Simulation of Swept–Wing Vortices Using Nonlinear Parabolized Stability Equations : Governing Equations

Tim S. Haynes

~~Member Technical Staff~~ SENIOR STAFF ENGINEER

~~Dynaes Engineering Co., Inc., Renton, Washington~~ Hughes Space and Communications, El Segundo, California

Helen L. Reed

Professor, Mechanical and Aerospace Engineering

Arizona State University, Tempe, Arizona

Nomenclature

c_p basic–state specific heat at constant pressure, dimensionless

C_p pressure coefficient

k spanwise Fourier index

k_1, k_2, k_3, k_4 curvature terms

L streamwise length scale

$\mathcal{L}_o$ Orr–Sommerfeld–type operator

$\mathcal{L}_1$ a PSE operator

$\mathcal{L}_2$ a PSE operator

$\mathcal{L}_3$ a PSE operator

M reference Mach number

n temporal Fourier index

N maximum number of temporal modes

$\mathcal{N}$ Nonlinear forcing term for PSE

P basic–state thermodynamic pressure, dimensionless

Pr reference Prandtl number

R_o Reynolds number based on δ_o and reference quantities

t time, dimensionless

T basic–state thermodynamic temperature, dimensionless

$\tilde{T}$ disturbance temperature shape function

T' disturbance thermodynamic temperature, dimensionless

$\tilde{u}, \tilde{v}, \tilde{w}$ disturbance velocity shape functions

u', v', w' disturbance velocity components in x, y, z directions, dimensionless

U, V, W basic–state velocity components in x, y, z directions, dimensionless

$\bar{\bar{V}}$ rescaled basic–state normal velocity component, dimensionless

$\bar{V}$ transformed basic–state normal velocity component, dimensionless

x, y, z orthogonal curvilinear coordinates, dimensionless

$\bar{x}$ rescaled streamwise coordinate, dimensionless

α	disturbance streamwise complex wavenumber
β	disturbance spanwise wavenumber
γ_r	reference specific heat ratio
δ_o	length scale
κ	basic–state coefficient of thermal conductivity, dimensionless
μ	basic–state coefficient of dynamic viscosity, dimensionless
ρ	basic–state mass density, dimensionless
$\tilde{\rho}$	disturbance density shape function
ρ'	disturbance mass density, dimensionless
ϕ'	disturbance vector (ρ', u', v', w', T')
ω	disturbance frequency

Disturbance Equations

The dimensionless compressible disturbance equations are presented below in body–intrinsic orthogonal curvilinear coordinates.

$$\mathcal{B}_o\frac{\partial \phi'}{\partial t} + \mathcal{B}_1\frac{\partial \phi'}{\partial x} + \mathcal{B}_2\frac{\partial \phi'}{\partial y} + \mathcal{B}_3\frac{\partial \phi'}{\partial z} + \mathcal{C}_1\frac{\partial^2 \phi'}{\partial x^2} + \mathcal{C}_2\frac{\partial^2 \phi'}{\partial y^2} + \mathcal{C}_3\frac{\partial^2 \phi'}{\partial z^2}$$

$$+\mathcal{D}_1\frac{\partial^2 \phi'}{\partial x \partial y} + \mathcal{D}_2\frac{\partial^2 \phi'}{\partial x \partial z} + \mathcal{D}_3\frac{\partial^2 \phi'}{\partial y \partial z} + \mathcal{D}_4\phi' = \mathcal{N} \tag{1}$$

The non–zero terms are:

$$\mathcal{B}_o(1,1) = R_o$$

$$\mathcal{B}_o(2,2) = R_o\rho$$

$$\mathcal{B}_o(3,3) = R_o\rho$$

$$\mathcal{B}_o(4,4) = R_o\rho$$

$$\mathcal{B}_o(5,1) = -(R_oT) + \frac{1}{\gamma_r}R_oT$$

$$\mathcal{B}_o(5,5) = -(R_o\rho) + \frac{1}{\gamma_r}R_o\rho + R_o c_p\rho$$

$$\mathcal{B}_1(1,1) = \frac{1}{k_1}R_oU$$

$$\mathcal{B}_1(1,2) = \frac{1}{k_1}R_o\rho$$

$$\mathcal{B}_1(2,1) = \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}R_oT$$

$$\mathcal{B}_1(2,2) = 2\frac{1}{k_1^3}\frac{1}{R_o}k_4\mu - \frac{2}{3}\frac{1}{k_1^3}\frac{1}{R_o}k_4\mu + \frac{1}{k_1}R_o\rho U - 2\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T}{\partial x} + \frac{2}{3}\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T}{\partial x}$$

$$\mathcal{B}_1(2,3) = -3\frac{1}{k_1}k_2\mu + \frac{2}{3}\frac{1}{k_1}k_2\mu - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_1(2,5) = \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}R_o\rho - 2\frac{1}{k_1}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{d\mu}{dT} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{d\mu}{dT}$$

$$- 2\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial U}{\partial x} + \frac{2}{3}\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial U}{\partial x} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{B}_1(3,2) = 3\frac{1}{k_1}k_2\mu - \frac{2}{3}\frac{1}{k_1}k_2\mu + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_1(3,3) = \frac{1}{k_1^3}\frac{1}{R_o}k_4\mu + \frac{1}{k_1}R_o\rho U - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T}{\partial x}$$

$$\mathcal{B}_1(3,5) = \frac{1}{k_1}k_2 U\frac{d\mu}{dT} - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial U}{\partial y} - \frac{1}{k_1^2}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial x}$$

$$\mathcal{B}_1(4,4) = \frac{1}{k_1^3}\frac{1}{R_o}k_4\mu + \frac{1}{k_1}R_o\rho U - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T}{\partial x}$$

$$\mathcal{B}_1(4,5) = -(\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial W}{\partial x})$$

$$\mathcal{B}_1(5,1) = -(\frac{1}{k_1}R_o TU) + \frac{1}{\gamma_r}\frac{1}{k_1}R_o TU$$

$$\mathcal{B}_1(5,2) = \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}k_2 M^2\mu\bar{\bar{V}} - \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2 M^2\mu\bar{\bar{V}} + \frac{8}{3}\frac{1}{k_1^2}M^2\mu\frac{\partial U}{\partial x}$$
$$\qquad - \frac{8}{3}\frac{1}{k_1^2}\gamma_r M^2\mu\frac{\partial U}{\partial x} - \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{B}_1(5,3) = -2\frac{1}{k_1}k_2 M^2\mu U + 2\frac{1}{k_1}\gamma_r k_2 M^2\mu U + 2\frac{1}{k_1}M^2\mu\frac{\partial U}{\partial y} - 2\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial U}{\partial y}$$
$$\qquad + 2\frac{1}{k_1^2}\frac{1}{R_o}M^2\mu\frac{\partial \bar{\bar{V}}}{\partial x} - 2\frac{1}{k_1^2}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial x}$$

$$\mathcal{B}_1(5,4) = 2\frac{1}{k_1^2}M^2\mu\frac{\partial W}{\partial x} - 2\frac{1}{k_1^2}\gamma_r M^2\mu\frac{\partial W}{\partial x}$$

$$\mathcal{B}_1(5,5) = \frac{1}{k_1^3}\frac{1}{Pr}\frac{1}{R_o}k_4\kappa - \frac{1}{k_1}R_o\rho U + \frac{1}{\gamma_r}\frac{1}{k_1}R_o\rho U + \frac{1}{k_1}R_o c_p\rho U - 2\frac{1}{k_1^2}\frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial T}{\partial x}$$

$$\mathcal{B}_2(1,1) = \bar{\bar{V}}$$

$$\mathcal{B}_2(1,3) = R_o\rho$$

$$\mathcal{B}_2(2,2) = -(k_2\mu) + \rho\bar{\bar{V}} - \frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_2(2,3) = \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T}{\partial x}$$

$$\mathcal{B}_2(2,5) = k_2 U\frac{d\mu}{dT} - \frac{d\mu}{dT}\frac{\partial U}{\partial y} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial x}$$

$$\mathcal{B}_2(3,1) = \frac{1}{\gamma_r}\frac{1}{M^2}R_o T$$

$$\mathcal{B}_2(3,2) = -\left(\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T}{\partial x}\right)$$

$$\mathcal{B}_2(3,3) = -2k_2\mu + \frac{2}{3}k_2\mu + \rho\bar{V} - 2\frac{d\mu}{dT}\frac{\partial T}{\partial y} + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_2(3,5) = \frac{1}{\gamma_r}\frac{1}{M^2}R_o\rho + \frac{2}{3}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{d\mu}{dT} + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial U}{\partial x} - 2\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y} + \frac{2}{3}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{B}_2(4,4) = -(k_2\mu) + \rho\bar{V} - \frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_2(4,5) = -\left(\frac{d\mu}{dT}\frac{\partial W}{\partial y}\right)$$

$$\mathcal{B}_2(5,1) = -(T\bar{\bar{V}}) + \frac{1}{\gamma_r}T\bar{\bar{V}}$$

$$\mathcal{B}_2(5,2) = -2k_2M^2\mu U + 2\gamma_r k_2 M^2\mu U + 2M^2\mu\frac{\partial U}{\partial y} - 2\gamma_r M^2\mu\frac{\partial U}{\partial y}$$
$$+ 2\frac{1}{k_1}\frac{1}{R_o}M^2\mu\frac{\partial \bar{\bar{V}}}{\partial x} - 2\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial x}$$

$$\mathcal{B}_2(5,3) = -\left(\frac{4}{3}\frac{1}{R_o}k_2 M^2\mu\bar{\bar{V}}\right) + \frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2\mu\bar{\bar{V}} - \frac{4}{3}\frac{1}{k_1}M^2\mu\frac{\partial U}{\partial x} + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial U}{\partial x}$$
$$+ \frac{8}{3}\frac{1}{R_o}M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y} - \frac{8}{3}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{B}_2(5,4) = 2M^2\mu\frac{\partial W}{\partial y} - 2\gamma_r M^2\mu\frac{\partial W}{\partial y}$$

$$\mathcal{B}_2(5,5) = -\left(\frac{1}{Pr}k_2\kappa\right) - \rho\bar{V} + \frac{1}{\gamma_r}\rho\bar{V} + c_p\rho\bar{V} - 2\frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_3(1,1) = R_o W$$

$$\mathcal{B}_3(1,4) = R_o\rho$$

$$\mathcal{B}_3(2,2) = R_o\rho W$$

$$\mathcal{B}_3(2,4) = \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T}{\partial x}$$

$$\mathcal{B}_3(2,5) = -\left(\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial W}{\partial x}\right)$$

$$\mathcal{B}_3(3,3) = R_o\rho W$$

$$\mathcal{B}_3(3,4) = \frac{2}{3}\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_3(3,5) = -\left(\frac{d\mu}{dT}\frac{\partial W}{\partial y}\right)$$

