5109))+13, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A8", BP(calBP(5105))+15, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A7", BP(calBP(5102))+35, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A6", BP(calBP(5097))+18, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A5", BP(calBP(5090))+20, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A4", BP(calBP(5088))+3, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A3", BP(calBP(5079))+46, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A2", BP(calBP(5070))+13, 25);
Interval (LnN(ln(a),ln(b)));
R_Date("A1", BP(calBP(5070))+20, 25);
};
};

```

```

Span("WL174PGapSim");
Boundary("EndWL174PGapSim");
};
};

```

Example OxCal program of Phase model SGWL174 Simulated to a Plateau

```

//A reasonable Interval uncertainty default is 10 and 2 for a and b
//or calculate a and b for the LnN(ln(a),ln(b)) term.
//Location parameter a=3*(max-min R_Date)/(No of dates)
//and Shape parameter b= 2 as default unless there is sufficient
//reason to change it from LnN(ln(a),ln(b))
//Set the variable values b, n, d, a, for each set of burials
Plot()
{
var(a);
// calculated from d and n below
var(b);
// shape nominally '2' but may be changed
var(d);
// difference between max and Min date in the Sequence
var(n);
// n is the number if R_Dates
Sequence("SGWL174_PPhaseNoGapSim")
{
Boundary("StartPPhaseNoGapsSim");
b=2;
n=22;
d=4540-4464;
a=3*d/n;
Phase()
{
KDE_Plot("KDEA_PPhaseNoGapSim", )
{
};
R_Date("A22", BP(calBP(5226))+4, 25);
R_Date("A21", BP(calBP(5222))+4, 25);
R_Date("A20", BP(calBP(5220))+21, 25);
R_Date("A19", BP(calBP(5202))+13, 25);
R_Date("A18", BP(calBP(5188))+4, 25);
R_Date("A17", BP(calBP(5185))+34, 25);
R_Date("A16", BP(calBP(5180))+18, 25);
R_Date("A15", BP(calBP(5162))+37, 25);
R_Date("A14", BP(calBP(5161))+26, 25);
R_Date("A13", BP(calBP(5120))+13, 25);
R_Date("A12", BP(calBP(5116))+9, 25);
R_Date("A11", BP(calBP(5113))+17, 25);
R_Date("A10", BP(calBP(5110))+25, 25);
R_Date("A9", BP(calBP(5109))+13, 25);
R_Date("A8", BP(calBP(5105))+15, 25);
R_Date("A7", BP(calBP(5102))+35, 25);
R_Date("A6", BP(calBP(5097))+18, 25);
R_Date("A5", BP(calBP(5090))+20, 25);
R_Date("A4", BP(calBP(5088))+3, 25);
R_Date("A3", BP(calBP(5079))+46, 25);
R_Date("A2", BP(calBP(5070))+13, 25);
R_Date("A1", BP(calBP(5070))+20, 25);
};
Span("WL174PPhaseNoGapSim");
Boundary("EndWL174PPhaseNoGapSim");
};
};

```