6

$$\mathcal{B}_3(4,1) = \frac{1}{\gamma_r}\frac{1}{M^2}R_oT$$

$$\mathcal{B}_3(4,2) = -\left(\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T}{\partial x}\right)$$

$$\mathcal{B}_3(4,3) = -(k_2\mu) + \frac{2}{3}k_2\mu - \frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{B}_3(4,4) = R_o\rho W$$

$$\mathcal{B}_3(4,5) = \frac{1}{\gamma_r}\frac{1}{M^2}R_o\rho + \frac{2}{3}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{d\mu}{dT} + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial U}{\partial x} + \frac{2}{3}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{B}_3(5,1) = -(R_oTW) + \frac{1}{\gamma_r}R_oTW$$

$$\mathcal{B}_3(5,2) = 2\frac{1}{k_1}M^2\mu\frac{\partial W}{\partial x} - 2\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial W}{\partial x}$$

$$\mathcal{B}_3(5,3) = 2M^2\mu\frac{\partial W}{\partial y} - 2\gamma_r M^2\mu\frac{\partial W}{\partial y}$$

$$\mathcal{B}_3(5,4) = -\left(\frac{4}{3}\frac{1}{R_o}k_2M^2\mu\bar{\bar{V}}\right) + \frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2\mu\bar{\bar{V}} - \frac{4}{3}\frac{1}{k_1}M^2\mu\frac{\partial U}{\partial x} + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial U}{\partial x}$$
$$- \frac{4}{3}\frac{1}{R_o}M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{B}_3(5,5) = -(R_o\rho W) + \frac{1}{\gamma_r}R_o\rho W + R_o c_p\rho W$$

$$\mathcal{C}_1(2,2) = -2\frac{1}{k_1^2}\mu + \frac{2}{3}\frac{1}{k_1^2}\mu$$

$$\mathcal{C}_1(3,3) = -\left(\frac{1}{k_1^2}\mu\right)$$

$$\mathcal{C}_1(4,4) = -\left(\frac{1}{k_1^2}\mu\right)$$

$$\mathcal{C}_1(5,5) = -\left(\frac{1}{k_1^2}\frac{1}{Pr}\kappa\right)$$

$$\mathcal{C}_2(2,2) = -\mu$$

$$\mathcal{C}_2(3,3) = -2\mu + \frac{2}{3}\mu$$

$$\mathcal{C}_2(4,4) = -\mu$$

$$\mathcal{C}_2(5,5) = -\left(\frac{1}{Pr}\kappa\right)$$

$$\mathcal{C}_3(2,2) = -\mu$$

$$\mathcal{C}_3(3,3) = -\mu$$

$$\mathcal{C}_3(4,4) = -2\mu + \frac{2}{3}\mu$$

$$\mathcal{C}_3(5,5) = -(\frac{1}{Pr}\kappa)$$

$$\mathcal{D}_1(2,3) = -(\frac{1}{k_1}\mu) + \frac{2}{3}\frac{1}{k_1}\mu$$

$$\mathcal{D}_1(3,2) = -(\frac{1}{k_1}\mu) + \frac{2}{3}\frac{1}{k_1}\mu$$

$$\mathcal{D}_2(2,4) = -(\frac{1}{k_1}\mu) + \frac{2}{3}\frac{1}{k_1}\mu$$

$$\mathcal{D}_2(4,2) = -(\frac{1}{k_1}\mu) + \frac{2}{3}\frac{1}{k_1}\mu$$

$$\mathcal{D}_3(3,4) = -\mu + \frac{2}{3}\mu$$

$$\mathcal{D}_3(4,3) = -\mu + \frac{2}{3}\mu$$

$$\mathcal{D}_4(1,1) = k_2\bar{\bar{V}} + \frac{1}{k_1}R_o\frac{\partial U}{\partial x} + \frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{D}_4(1,2) = \frac{1}{k_1}R_o\frac{\partial \rho}{\partial x}$$

$$\mathcal{D}_4(1,3) = k_2 R_o\rho + R_o\frac{\partial \rho}{\partial y}$$

$$\mathcal{D}_4(2,1) = k_2 U\bar{\bar{V}} + \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}R_o\frac{\partial T}{\partial x} + \frac{1}{k_1}R_o U\frac{\partial U}{\partial x} + \bar{V}\frac{\partial U}{\partial y}$$

$$\mathcal{D}_4(2,2) = k_2\rho\bar{\bar{V}} + k_2\frac{d\mu}{dT}\frac{\partial T}{\partial y} + \frac{1}{k_1}R_o\rho\frac{\partial U}{\partial x} + \mu(k_2)^2$$

$$\begin{aligned}\mathcal{D}_4(2,3) = & -2\frac{1}{k_1^2}\frac{1}{R_o}k_3\mu + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_3\mu + 2\frac{1}{k_1^2}\frac{1}{R_o}k_2 k_4\mu - \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_2 k_4\mu + k_2 R_o\rho U\\ & -2\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial x} + \frac{2}{3}\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial x} + R_o\rho\frac{\partial U}{\partial y}\end{aligned}$$

$$\mathcal{D}_4(2,5) \;=\; -2\frac{1}{k_1^2}\frac{1}{R_o^2}k_3\bar{\bar{V}}\frac{d\mu}{dT} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}k_3\bar{\bar{V}}\frac{d\mu}{dT} + 2\frac{1}{k_1^2}\frac{1}{R_o^2}k_2k_4\bar{\bar{V}}\frac{d\mu}{dT} - \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}k_2k_4\bar{\bar{V}}\frac{d\mu}{dT}$$

$$+\; \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}R_o\frac{\partial\rho}{\partial x} + 2\frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial U}{\partial x} - \frac{2}{3}\frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial U}{\partial x} - k_2\frac{d\mu}{dT}\frac{\partial U}{\partial y}$$

$$-\; 3\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial x} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial\bar{V}}{\partial x} - 2\frac{1}{k_1}\frac{1}{R_o}k_2\bar{V}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}$$

$$+\; \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2} + k_2 U\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2} - 2\frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{\partial U}{\partial x}\frac{d^2\mu}{dT^2}$$

$$+\; \frac{2}{3}\frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{\partial U}{\partial x}\frac{d^2\mu}{dT^2} - \frac{\partial T}{\partial y}\frac{\partial U}{\partial y}\frac{d^2\mu}{dT^2} - \frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial x}\frac{d^2\mu}{dT^2}$$

$$+\; \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial x}\frac{\partial\bar{V}}{\partial y}\frac{d^2\mu}{dT^2} - 2\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial x^2} + \frac{2}{3}\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial x^2}$$

$$-\; \frac{d\mu}{dT}\frac{\partial^2 U}{\partial y^2} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{V}}{\partial x\partial y} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial x\partial y} + U\frac{d\mu}{dT}(k_2)^2$$

$$\mathcal{D}_4(3,1) \;=\; \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial T}{\partial y} + \frac{1}{k_1}U\frac{\partial\bar{\bar{V}}}{\partial x} + \frac{1}{R_o}\bar{\bar{V}}\frac{\partial\bar{\bar{V}}}{\partial y} - k_2 R_o(U)^2$$

$$\mathcal{D}_4(3,2) \;=\; \frac{1}{k_1^2}\frac{1}{R_o}k_3\mu - \frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\mu - 2k_2 R_o\rho U + \frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial x} + \frac{1}{k_1}\rho\frac{\partial\bar{\bar{V}}}{\partial x}$$

$$\mathcal{D}_4(3,3) \;=\; \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial y} + \rho\frac{\partial\bar{\bar{V}}}{\partial y} + 2\mu(k_2)^2 - \frac{2}{3}\mu(k_2)^2$$

$$\mathcal{D}_4(3,5) \;=\; \frac{1}{k_1^2}\frac{1}{R_o}k_3 U\frac{d\mu}{dT} - \frac{1}{k_1^2}\frac{1}{R_o}k_2k_4 U\frac{d\mu}{dT} + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial\rho}{\partial y} + 3\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial U}{\partial x}$$

$$-\; \frac{2}{3}\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial U}{\partial x} + \frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial x} - 2\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y} + \frac{2}{3}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$+\; \frac{1}{k_1}k_2 U\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2} + \frac{2}{3}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2} + \frac{2}{3}\frac{1}{k_1}\frac{\partial T}{\partial y}\frac{\partial U}{\partial x}\frac{d^2\mu}{dT^2}$$

$$-\; \frac{1}{k_1}\frac{\partial T}{\partial x}\frac{\partial U}{\partial y}\frac{d^2\mu}{dT^2} - \frac{1}{k_1^2}\frac{1}{R_o}\frac{\partial T}{\partial x}\frac{\partial\bar{\bar{V}}}{\partial x}\frac{d^2\mu}{dT^2} - 2\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial y}\frac{d^2\mu}{dT^2}$$

$$+\; \frac{2}{3}\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial y}\frac{d^2\mu}{dT^2} - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial x\partial y} + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial x\partial y}$$

$$-\; \frac{1}{k_1^2}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial x^2} - 2\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial y^2} + \frac{2}{3}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial y^2}$$

$$+\; 2\frac{1}{R_o}\bar{\bar{V}}\frac{d\mu}{dT}(k_2)^2 - \frac{2}{3}\frac{1}{R_o}\bar{\bar{V}}\frac{d\mu}{dT}(k_2)^2$$

$$\mathcal{D}_4(4,1) \;=\; \frac{1}{k_1}R_o U\frac{\partial W}{\partial x} + \bar{V}\frac{\partial W}{\partial y}$$

$$\mathcal{D}_4(4,2) \;=\; \frac{1}{k_1}R_o\rho\frac{\partial W}{\partial x}$$

$$\mathcal{D}_4(4,3) \;=\; R_o\rho\frac{\partial W}{\partial y}$$

$$\mathcal{D}_4(4,5) \;=\; \frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial W}{\partial x} - k_2\frac{d\mu}{dT}\frac{\partial W}{\partial y} - \frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{\partial W}{\partial x}\frac{d^2\mu}{dT^2} - \frac{\partial T}{\partial y}\frac{\partial W}{\partial y}\frac{d^2\mu}{dT^2}$$
$$\qquad - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 W}{\partial x^2} - \frac{d\mu}{dT}\frac{\partial^2 W}{\partial y^2}$$

$$\mathcal{D}_4(5,1) \;=\; -(\frac{1}{k_1}R_o U\frac{\partial T}{\partial x}) + \frac{1}{\gamma_r}\frac{1}{k_1}R_o U\frac{\partial T}{\partial x} + \frac{1}{k_1}R_o c_p U\frac{\partial T}{\partial x} - \bar{\bar{V}}\frac{\partial T}{\partial y}$$
$$\qquad + \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial T}{\partial y} + c_p\bar{\bar{V}}\frac{\partial T}{\partial y}$$

$$\mathcal{D}_4(5,2) \;=\; -(\frac{1}{k_1}R_o\frac{\partial P}{\partial x}) + \frac{1}{\gamma_r}\frac{1}{k_1}R_o\frac{\partial P}{\partial x} + \frac{1}{k_1}R_o c_p\rho\frac{\partial T}{\partial x} - 2k_2 M^2\mu\frac{\partial U}{\partial y}$$
$$\qquad + 2\gamma_r k_2 M^2\mu\frac{\partial U}{\partial y} - 2\frac{1}{k_1}\frac{1}{R_o}k_2 M^2\mu\frac{\partial\bar{\bar{V}}}{\partial x} + 2\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2 M^2\mu\frac{\partial\bar{\bar{V}}}{\partial x}$$
$$\qquad + 2M^2\mu U\left(k_2\right)^2 - 2\gamma_r M^2\mu U\left(k_2\right)^2$$

$$\mathcal{D}_4(5,3) \;=\; -(R_o\frac{\partial P}{\partial y}) + \frac{1}{\gamma_r}R_o\frac{\partial P}{\partial y} + R_o c_p\rho\frac{\partial T}{\partial y} + \frac{8}{3}\frac{1}{k_1}k_2 M^2\mu\frac{\partial U}{\partial x}$$
$$\qquad - \frac{8}{3}\frac{1}{k_1}\gamma_r k_2 M^2\mu\frac{\partial U}{\partial x} - \frac{4}{3}\frac{1}{R_o}k_2 M^2\mu\frac{\partial\bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2\mu\frac{\partial\bar{\bar{V}}}{\partial y}$$
$$\qquad + \frac{8}{3}\frac{1}{R_o}M^2\mu\bar{\bar{V}}\left(k_2\right)^2 - \frac{8}{3}\frac{1}{R_o}\gamma_r M^2\mu\bar{\bar{V}}\left(k_2\right)^2$$

10

$$
\begin{aligned}
\mathcal{D}_4(5,5) \;=\; & -\Big(\frac{1}{k_1}R_oU\frac{\partial\rho}{\partial x}\Big) + \frac{1}{\gamma_r}\frac{1}{k_1}R_oU\frac{\partial\rho}{\partial x} - \bar{\bar{V}}\frac{\partial\rho}{\partial y} + \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial\rho}{\partial y} + \frac{1}{k_1^3}\frac{1}{Pr}\frac{1}{R_o}k_4\frac{d\kappa}{dT}\frac{\partial T}{\partial x} \\
& + \frac{1}{k_1}R_o\rho U\frac{dc_p}{dT}\frac{\partial T}{\partial x} - \frac{1}{Pr}k_2\frac{d\kappa}{dT}\frac{\partial T}{\partial y} + \rho\bar{\bar{V}}\frac{dc_p}{dT}\frac{\partial T}{\partial y} + \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}k_2M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial U}{\partial x} \\
& - \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2M^2 U\frac{d\mu}{dT}\frac{\partial U}{\partial x} - 2k_2M^2U\frac{d\mu}{dT}\frac{\partial U}{\partial y} + 2\gamma_r k_2M^2U\frac{d\mu}{dT}\frac{\partial U}{\partial y} \\
& - 2\frac{1}{k_1}\frac{1}{R_o}k_2M^2U\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial x} + 2\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2M^2U\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial x} \\
& + 2\frac{1}{k_1}\frac{1}{R_o}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial x} - 2\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial x} \\
& - \frac{4}{3}\frac{1}{R_o^2}k_2M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{R_o^2}\gamma_r k_2M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y} \\
& - \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial x}\frac{\partial\bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial U}{\partial x}\frac{\partial\bar{\bar{V}}}{\partial y} \\
& - \frac{1}{k_1^2}\frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 T}{\partial x^2} - \frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 T}{\partial y^2} + M^2\frac{d\mu}{dT}(k_2)^2(U)^2 \\
& - \gamma_r M^2\frac{d\mu}{dT}(k_2)^2(U)^2 + \frac{4}{3}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}(k_2)^2(\bar{\bar{V}})^2 \\
& - \frac{4}{3}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}(k_2)^2(\bar{\bar{V}})^2 - \frac{1}{k_1^2}\frac{1}{Pr}\frac{d^2\kappa}{dT^2}\Big(\frac{\partial T}{\partial x}\Big)^2 - \frac{1}{Pr}\frac{d^2\kappa}{dT^2}\Big(\frac{\partial T}{\partial y}\Big)^2 \\
& + \frac{4}{3}\frac{1}{k_1^2}M^2\frac{d\mu}{dT}\Big(\frac{\partial U}{\partial x}\Big)^2 - \frac{4}{3}\frac{1}{k_1^2}\gamma_r M^2\frac{d\mu}{dT}\Big(\frac{\partial U}{\partial x}\Big)^2 + M^2\frac{d\mu}{dT}\Big(\frac{\partial U}{\partial y}\Big)^2 \\
& - \gamma_r M^2\frac{d\mu}{dT}\Big(\frac{\partial U}{\partial y}\Big)^2 + \frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\Big(\frac{\partial\bar{\bar{V}}}{\partial x}\Big)^2 - \frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\Big(\frac{\partial\bar{\bar{V}}}{\partial x}\Big)^2 \\
& + \frac{4}{3}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\Big(\frac{\partial\bar{\bar{V}}}{\partial y}\Big)^2 - \frac{4}{3}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\Big(\frac{\partial\bar{\bar{V}}}{\partial y}\Big)^2 + \frac{1}{k_1^2}M^2\frac{d\mu}{dT}\Big(\frac{\partial W}{\partial x}\Big)^2 \\
& - \frac{1}{k_1^2}\gamma_r M^2\frac{d\mu}{dT}\Big(\frac{\partial W}{\partial x}\Big)^2 + M^2\frac{d\mu}{dT}\Big(\frac{\partial W}{\partial y}\Big)^2 - \gamma_r M^2\frac{d\mu}{dT}\Big(\frac{\partial W}{\partial y}\Big)^2 \\[4pt]
\mathcal{N}(1) \;=\; & \frac{1}{k_1}R_o\frac{\partial u'}{\partial x}\rho' + R_o\frac{\partial v'}{\partial y}\rho' + R_o\frac{\partial w'}{\partial z}\rho' + \frac{1}{k_1}R_o\frac{\partial\rho'}{\partial x}u' \\
& + R_o\frac{\partial\rho'}{\partial y}v' + k_2R_o\rho'v' + R_o\frac{\partial\rho'}{\partial z}w'
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}(2) \;=\; & -2\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial u'}{\partial x} + \frac{2}{3}\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial u'}{\partial x} - \frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial u'}{\partial y} - \frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial u'}{\partial z} \\
& -\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial v'}{\partial x} + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial v'}{\partial y} - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial w'}{\partial x} + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial w'}{\partial z} \\
& +\frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}R_o\frac{\partial T'}{\partial x}\rho' + R_o\frac{\partial u'}{\partial t}\rho' + \frac{1}{k_1}R_oU\frac{\partial u'}{\partial x}\rho' + \bar{\bar{V}}\frac{\partial u'}{\partial y}\rho' + R_oW\frac{\partial u'}{\partial z}\rho' \\
& -2\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 u'}{\partial x^2}T' + \frac{2}{3}\frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 u'}{\partial x^2}T' - \frac{d\mu}{dT}\frac{\partial^2 u'}{\partial y^2}T' - \frac{d\mu}{dT}\frac{\partial^2 u'}{\partial z^2}T' \\
& +\frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 v'}{\partial x\partial y}T' - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 v'}{\partial x\partial y}T' + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 w'}{\partial x\partial z}T' \\
& -\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 w'}{\partial x\partial z}T' + \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}R_o\frac{\partial \rho'}{\partial x}T' + 2\frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial u'}{\partial x}T' \\
& -\frac{2}{3}\frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial u'}{\partial x}T' - 2\frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial u'}{\partial x}T' + \frac{2}{3}\frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial u'}{\partial x}T' \\
& -k_2\frac{d\mu}{dT}\frac{\partial u'}{\partial y}T' - \frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial u'}{\partial y}T' - 3\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial v'}{\partial x}T' \\
& +\frac{2}{3}\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial v'}{\partial x}T' - \frac{1}{k_1}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial v'}{\partial x}T' + \frac{2}{3}\frac{1}{k_1}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial v'}{\partial y}T' \\
& +\frac{2}{3}\frac{1}{k_1}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial w'}{\partial z}T' + k_2\frac{d\mu}{dT}\frac{\partial T'}{\partial y}u' + \frac{1}{k_1}R_o\rho\frac{\partial u'}{\partial x}u' \\
& +k_2\bar{\bar{V}}\rho'u' + \frac{1}{k_1}R_o\frac{\partial U}{\partial x}\rho'u' + \frac{1}{k_1}R_o\frac{\partial u'}{\partial x}\rho'u' \\
& +k_2\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}T'u' - 2\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial T'}{\partial x}v' + \frac{2}{3}\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial T'}{\partial x}v' \\
& +R_o\rho\frac{\partial u'}{\partial y}v' + k_2R_oU\rho'v' + R_o\frac{\partial U}{\partial y}\rho'v' + R_o\frac{\partial u'}{\partial y}\rho'v' \\
& -2\frac{1}{k_1^2}\frac{1}{R_o}k_3\frac{d\mu}{dT}T'v' + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_3\frac{d\mu}{dT}T'v' + 2\frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\frac{d\mu}{dT}T'v' \\
& -\frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\frac{d\mu}{dT}T'v' - 2\frac{1}{k_1}k_2\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}T'v' + \frac{2}{3}\frac{1}{k_1}k_2\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}T'v' \\
& +k_2R_o\rho u'v' + k_2R_o\rho'u'v' + R_o\rho\frac{\partial u'}{\partial z}w' + R_o\frac{\partial u'}{\partial z}\rho'w' + \frac{d\mu}{dT}T'u'\left(k_2\right)^2
\end{aligned}
$$

$$\mathcal{N}(3) = \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial u'}{\partial x} - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial u'}{\partial y} - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial v'}{\partial x}$$

$$- 2\frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial v'}{\partial y} + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial v'}{\partial y} - \frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial v'}{\partial z}$$

$$- \frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial w'}{\partial y} + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial w'}{\partial z} + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial T'}{\partial y}\rho'$$

$$+ R_o\frac{\partial v'}{\partial t}\rho' + \frac{1}{k_1}R_oU\frac{\partial v'}{\partial x}\rho' + \bar{\bar{V}}\frac{\partial v'}{\partial y}\rho' + R_oW\frac{\partial v'}{\partial z}\rho' - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 u'}{\partial x\partial y}T'$$

$$+ \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 u'}{\partial x\partial y}T' - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 v'}{\partial x^2}T' - 2\frac{d\mu}{dT}\frac{\partial^2 v'}{\partial y^2}T'$$

$$+ \frac{2}{3}\frac{d\mu}{dT}\frac{\partial^2 v'}{\partial y^2}T' - \frac{d\mu}{dT}\frac{\partial^2 v'}{\partial z^2}T' + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial^2 w'}{\partial y\partial z}T'$$

$$- \frac{d\mu}{dT}\frac{\partial^2 w'}{\partial y\partial z}T' + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial \rho'}{\partial y}T' + 3\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial u'}{\partial x}T'$$

$$- \frac{2}{3}\frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial u'}{\partial x}T' + \frac{2}{3}\frac{1}{k_1}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial u'}{\partial x}T' - \frac{1}{k_1}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial u'}{\partial y}T'$$

$$+ \frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial v'}{\partial x}T' - \frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial v'}{\partial x}T' - 2k_2\frac{d\mu}{dT}\frac{\partial v'}{\partial y}T'$$

$$+ \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial v'}{\partial y}T' - 2\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial v'}{\partial y}T' + \frac{2}{3}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial v'}{\partial y}T'$$

$$+ \frac{2}{3}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial w'}{\partial z}T' + \frac{1}{k_1}k_2\frac{d\mu}{dT}\frac{\partial T'}{\partial x}u'$$

$$+ \frac{1}{k_1}R_o\rho\frac{\partial v'}{\partial x}u' - 2k_2R_oU\rho'u' + \frac{1}{k_1}\frac{\partial \bar{\bar{V}}}{\partial x}\rho'u'$$

$$+ \frac{1}{k_1}R_o\frac{\partial v'}{\partial x}\rho'u' + \frac{1}{k_1^2}\frac{1}{R_o}k_3\frac{d\mu}{dT}T'u' - \frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\frac{d\mu}{dT}T'u'$$

$$+ \frac{1}{k_1}k_2\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}T'u' + \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial T'}{\partial y}v' + R_o\rho\frac{\partial v'}{\partial y}v'$$

$$+ \frac{\partial \bar{\bar{V}}}{\partial y}\rho'v' + R_o\frac{\partial v'}{\partial y}\rho'v' + \frac{2}{3}k_2\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}T'v'$$

$$+ R_o\rho\frac{\partial v'}{\partial z}w' + R_o\frac{\partial v'}{\partial z}\rho'w' + 2\frac{d\mu}{dT}T'v'\left(k_2\right)^2$$

$$- \frac{2}{3}\frac{d\mu}{dT}T'v'\left(k_2\right)^2 - k_2R_o\rho\left(u'\right)^2 - k_2R_o\rho'\left(u'\right)^2$$

13

$$
\begin{aligned}
\mathcal{N}(4) = \ & \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial u'}{\partial x} - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial u'}{\partial z} + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial v'}{\partial y} \\
& - \frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial v'}{\partial z} - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial T'}{\partial x}\frac{\partial w'}{\partial x} - \frac{d\mu}{dT}\frac{\partial T'}{\partial y}\frac{\partial w'}{\partial y} \\
& - 2\frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial w'}{\partial z} + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial T'}{\partial z}\frac{\partial w'}{\partial z} + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial T'}{\partial z}\rho' \\
& + R_o\frac{\partial w'}{\partial t}\rho' + \frac{1}{k_1}R_oU\frac{\partial w'}{\partial x}\rho' + \bar{\bar{V}}\frac{\partial w'}{\partial y}\rho' + R_oW\frac{\partial w'}{\partial z}\rho' - \frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 u'}{\partial x\partial z}T' \\
& + \frac{2}{3}\frac{1}{k_1}\frac{d\mu}{dT}\frac{\partial^2 u'}{\partial x\partial z}T' - \frac{d\mu}{dT}\frac{\partial^2 v'}{\partial y\partial z}T' + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial^2 v'}{\partial y\partial z}T' \\
& - \frac{1}{k_1^2}\frac{d\mu}{dT}\frac{\partial^2 w'}{\partial x^2}T' - \frac{d\mu}{dT}\frac{\partial^2 w'}{\partial y^2}T' - 2\frac{d\mu}{dT}\frac{\partial^2 w'}{\partial z^2}T' \\
& + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial^2 w'}{\partial z^2}T' + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial \rho'}{\partial z}T' - \frac{1}{k_1}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial u'}{\partial z}T' \\
& - k_2\frac{d\mu}{dT}\frac{\partial v'}{\partial z}T' + \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial v'}{\partial z}T' - \frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial v'}{\partial z}T' \\
& + \frac{1}{k_1^3}\frac{1}{R_o}k_4\frac{d\mu}{dT}\frac{\partial w'}{\partial x}T' - \frac{1}{k_1^2}\frac{\partial T}{\partial x}\frac{d^2\mu}{dT^2}\frac{\partial w'}{\partial x}T' - k_2\frac{d\mu}{dT}\frac{\partial w'}{\partial y}T' \\
& - \frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial w'}{\partial y}T' + \frac{1}{k_1}R_o\rho\frac{\partial w'}{\partial x}u' + \frac{1}{k_1}R_o\frac{\partial W}{\partial x}\rho'u' \\
& + \frac{1}{k_1}R_o\frac{\partial w'}{\partial x}\rho'u' + \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial T'}{\partial z}v' + R_o\rho\frac{\partial w'}{\partial y}v' \\
& + R_o\frac{\partial W}{\partial y}\rho'v' + R_o\frac{\partial w'}{\partial y}\rho'v' + R_o\rho\frac{\partial w'}{\partial z}w' + R_o\frac{\partial w'}{\partial z}\rho'w'
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}(5) =\ & 2\frac{1}{k_1}M^2\mu\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} - 2\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial u'}{\partial y}\frac{\partial v'}{\partial x} - \frac{4}{3}\frac{1}{k_1}M^2\mu\frac{\partial u'}{\partial x}\frac{\partial v'}{\partial y} \\
& + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial u'}{\partial x}\frac{\partial v'}{\partial y} + 2\frac{1}{k_1}M^2\mu\frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} - 2\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial u'}{\partial z}\frac{\partial w'}{\partial x} \\
& + 2M^2\mu\frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y} - 2\gamma_r M^2\mu\frac{\partial v'}{\partial z}\frac{\partial w'}{\partial y} - \frac{4}{3}\frac{1}{k_1}M^2\mu\frac{\partial u'}{\partial x}\frac{\partial w'}{\partial z} \\
& + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2\mu\frac{\partial u'}{\partial x}\frac{\partial w'}{\partial z} - \frac{4}{3}M^2\mu\frac{\partial v'}{\partial y}\frac{\partial w'}{\partial z} + \frac{4}{3}\gamma_r M^2\mu\frac{\partial v'}{\partial y}\frac{\partial w'}{\partial z} \\
& - R_o\frac{\partial T'}{\partial t}\rho' + \frac{1}{\gamma_r}R_o\frac{\partial T'}{\partial t}\rho' + R_o c_p\frac{\partial T'}{\partial t}\rho' - \frac{1}{k_1}R_o U\frac{\partial T'}{\partial x}\rho' \\
& + \frac{1}{\gamma_r}\frac{1}{k_1}R_o U\frac{\partial T'}{\partial x}\rho' + \frac{1}{k_1}R_o c_p U\frac{\partial T'}{\partial x}\rho' - \bar{\bar{V}}\frac{\partial T'}{\partial y}\rho' \\
& + \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial T'}{\partial y}\rho' + c_p\bar{\bar{V}}\frac{\partial T'}{\partial y}\rho' - R_o W\frac{\partial T'}{\partial z}\rho' \\
& + \frac{1}{\gamma_r}R_o W\frac{\partial T'}{\partial z}\rho' + R_o c_p W\frac{\partial T'}{\partial z}\rho' - \frac{1}{k_1^2}\frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 T'}{\partial x^2}T' \\
& - \frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 T'}{\partial y^2}T' - \frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 T'}{\partial z^2}T' - R_o\frac{\partial \rho'}{\partial t}T' \\
& + \frac{1}{\gamma_r}R_o\frac{\partial \rho'}{\partial t}T' - \frac{1}{k_1}R_o U\frac{\partial \rho'}{\partial x}T' + \frac{1}{\gamma_r}\frac{1}{k_1}R_o U\frac{\partial \rho'}{\partial x}T' \\
& - \bar{\bar{V}}\frac{\partial \rho'}{\partial y}T' + \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial \rho'}{\partial y}T' - R_o W\frac{\partial \rho'}{\partial z}T' \\
& + \frac{1}{\gamma_r}R_o W\frac{\partial \rho'}{\partial z}T' + R_o\rho\frac{dc_p}{dT}\frac{\partial T'}{\partial t}T' + \frac{1}{k_1^3}\frac{1}{Pr}\frac{1}{R_o}k_4\frac{d\kappa}{dT}\frac{\partial T'}{\partial x}T' \\
& + \frac{1}{k_1}R_o\rho U\frac{dc_p}{dT}\frac{\partial T'}{\partial x}T' - \frac{1}{k_1^2}\frac{1}{Pr}\frac{\partial T}{\partial x}\frac{d^2\kappa}{dT^2}\frac{\partial T'}{\partial x}T' - \frac{1}{Pr}k_2\frac{d\kappa}{dT}\frac{\partial T'}{\partial y}T' \\
& + \rho\bar{\bar{V}}\frac{dc_p}{dT}\frac{\partial T'}{\partial y}T' - \frac{1}{Pr}\frac{\partial T}{\partial y}\frac{d^2\kappa}{dT^2}\frac{\partial T'}{\partial y}T' + R_o\rho W\frac{dc_p}{dT}\frac{\partial T'}{\partial z}T' \\
& + \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}k_2 M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial u'}{\partial x}T' - \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2 M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial u'}{\partial x}T' + \frac{8}{3}\frac{1}{k_1^2}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial x}\frac{\partial u'}{\partial x}T' \\
& - \frac{8}{3}\frac{1}{k_1^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial U}{\partial x}\frac{\partial u'}{\partial x}T' - \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}\frac{\partial u'}{\partial x}T' + \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}\frac{\partial u'}{\partial x}T' \\
& - 2k_2 M^2 U\frac{d\mu}{dT}\frac{\partial u'}{\partial y}T' + 2\gamma_r k_2 M^2 U\frac{d\mu}{dT}\frac{\partial u'}{\partial y}T' + 2M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial u'}{\partial y}T'
\end{aligned}
$$

15

$$- 2\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial y} \frac{\partial u'}{\partial y} T' + 2\frac{1}{k_1}\frac{1}{R_o} M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial x} \frac{\partial u'}{\partial y} T' - 2\frac{1}{k_1}\frac{1}{R_o} \dot{\gamma_r} M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial x} \frac{\partial u'}{\partial y} T'$$

$$+ 2\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial x} \frac{\partial u'}{\partial z} T' - 2\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial x} \frac{\partial u'}{\partial z} T' - 2\frac{1}{k_1} k_2 M^2 U \frac{d\mu}{dT} \frac{\partial v'}{\partial x} T'$$

$$+ 2\frac{1}{k_1}\gamma_r k_2 M^2 U \frac{d\mu}{dT} \frac{\partial v'}{\partial x} T' + 2\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial y} \frac{\partial v'}{\partial x} T' - 2\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial y} \frac{\partial v'}{\partial x} T'$$

$$+ 2\frac{1}{k_1^2}\frac{1}{R_o} M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial x} \frac{\partial v'}{\partial x} T' - 2\frac{1}{k_1^2}\frac{1}{R_o}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial x} \frac{\partial v'}{\partial x} T' + 2\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial y} \frac{\partial v'}{\partial x} T'$$

$$- 2\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial y} \frac{\partial v'}{\partial x} T' - \frac{4}{3}\frac{1}{R_o} k_2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} \frac{\partial v'}{\partial y} T' + \frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} \frac{\partial v'}{\partial y} T'$$

$$- \frac{4}{3}\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial x} \frac{\partial v'}{\partial y} T' + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial x} \frac{\partial v'}{\partial y} T' + \frac{8}{3}\frac{1}{R_o} M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \frac{\partial v'}{\partial y} T'$$

$$- \frac{8}{3}\frac{1}{R_o}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \frac{\partial v'}{\partial y} T' - \frac{4}{3}\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial y} T' + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial y} T'$$

$$+ 2M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial y} \frac{\partial v'}{\partial z} T' - 2\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial y} \frac{\partial v'}{\partial z} T' + 2\frac{1}{k_1^2} M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial x} \frac{\partial w'}{\partial x} T'$$

$$- 2\frac{1}{k_1^2}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial x} \frac{\partial w'}{\partial x} T' + 2\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial z} \frac{\partial w'}{\partial x} T' - 2\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial z} \frac{\partial w'}{\partial x} T'$$

$$+ 2M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial y} \frac{\partial w'}{\partial y} T' - 2\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial y} \frac{\partial w'}{\partial y} T' + 2M^2 \frac{d\mu}{dT} \frac{\partial v'}{\partial z} \frac{\partial w'}{\partial y} T'$$

$$- 2\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial v'}{\partial z} \frac{\partial w'}{\partial y} T' - \frac{4}{3}\frac{1}{R_o} k_2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} \frac{\partial w'}{\partial z} T' + \frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} \frac{\partial w'}{\partial z} T'$$

$$- \frac{4}{3}\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial x} \frac{\partial w'}{\partial z} T' + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial x} \frac{\partial w'}{\partial z} T' - \frac{4}{3}\frac{1}{R_o} M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \frac{\partial w'}{\partial z} T'$$

$$+ \frac{4}{3}\frac{1}{R_o}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \frac{\partial w'}{\partial z} T' - \frac{4}{3}\frac{1}{k_1} M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial x} \frac{\partial w'}{\partial z} T' + \frac{4}{3}\frac{1}{k_1}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial x} \frac{\partial w'}{\partial z} T'$$

$$- \frac{4}{3} M^2 \frac{d\mu}{dT} \frac{\partial v'}{\partial y} \frac{\partial w'}{\partial z} T' + \frac{4}{3}\gamma_r M^2 \frac{d\mu}{dT} \frac{\partial v'}{\partial y} \frac{\partial w'}{\partial z} T' + \frac{1}{k_1} R_o U \frac{dc_p}{dT} \frac{\partial T}{\partial x} \rho' T'$$

$$+ \bar{\bar{V}} \frac{dc_p}{dT} \frac{\partial T}{\partial y} \rho' T' + R_o \frac{dc_p}{dT} \frac{\partial T'}{\partial t} \rho' T' + \frac{1}{k_1} R_o U \frac{dc_p}{dT} \frac{\partial T'}{\partial x} \rho' T'$$

$$+ \bar{\bar{V}} \frac{dc_p}{dT} \frac{\partial T'}{\partial y} \rho' T' + R_o W \frac{dc_p}{dT} \frac{\partial T'}{\partial z} \rho' T' - \frac{1}{k_1} R_o T \frac{\partial \rho'}{\partial x} u'$$

16

$$+ \frac{1}{\gamma_r}\frac{1}{k_1}R_oT\frac{\partial \rho'}{\partial x}u' - \frac{1}{k_1}R_o\rho\frac{\partial T'}{\partial x}u' + \frac{1}{\gamma_r}\frac{1}{k_1}R_o\rho\frac{\partial T'}{\partial x}u'$$

$$+ \frac{1}{k_1}R_oc_p\rho\frac{\partial T'}{\partial x}u' - 2k_2M^2\mu\frac{\partial u'}{\partial y}u' + 2\gamma_r k_2M^2\mu\frac{\partial u'}{\partial y}u'$$

$$- 2\frac{1}{k_1}k_2M^2\mu\frac{\partial v'}{\partial x}u' + 2\frac{1}{k_1}\gamma_r k_2M^2\mu\frac{\partial v'}{\partial x}u' - \frac{1}{k_1}R_o\frac{\partial T}{\partial x}\rho'u'$$

$$+ \frac{1}{\gamma_r}\frac{1}{k_1}R_o\frac{\partial T}{\partial x}\rho'u' + \frac{1}{k_1}R_oc_p\frac{\partial T}{\partial x}\rho'u' - \frac{1}{k_1}R_o\frac{\partial T'}{\partial x}\rho'u'$$

$$+ \frac{1}{\gamma_r}\frac{1}{k_1}R_o\frac{\partial T'}{\partial x}\rho'u' + \frac{1}{k_1}R_oc_p\frac{\partial T'}{\partial x}\rho'u' - \frac{1}{k_1}R_o\frac{\partial \rho}{\partial x}T'u'$$

$$+ \frac{1}{\gamma_r}\frac{1}{k_1}R_o\frac{\partial \rho}{\partial x}T'u' + \frac{1}{k_1}R_o\rho\frac{dc_p}{dT}\frac{\partial T}{\partial x}T'u' - 2k_2M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}T'u'$$

$$+ 2\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}T'u' - 2\frac{1}{k_1}\frac{1}{R_o}k_2M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial x}T'u' + 2\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial x}T'u'$$

$$- \frac{1}{k_1}R_o\frac{\partial \rho'}{\partial x}T'u' + \frac{1}{\gamma_r}\frac{1}{k_1}R_o\frac{\partial \rho'}{\partial x}T'u' + \frac{1}{k_1}R_o\rho\frac{dc_p}{dT}\frac{\partial T'}{\partial x}T'u'$$

$$- 2k_2M^2\frac{d\mu}{dT}\frac{\partial u'}{\partial y}T'u' + 2\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial u'}{\partial y}T'u' - 2\frac{1}{k_1}k_2M^2\frac{d\mu}{dT}\frac{\partial v'}{\partial x}T'u'$$

$$+ 2\frac{1}{k_1}\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial v'}{\partial x}T'u' + \frac{1}{k_1}R_o\frac{dc_p}{dT}\frac{\partial T}{\partial x}\rho'T'u' + \frac{1}{k_1}R_o\frac{dc_p}{dT}\frac{\partial T'}{\partial x}\rho'T'u'$$

$$- R_oT\frac{\partial \rho'}{\partial y}v' + \frac{1}{\gamma_r}R_oT\frac{\partial \rho'}{\partial y}v' - R_o\rho\frac{\partial T'}{\partial y}v' + \frac{1}{\gamma_r}R_o\rho\frac{\partial T'}{\partial y}v'$$

$$+ R_oc_p\rho\frac{\partial T'}{\partial y}v' + \frac{8}{3}\frac{1}{k_1}k_2M^2\mu\frac{\partial u'}{\partial x}v' - \frac{8}{3}\frac{1}{k_1}\gamma_r k_2M^2\mu\frac{\partial u'}{\partial x}v'$$

$$- \frac{4}{3}k_2M^2\mu\frac{\partial v'}{\partial y}v' + \frac{4}{3}\gamma_r k_2M^2\mu\frac{\partial v'}{\partial y}v' - \frac{4}{3}k_2M^2\mu\frac{\partial w'}{\partial z}v' + \frac{4}{3}\gamma_r k_2M^2\mu\frac{\partial w'}{\partial z}v'$$

$$- R_o\frac{\partial T}{\partial y}\rho'v' + \frac{1}{\gamma_r}R_o\frac{\partial T}{\partial y}\rho'v' + R_oc_p\frac{\partial T}{\partial y}\rho'v' - R_o\frac{\partial T'}{\partial y}\rho'v' + \frac{1}{\gamma_r}R_o\frac{\partial T'}{\partial y}\rho'v'$$

$$+ R_oc_p\frac{\partial T'}{\partial y}\rho'v' - R_o\frac{\partial \rho}{\partial y}T'v' + \frac{1}{\gamma_r}R_o\frac{\partial \rho}{\partial y}T'v' + R_o\rho\frac{dc_p}{dT}\frac{\partial T}{\partial y}T'v'$$

$$+ \frac{8}{3}\frac{1}{k_1}k_2M^2\frac{d\mu}{dT}\frac{\partial U}{\partial x}T'v' - \frac{8}{3}\frac{1}{k_1}\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial U}{\partial x}T'v' - \frac{4}{3}\frac{1}{R_o}k_2M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}T'v'$$

$$+ \frac{4}{3}\frac{1}{R_o}\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}T'v' - R_o\frac{\partial \rho'}{\partial y}T'v' + \frac{1}{\gamma_r}R_o\frac{\partial \rho'}{\partial y}T'v'$$

17

$$+ R_o \rho \frac{dc_p}{dT} \frac{\partial T'}{\partial y} T'v' + \frac{8}{3} \frac{1}{k_1} k_2 M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial x} T'v' - \frac{8}{3} \frac{1}{k_1} \gamma_r k_2 M^2 \frac{d\mu}{dT} \frac{\partial u'}{\partial x} T'v'$$

$$- \frac{4}{3} k_2 M^2 \frac{d\mu}{dT} \frac{\partial v'}{\partial y} T'v' + \frac{4}{3} \gamma_r k_2 M^2 \frac{d\mu}{dT} \frac{\partial v'}{\partial y} T'v' - \frac{4}{3} k_2 M^2 \frac{d\mu}{dT} \frac{\partial w'}{\partial z} T'v'$$

$$+ \frac{4}{3} \gamma_r k_2 M^2 \frac{d\mu}{dT} \frac{\partial w'}{\partial z} T'v' + R_o \frac{dc_p}{dT} \frac{\partial T}{\partial y} \rho'T'v' + R_o \frac{dc_p}{dT} \frac{\partial T'}{\partial y} \rho'T'v' - R_o T \frac{\partial \rho'}{\partial z} w'$$

$$+ \frac{1}{\gamma_r} R_o T \frac{\partial \rho'}{\partial z} w' - R_o \rho \frac{\partial T'}{\partial z} w' + \frac{1}{\gamma_r} R_o \rho \frac{\partial T'}{\partial z} w' + R_o c_p \rho \frac{\partial T'}{\partial z} w' - R_o \frac{\partial T'}{\partial z} \rho'w'$$

$$+ \frac{1}{\gamma_r} R_o \frac{\partial T'}{\partial z} \rho'w' + R_o c_p \frac{\partial T'}{\partial z} \rho'w' - R_o \frac{\partial \rho'}{\partial z} T'w' + \frac{1}{\gamma_r} R_o \frac{\partial \rho'}{\partial z} T'w' + R_o \rho \frac{dc_p}{dT} \frac{\partial T'}{\partial z} T'w'$$

$$+ R_o \frac{dc_p}{dT} \frac{\partial T'}{\partial z} \rho'T'w' + 2M^2 U \frac{d\mu}{dT} T'u'(k_2)^2 - 2\gamma_r M^2 U \frac{d\mu}{dT} T'u'(k_2)^2$$

$$+ \frac{8}{3} \frac{1}{R_o} M^2 \bar{\bar{V}} \frac{d\mu}{dT} T'v'(k_2)^2 - \frac{8}{3} \frac{1}{R_o} \gamma_r M^2 \bar{\bar{V}} \frac{d\mu}{dT} T'v'(k_2)^2 - \frac{1}{k_1^2} \frac{1}{Pr} \frac{d\kappa}{dT} \left(\frac{\partial T'}{\partial x}\right)^2$$

$$- \frac{1}{Pr} \frac{d\kappa}{dT} \left(\frac{\partial T'}{\partial y}\right)^2 - \frac{1}{Pr} \frac{d\kappa}{dT} \left(\frac{\partial T'}{\partial z}\right)^2 + \frac{4}{3} \frac{1}{k_1^2} M^2 \mu \left(\frac{\partial u'}{\partial x}\right)^2$$

$$- \frac{4}{3} \frac{1}{k_1^2} \gamma_r M^2 \mu \left(\frac{\partial u'}{\partial x}\right)^2 + \frac{4}{3} \frac{1}{k_1^2} M^2 \frac{d\mu}{dT} T' \left(\frac{\partial u'}{\partial x}\right)^2 - \frac{4}{3} \frac{1}{k_1^2} \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial u'}{\partial x}\right)^2$$

$$+ M^2 \mu \left(\frac{\partial u'}{\partial y}\right)^2 - \gamma_r M^2 \mu \left(\frac{\partial u'}{\partial y}\right)^2 + M^2 \frac{d\mu}{dT} T' \left(\frac{\partial u'}{\partial y}\right)^2$$

$$- \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial u'}{\partial y}\right)^2 + M^2 \mu \left(\frac{\partial u'}{\partial z}\right)^2 - \gamma_r M^2 \mu \left(\frac{\partial u'}{\partial z}\right)^2$$

$$+ M^2 \frac{d\mu}{dT} T' \left(\frac{\partial u'}{\partial z}\right)^2 - \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial u'}{\partial z}\right)^2 + \frac{1}{k_1^2} M^2 \mu \left(\frac{\partial v'}{\partial x}\right)^2$$

$$- \frac{1}{k_1^2} \gamma_r M^2 \mu \left(\frac{\partial v'}{\partial x}\right)^2 + \frac{1}{k_1^2} M^2 \frac{d\mu}{dT} T' \left(\frac{\partial v'}{\partial x}\right)^2 - \frac{1}{k_1^2} \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial v'}{\partial x}\right)^2$$

$$+ \frac{4}{3} M^2 \mu \left(\frac{\partial v'}{\partial y}\right)^2 - \frac{4}{3} \gamma_r M^2 \mu \left(\frac{\partial v'}{\partial y}\right)^2 + \frac{4}{3} M^2 \frac{d\mu}{dT} T' \left(\frac{\partial v'}{\partial y}\right)^2$$

$$- \frac{4}{3} \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial v'}{\partial y}\right)^2 + M^2 \mu \left(\frac{\partial v'}{\partial z}\right)^2 - \gamma_r M^2 \mu \left(\frac{\partial v'}{\partial z}\right)^2$$

$$+ M^2 \frac{d\mu}{dT} T' \left(\frac{\partial v'}{\partial z}\right)^2 - \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial v'}{\partial z}\right)^2 + \frac{1}{k_1^2} M^2 \mu \left(\frac{\partial w'}{\partial x}\right)^2$$

$$- \frac{1}{k_1^2} \gamma_r M^2 \mu \left(\frac{\partial w'}{\partial x}\right)^2 + \frac{1}{k_1^2} M^2 \frac{d\mu}{dT} T' \left(\frac{\partial w'}{\partial x}\right)^2 - \frac{1}{k_1^2} \gamma_r M^2 \frac{d\mu}{dT} T' \left(\frac{\partial w'}{\partial x}\right)^2$$

$$+ M^2 \mu \Big(\frac{\partial w'}{\partial y} \Big)^2 - \gamma_r M^2 \mu \Big(\frac{\partial w'}{\partial y} \Big)^2 + M^2 \frac{d\mu}{dT} T' \Big(\frac{\partial w'}{\partial y} \Big)^2$$

$$- \gamma_r M^2 \frac{d\mu}{dT} T' \Big(\frac{\partial w'}{\partial y} \Big)^2 + \frac{4}{3} M^2 \mu \Big(\frac{\partial w'}{\partial z} \Big)^2 - \frac{4}{3} \gamma_r M^2 \mu \Big(\frac{\partial w'}{\partial z} \Big)^2$$

$$+ \frac{4}{3} M^2 \frac{d\mu}{dT} T' \Big(\frac{\partial w'}{\partial z} \Big)^2 - \frac{4}{3} \gamma_r M^2 \frac{d\mu}{dT} T' \Big(\frac{\partial w'}{\partial z} \Big)^2 + M^2 \mu (k_2)^2 (u')^2$$

$$- \gamma_r M^2 \mu (k_2)^2 (u')^2 + M^2 \frac{d\mu}{dT} T' (k_2)^2 (u')^2 - \gamma_r M^2 \frac{d\mu}{dT} T' (k_2)^2 (u')^2$$

$$+ \frac{4}{3} M^2 \mu (k_2)^2 (v')^2 - \frac{4}{3} \gamma_r M^2 \mu (k_2)^2 (v')^2 + \frac{4}{3} M^2 \frac{d\mu}{dT} T' (k_2)^2 (v')^2$$

$$- \frac{4}{3} \gamma_r M^2 \frac{d\mu}{dT} T' (k_2)^2 (v')^2$$

Nonlinear Parabolized Stability Equations

The dimensionless compressible Nonlinear Parabolized Stability Equations (NPSE) are presented below in body–intrinsic orthogonal curvilinear coordinates. The corresponding Linear Parabolized Stability Equations (LPSE) and Linear Stability Theory (LST) equations are obtained by setting certain terms to zero in the NPSE. For LPSE the right–hand side terms representing the nonlinear forcing are set to zero. The LST equations are obtained by neglecting the nonlinear terms as well as the nonparallel terms ($\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$). For incompressible Mach numbers only quadratic nonlinear terms are retained.

The detailed form of the equations presented below was derived with the aid of a symbol manipulator. The equations were then systematically converted to LaTeX using several programs designed by the author. This results in the expanded and somewhat inelegant form. The C++ program to solve these equations was generated in a similar fashion.

$$
\left\{ (\mathcal{L}_o + \mathcal{L}_1)\tilde{\phi} + \mathcal{L}_2 \frac{\partial \tilde{\phi}}{\partial \bar{x}} + \frac{\partial \alpha}{\partial \bar{x}} \mathcal{L}_3 \tilde{\phi} \right\}_{(n,k)} \mathcal{A}_{(n,k)} =
$$

$$
\sum_{\substack{n=n_2+n_3 \\ |n_2|\leq N \\ |n_3|\leq N}} \sum_{\substack{k=k_2+k_3 \\ |k_2|\leq K \\ |k_3|\leq K}} \hat{\mathcal{N}}^{(quad)}_{(n_2,k_2,n_3,k_3)} + \sum_{\substack{n=n_2+\ldots \\ |n_2|\leq N \\ |n_3|\leq N \\ |n_4|\leq N}} \sum_{\substack{k=k_2+\ldots \\ |k_2|\leq K \\ |k_3|\leq K \\ |k_4|\leq K}} \hat{\mathcal{N}}^{(cub)}_{(n_2,\ldots)} +
$$

$$
\sum_{\substack{n=n_2+\ldots \\ |n_2|\leq N \\ |n_3|\leq N \\ |n_4|\leq N \\ |n_5|\leq N}} \sum_{\substack{k=k_2+\ldots \\ |k_2|\leq K \\ |k_3|\leq K \\ |k_4|\leq K \\ |k_5|\leq N}} \hat{\mathcal{N}}^{(quart)}_{(n_2,\ldots)}
$$

$$
\mathcal{L}_o(1,1) = -(in\omega_o R_o) + i\frac{1}{k_1}\alpha R_o U + ik\beta_o R_o W
$$

$$
\mathcal{L}_o(1,2) = i\frac{1}{k_1}\alpha R_o \rho + \frac{1}{k_1}\frac{\partial \rho}{\partial \bar{x}}
$$

$$
\mathcal{L}_o(1,3) = (R_o\rho)\frac{\partial}{\partial y} + k_2 R_o \rho + R_o \frac{\partial \rho}{\partial y}
$$

$$
\mathcal{L}_o(1,4) = ik\beta_o R_o \rho
$$

$$
\mathcal{L}_o(2,1) = \frac{1}{\gamma_r} i \frac{1}{k_1} \frac{1}{M^2} \alpha R_o T + \frac{1}{\gamma_r} \frac{1}{k_1} \frac{1}{M^2} \frac{\partial T}{\partial \bar{x}}
$$

$$\mathcal{L}_o(2,2) = 2\frac{1}{k_1^2}\alpha^2\mu - \frac{2}{3}\frac{1}{k_1^2}\alpha^2\mu + (k\beta_o)^2\mu + (-\mu)\frac{\partial^2}{\partial y^2}$$
$$+ (-(k_2\mu))\frac{\partial}{\partial y} + k_2^2\mu + \frac{4}{3}i\frac{1}{k_1^3}\frac{1}{R_o}\alpha k_4\mu - in\omega_o R_o\rho$$
$$+ i\frac{1}{k_1}\alpha R_o\rho U + ik\beta_o R_o\rho W - \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}} + (-(\frac{d\mu}{dT}\frac{\partial T}{\partial y}))\frac{\partial}{\partial y}$$
$$+ k_2\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{L}_o(2,3) = (-(i\frac{1}{k_1}\alpha\mu)/3)\frac{\partial}{\partial y} - frac73 i\frac{1}{k_1}\alpha k_2\mu - 2\frac{1}{k_1^2}\frac{1}{R_o}k_3\mu + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_3\mu$$
$$+ 2\frac{1}{k_1^2}\frac{1}{R_o}k_2 k_4\mu - \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_2 k_4\mu + k_2 R_o\rho U + (\frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}})\frac{\partial}{\partial y}$$
$$- 2\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}} - i\frac{1}{k_1}\alpha\frac{d\mu}{dT}\frac{\partial T}{\partial y} + R_o\rho\frac{\partial U}{\partial y}$$

$$\mathcal{L}_o(2,4) = \frac{1}{k_1}\alpha k\beta_o\mu - \frac{2}{3}\frac{1}{k_1}\alpha k\beta_o\mu + \frac{2}{3}i\frac{1}{k_1}\frac{1}{R_o}k\beta_o\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}}$$

$$\mathcal{L}_o(2,5) = \frac{1}{\gamma_r}i\frac{1}{k_1}\frac{1}{M^2}\alpha R_o\rho + (k_2 U\frac{d\mu}{dT})\frac{\partial}{\partial y} + k_2^2 U\frac{d\mu}{dT} + \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}\frac{\partial\rho}{\partial \bar{x}}$$
$$+ (-(\frac{d\mu}{dT}\frac{\partial U}{\partial y}))\frac{\partial}{\partial y} - k_2\frac{d\mu}{dT}\frac{\partial U}{\partial y} - i\frac{1}{k_1}\frac{1}{R_o}k\beta_o\frac{d\mu}{dT}\frac{\partial W}{\partial \bar{x}} + k_2 U\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}$$
$$- \frac{\partial T}{\partial y}\frac{\partial U}{\partial y}\frac{d^2\mu}{dT^2} - \frac{d\mu}{dT}\frac{\partial^2 U}{\partial y^2}$$

$$\mathcal{L}_o(3,1) = (\frac{1}{\gamma_r}\frac{1}{M^2}R_o T)\frac{\partial}{\partial y} - k_2 R_o U^2 + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial T}{\partial y}$$

$$\mathcal{L}_o(3,2) = (-(i\frac{1}{k_1}\alpha\mu)/3)\frac{\partial}{\partial y} + frac73 i\frac{1}{k_1}\alpha k_2\mu + \frac{1}{k_1^2}\frac{1}{R_o}k_3\mu - \frac{1}{k_1^2}\frac{1}{R_o}k_2 k_4\mu$$
$$- 2k_2 R_o\rho U + (-(\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}}))\frac{\partial}{\partial y} + \frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}} + \frac{2}{3}i\frac{1}{k_1}\alpha\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{L}_o(3,3) = \frac{1}{k_1^2}\alpha^2\mu + (k\beta_o)^2\mu + (-2\mu)\frac{\partial^2}{\partial y^2} + (\frac{2}{3}\mu)\frac{\partial^2}{\partial y^2}$$
$$+ (-2k_2\mu)\frac{\partial}{\partial y} + (\frac{2}{3}k_2\mu)\frac{\partial}{\partial y} + 2k_2^2\mu - \frac{2}{3}k_2^2\mu$$
$$+ i\frac{1}{k_1^3}\frac{1}{R_o}\alpha k_4\mu - in\omega_o R_o\rho + i\frac{1}{k_1}\alpha R_o\rho U + ik\beta_o R_o\rho W$$
$$- i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\frac{d\mu}{dT}\frac{\partial T}{\partial \bar{x}} + (-2\frac{d\mu}{dT}\frac{\partial T}{\partial y})\frac{\partial}{\partial y} + (\frac{2}{3}\frac{d\mu}{dT}\frac{\partial T}{\partial y})\frac{\partial}{\partial y} + \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{L}_o(3,4) = (-(ik\beta_o\mu)/3)\frac{\partial}{\partial y} + \frac{2}{3}ik\beta_o\frac{d\mu}{dT}\frac{\partial T}{\partial y}$$

$$\mathcal{L}_o(3,5) = (\frac{1}{\gamma_r}\frac{1}{M^2}R_o\rho)\frac{\partial}{\partial y} + i\frac{1}{k_1}\alpha k_2 U\frac{d\mu}{dT} + \frac{1}{k_1^2}\frac{1}{R_o}k_3 U\frac{d\mu}{dT} - \frac{1}{k_1^2}\frac{1}{R_o}k_2 k_4 U\frac{d\mu}{dT}$$
$$+ \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial\rho}{\partial y} - i\frac{1}{k_1}\alpha\frac{d\mu}{dT}\frac{\partial U}{\partial y} - ik\beta_o\frac{d\mu}{dT}\frac{\partial W}{\partial y} + \frac{1}{k_1}\frac{1}{R_o}k_2 U\frac{\partial T}{\partial\bar{x}}\frac{d^2\mu}{dT^2}$$
$$- \frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial\bar{x}}\frac{\partial U}{\partial y}\frac{d^2\mu}{dT^2} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial\bar{x}\partial y}$$

$$\mathcal{L}_o(4,1) = \frac{1}{\gamma_r}i\frac{1}{M^2}k\beta_o R_o T + \frac{1}{k_1}U\frac{\partial W}{\partial\bar{x}}$$

$$\mathcal{L}_o(4,2) = \frac{1}{k_1}\alpha k\beta_o\mu - \frac{2}{3}\frac{1}{k_1}\alpha k\beta_o\mu - i\frac{1}{k_1}\frac{1}{R_o}k\beta_o\frac{d\mu}{dT}\frac{\partial T}{\partial\bar{x}} + \frac{1}{k_1}\rho\frac{\partial W}{\partial\bar{x}}$$

$$\mathcal{L}_o(4,3) = (-(ik\beta_o\mu)/3)\frac{\partial}{\partial y} - (ik\beta_o k_2\mu)/3 - ik\beta_o\frac{d\mu}{dT}\frac{\partial T}{\partial y} + R_o\rho\frac{\partial W}{\partial y}$$

$$\mathcal{L}_o(4,4) = \frac{1}{k_1^2}\alpha^2\mu + 2(k\beta_o)^2\mu - \frac{2}{3}(k\beta_o)^2\mu + (-\mu)\frac{\partial^2}{\partial y^2}$$
$$+ (-(k_2\mu))\frac{\partial}{\partial y} + i\frac{1}{k_1^3}\frac{1}{R_o}\alpha k_4\mu - in\omega_o R_o\rho + i\frac{1}{k_1}\alpha R_o\rho U$$
$$+ ik\beta_o R_o\rho W - i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\frac{d\mu}{dT}\frac{\partial T}{\partial\bar{x}} + (-(\frac{d\mu}{dT}\frac{\partial T}{\partial y}))\frac{\partial}{\partial y}$$

$$\mathcal{L}_o(4,5) = \frac{1}{\gamma_r}i\frac{1}{M^2}k\beta_o R_o\rho - i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\frac{d\mu}{dT}\frac{\partial W}{\partial\bar{x}} + \frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial W}{\partial\bar{x}} + (-(\frac{d\mu}{dT}\frac{\partial W}{\partial y}))\frac{\partial}{\partial y}$$
$$- k_2\frac{d\mu}{dT}\frac{\partial W}{\partial y} - \frac{1}{k_1^2}\frac{1}{R_o^2}\frac{\partial T}{\partial\bar{x}}\frac{\partial W}{\partial\bar{x}}\frac{d^2\mu}{dT^2} - \frac{\partial T}{\partial y}\frac{\partial W}{\partial y}\frac{d^2\mu}{dT^2} - \frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 W}{\partial\bar{x}^2}$$
$$- \frac{d\mu}{dT}\frac{\partial^2 W}{\partial y^2}$$

$$\mathcal{L}_o(5,1) = in\omega_o R_o T - \frac{1}{\gamma_r}in\omega_o R_o T - i\frac{1}{k_1}\alpha R_o TU + \frac{1}{\gamma_r}i\frac{1}{k_1}\alpha R_o TU$$
$$- ik\beta_o R_o TW + \frac{1}{\gamma_r}ik\beta_o R_o TW - \frac{1}{k_1}U\frac{\partial T}{\partial\bar{x}} + \frac{1}{\gamma_r}\frac{1}{k_1}U\frac{\partial T}{\partial\bar{x}}$$
$$+ \frac{1}{k_1}c_p U\frac{\partial T}{\partial\bar{x}}$$

$$\mathcal{L}_o(5,2) = (-2k_2 M^2 \mu U)\frac{\partial}{\partial y} + (2\gamma_r k_2 M^2 \mu U)\frac{\partial}{\partial y} + 2k_2^2 M^2 \mu U - 2\gamma_r k_2^2 M^2 \mu U$$

$$- \frac{1}{k_1}\frac{\partial bas[p]}{\partial \bar{x}} + \frac{1}{\gamma_r}\frac{1}{k_1}\frac{\partial bas[p]}{\partial \bar{x}} + \frac{1}{k_1}c_p \rho \frac{\partial T}{\partial \bar{x}} + (2M^2 \mu \frac{\partial U}{\partial y})\frac{\partial}{\partial y}$$

$$+ (-2\gamma_r M^2 \mu \frac{\partial U}{\partial y})\frac{\partial}{\partial y} - 2k_2 M^2 \mu \frac{\partial U}{\partial y} + 2\gamma_r k_2 M^2 \mu \frac{\partial U}{\partial y} + 2i\frac{1}{k_1}\frac{1}{R_o}k\beta_o M^2 \mu \frac{\partial W}{\partial \bar{x}}$$

$$- 2i\frac{1}{k_1}\frac{1}{R_o}k\beta_o \gamma_r M^2 \mu \frac{\partial W}{\partial \bar{x}}$$

$$\mathcal{L}_o(5,3) = -2i\frac{1}{k_1}\alpha k_2 M^2 \mu U + 2i\frac{1}{k_1}\alpha\gamma_r k_2 M^2 \mu U + R_o c_p \rho \frac{\partial T}{\partial y} + 2i\frac{1}{k_1}\alpha M^2 \mu \frac{\partial U}{\partial y}$$

$$- 2i\frac{1}{k_1}\alpha\gamma_r M^2 \mu \frac{\partial U}{\partial y} + 2ik\beta_o M^2 \mu \frac{\partial W}{\partial y} - 2ik\beta_o \gamma_r M^2 \mu \frac{\partial W}{\partial y}$$

$$\mathcal{L}_o(5,4) = 2i\frac{1}{k_1^2}\frac{1}{R_o}\alpha M^2 \mu \frac{\partial W}{\partial \bar{x}} - 2i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\gamma_r M^2 \mu \frac{\partial W}{\partial \bar{x}} + (2M^2 \mu \frac{\partial W}{\partial y})\frac{\partial}{\partial y} + (-2\gamma_r M^2 \mu \frac{\partial W}{\partial y})\frac{\partial}{\partial y}$$

$$\mathcal{L}_o(5,5) = \frac{1}{k_1^2}\frac{1}{Pr}\alpha^2 \kappa + \frac{1}{Pr}(k\beta_o)^2 \kappa + (-(\frac{1}{Pr}\kappa))\frac{\partial^2}{\partial y^2} + (-(\frac{1}{Pr}k_2\kappa))\frac{\partial}{\partial y}$$

$$+ i\frac{1}{k_1^3}\frac{1}{Pr}\frac{1}{R_o}\alpha k_4 \kappa + in\omega_o R_o \rho - \frac{1}{\gamma_r}in\omega_o R_o \rho - in\omega_o R_o c_p \rho$$

$$- i\frac{1}{k_1}\alpha R_o \rho U + \frac{1}{\gamma_r}i\frac{1}{k_1}\alpha R_o \rho U + i\frac{1}{k_1}\alpha R_o c_p \rho U - ik\beta_o R_o \rho W$$

$$+ \frac{1}{\gamma_r}ik\beta_o R_o \rho W + ik\beta_o R_o c_p \rho W + k_2^2 M^2 U^2 \frac{d\mu}{dT} - \gamma_r k_2^2 M^2 U^2 \frac{d\mu}{dT}$$

$$- \frac{1}{k_1}U\frac{\partial \rho}{\partial \bar{x}} + \frac{1}{\gamma_r}\frac{1}{k_1}U\frac{\partial \rho}{\partial \bar{x}} - 2i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\alpha\frac{d\kappa}{dT}\frac{\partial T}{\partial \bar{x}} + \frac{1}{k_1^3}\frac{1}{Pr}\frac{1}{R_o^2}k_4\frac{d\kappa}{dT}\frac{\partial T}{\partial \bar{x}}$$

$$+ \frac{1}{k_1}\rho U\frac{dc_p}{dT}\frac{\partial T}{\partial \bar{x}} + (-2\frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial T}{\partial y})\frac{\partial}{\partial y} - \frac{1}{Pr}k_2\frac{d\kappa}{dT}\frac{\partial T}{\partial y} - 2k_2 M^2 U\frac{d\mu}{dT}\frac{\partial U}{\partial y}$$

$$+ 2\gamma_r k_2 M^2 U\frac{d\mu}{dT}\frac{\partial U}{\partial y} + M^2\frac{d\mu}{dT}\frac{\partial U^2}{\partial y} - \gamma_r M^2\frac{d\mu}{dT}\frac{\partial U^2}{\partial y} + \frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial W^2}{\partial \bar{x}}$$

$$- \frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial W^2}{\partial \bar{x}} + M^2\frac{d\mu}{dT}\frac{\partial W^2}{\partial y} - \gamma_r M^2\frac{d\mu}{dT}\frac{\partial W^2}{\partial y} - \frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o^2}\frac{\partial T^2}{\partial \bar{x}}\frac{d^2\kappa}{dT^2}$$

$$- \frac{1}{Pr}\frac{\partial T^2}{\partial y}\frac{d^2\kappa}{dT^2} - \frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o^2}\frac{d\kappa}{dT}\frac{\partial^2 T}{\partial \bar{x}^2} - \frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 T}{\partial y^2}$$

23

$$\mathcal{L}_1(1,1) = \bar{\bar{V}}\frac{\partial}{\partial y} + k_2\bar{\bar{V}} + \frac{1}{k_1}\frac{\partial U}{\partial \bar{x}} + \frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{L}_1(2,1) = k_2 U\bar{\bar{V}} + \frac{1}{k_1}U\frac{\partial U}{\partial \bar{x}} + \bar{\bar{V}}\frac{\partial U}{\partial y}$$

$$\mathcal{L}_1(2,2) = \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\mu + (\rho\bar{\bar{V}})\frac{\partial}{\partial y} + k_2\rho\bar{\bar{V}} - \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\mu\frac{d\alpha}{d\bar{x}}$$
$$+ \frac{1}{k_1}\rho\frac{\partial U}{\partial \bar{x}}$$

$$\mathcal{L}_1(2,5) = -(\frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\alpha k_2\bar{\bar{V}}\frac{d\mu}{dT}) - 2\frac{1}{k_1^2}\frac{1}{R_o^2}k_3\bar{\bar{V}}\frac{d\mu}{dT} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}k_3\bar{\bar{V}}\frac{d\mu}{dT} + 2\frac{1}{k_1^2}\frac{1}{R_o^2}k_2 k_4\bar{\bar{V}}\frac{d\mu}{dT}$$
$$- \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}k_2 k_4\bar{\bar{V}}\frac{d\mu}{dT} - \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}} + 2\frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}} - \frac{2}{3}\frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}$$
$$+ (-(\frac{1}{k_1}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial \bar{x}}))\frac{\partial}{\partial y} - 3\frac{1}{k_1}\frac{1}{R_o^2}k_2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial \bar{x}} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o^2}k_2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial \bar{x}}$$
$$+ \frac{2}{3}i\frac{1}{k_1}\frac{1}{R_o}\alpha\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y} - 2\frac{1}{k_1}\frac{1}{R_o^2}k_2\bar{\bar{V}}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o^2}k_2\bar{\bar{V}}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}$$
$$- 2\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{\partial U}{\partial \bar{x}}\frac{d^2\mu}{dT^2} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{\partial U}{\partial \bar{x}}\frac{d^2\mu}{dT^2} - \frac{1}{k_1}\frac{1}{R_o^2}\frac{\partial T}{\partial y}\frac{\partial \bar{\bar{V}}}{\partial \bar{x}}\frac{d^2\mu}{dT^2}$$
$$+ \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{\partial \bar{\bar{V}}}{\partial y}\frac{d^2\mu}{dT^2} - 2\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial \bar{x}^2} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial \bar{x}^2}$$
$$+ \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 \bar{\bar{V}}}{\partial \bar{x}\partial y} - \frac{1}{k_1}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 \bar{\bar{V}}}{\partial \bar{x}\partial y}$$

$$\mathcal{L}_1(3,1) = \frac{1}{k_1}\frac{1}{R_o}U\frac{\partial \bar{\bar{V}}}{\partial \bar{x}} + \frac{1}{R_o}\bar{\bar{V}}\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{L}_1(3,2) = \frac{1}{k_1}\frac{1}{R_o}\rho\frac{\partial \bar{\bar{V}}}{\partial \bar{x}}$$

$$\mathcal{L}_1(3,3) = i\frac{1}{k_1^2}\frac{1}{R_o}\mu + (\rho\bar{\bar{V}})\frac{\partial}{\partial y} - i\frac{1}{k_1^2}\frac{1}{R_o}\mu\frac{d\alpha}{d\bar{x}} + \rho\frac{\partial \bar{\bar{V}}}{\partial y}$$

24

$$\mathcal{L}_1(3,5) = \left(\frac{2}{3}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{d\mu}{dT}\right)\frac{\partial}{\partial y} + 2\frac{1}{R_o}k_2^2\bar{\bar{V}}\frac{d\mu}{dT} - \frac{2}{3}\frac{1}{R_o}k_2^2\bar{\bar{V}}\frac{d\mu}{dT} + \left(\frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial U}{\partial\bar{x}}\right)\frac{\partial}{\partial y}$$

$$+ 3\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial U}{\partial\bar{x}} - \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial U}{\partial\bar{x}} - i\frac{1}{k_1^2}\frac{1}{R_o^2}\alpha\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}} + \frac{1}{k_1^3}\frac{1}{R_o}3k_4\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}$$

$$+ \left(-2\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}\right)\frac{\partial}{\partial y} + \left(\frac{2}{3}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}\right)\frac{\partial}{\partial y} - 2\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y} + \frac{2}{3}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$+ \frac{2}{3}\frac{1}{R_o}k_2\bar{\bar{V}}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{\partial U}{\partial\bar{x}}\frac{d^2\mu}{dT^2} - \frac{1}{k_1^2}\frac{1}{R_o}3\frac{\partial T}{\partial\bar{x}}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\frac{d^2\mu}{dT^2} - 2\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial y}\frac{d^2\mu}{dT^2}$$

$$+ \frac{2}{3}\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial y}\frac{d^2\mu}{dT^2} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2 U}{\partial\bar{x}\partial y} - \frac{1}{k_1^2}\frac{1}{R_o}3\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial\bar{x}^2} - 2\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial y^2}$$

$$+ \frac{2}{3}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2\bar{\bar{V}}}{\partial y^2}$$

$$\mathcal{L}_1(4,1) = \bar{\bar{V}}\frac{\partial W}{\partial y}$$

$$\mathcal{L}_1(4,4) = i\frac{1}{k_1^2}\frac{1}{R_o}\mu + (\rho\bar{\bar{V}})\frac{\partial}{\partial y} - i\frac{1}{k_1^2}\frac{1}{R_o}\mu\frac{d\alpha}{d\bar{x}}$$

$$\mathcal{L}_1(4,5) = \frac{2}{3}i\frac{1}{R_o}k\beta_o k_2\bar{\bar{V}}\frac{d\mu}{dT} + \frac{2}{3}i\frac{1}{k_1}\frac{1}{R_o}k\beta_o\frac{d\mu}{dT}\frac{\partial U}{\partial\bar{x}} + \frac{2}{3}i\frac{1}{R_o}k\beta_o\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$\mathcal{L}_1(5,1) = (-(T\bar{\bar{V}}))\frac{\partial}{\partial y} + \left(\frac{1}{\gamma_r}T\bar{\bar{V}}\right)\frac{\partial}{\partial y} - \bar{\bar{V}}\frac{\partial T}{\partial y} + \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial T}{\partial y}$$

$$+ c_p\bar{\bar{V}}\frac{\partial T}{\partial y}$$

$$\mathcal{L}_1(5,2) = \frac{8}{3}i\frac{1}{k_1}\frac{1}{R_o}\alpha k_2 M^2\mu\bar{\bar{V}} - \frac{8}{3}i\frac{1}{k_1}\frac{1}{R_o}\alpha\gamma_r k_2 M^2\mu\bar{\bar{V}} + \frac{8}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha M^2\mu\frac{\partial U}{\partial\bar{x}}$$

$$- \frac{8}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha\gamma_r M^2\mu\frac{\partial U}{\partial\bar{x}} + \left(2\frac{1}{k_1}\frac{1}{R_o^2}M^2\mu\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\right)\frac{\partial}{\partial y} + \left(-2\frac{1}{k_1}\frac{1}{R_o^2}\gamma_r M^2\mu\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\right)\frac{\partial}{\partial y}$$

$$- 2\frac{1}{k_1}\frac{1}{R_o^2}k_2 M^2\mu\frac{\partial\bar{\bar{V}}}{\partial\bar{x}} + 2\frac{1}{k_1}\frac{1}{R_o^2}\gamma_r k_2 M^2\mu\frac{\partial\bar{\bar{V}}}{\partial\bar{x}} - \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\alpha M^2\mu\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$+ \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\alpha\gamma_r M^2\mu\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$\mathcal{L}_1(5,3) = -(\frac{4}{3}\frac{1}{R_o}k_2M^2\mu\bar{\bar{V}})\frac{\partial}{\partial y} + (\frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2\mu\bar{\bar{V}})\frac{\partial}{\partial y} + \frac{8}{3}\frac{1}{R_o}k_2^2 M^2\mu\bar{V}$$

$$- \frac{8}{3}\frac{1}{R_o}\gamma_r k_2^2 M^2\mu\bar{V} + -(\frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}M^2\mu\frac{\partial U}{\partial \bar{x}})\frac{\partial}{\partial y} + (\frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial U}{\partial \bar{x}})\frac{\partial}{\partial y}$$

$$+ \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}k_2 M^2\mu\frac{\partial U}{\partial \bar{x}} - \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2 M^2\mu\frac{\partial U}{\partial \bar{x}} + 2i\frac{1}{k_1^2}\frac{1}{R_o^2}\alpha M^2\mu\frac{\partial \bar{\bar{V}}}{\partial \bar{x}}$$

$$- 2i\frac{1}{k_1^2}\frac{1}{R_o^2}\alpha\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial \bar{x}} + (\frac{8}{3}\frac{1}{R_o}M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y})\frac{\partial}{\partial y} + -(\frac{8}{3}\frac{1}{R_o}\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y})\frac{\partial}{\partial y}$$

$$- \frac{4}{3}\frac{1}{R_o}k_2 M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{R_o}\gamma_r k_2 M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{L}_1(5,4) = -(\frac{4}{3}i\frac{1}{R_o}k\beta_o k_2 M^2\mu\bar{\bar{V}}) + \frac{4}{3}i\frac{1}{R_o}k\beta_o\gamma_r k_2 M^2\mu\bar{V}$$

$$- \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}k\beta_o M^2\mu\frac{\partial U}{\partial \bar{x}} + \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}k\beta_o\gamma_r M^2\mu\frac{\partial U}{\partial \bar{x}} - \frac{4}{3}i\frac{1}{R_o}k\beta_o M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$+ \frac{4}{3}i\frac{1}{R_o}k\beta_o\gamma_r M^2\mu\frac{\partial \bar{\bar{V}}}{\partial y}$$

$$\mathcal{L}_1(5,5) = i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\kappa + (-(\rho\bar{V}))\frac{\partial}{\partial y} + (\frac{1}{\gamma_r}\rho\bar{V})\frac{\partial}{\partial y} + (c_p\rho\bar{V})\frac{\partial}{\partial y}$$

$$- i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\kappa\frac{d\alpha}{d\bar{x}} + \frac{4}{3}\frac{1}{R_o^2}k_2^2 M^2\bar{\bar{V}}^2\frac{d\mu}{dT} - \frac{4}{3}\frac{1}{R_o^2}\gamma_r k_2^2 M^2\bar{V}^2\frac{d\mu}{dT} - \bar{V}\frac{\partial\rho}{\partial y}$$

$$+ \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial\rho}{\partial y} + \rho\bar{V}\frac{dc_p}{dT}\frac{\partial T}{\partial y} + \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o^2}k_2 M^2\bar{V}\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}} - \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o^2}\gamma_r k_2 M^2\bar{V}\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}$$

$$+ \frac{4}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}^2 - \frac{4}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}^2 - 2\frac{1}{k_1}\frac{1}{R_o^2}k_2 M^2 U\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial \bar{x}}$$

$$+ 2\frac{1}{k_1}\frac{1}{R_o^2}\gamma_r k_2 M^2 U\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial \bar{x}} + 2\frac{1}{k_1}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial \bar{x}} - 2\frac{1}{k_1}\frac{1}{R_o^2}\dot{\gamma}_r M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial\bar{\bar{V}}}{\partial \bar{x}}$$

$$+ (\frac{1}{k_1^2}M^2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial \bar{x}}^2)/R_o{}^4 - (\frac{1}{k_1^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial \bar{x}}^2)/R_o{}^4 - \frac{4}{3}\frac{1}{R_o^2}k_2 M^2\bar{V}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$+ \frac{4}{3}\frac{1}{R_o^2}\gamma_r k_2 M^2\bar{V}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y} - \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}\frac{\partial\bar{\bar{V}}}{\partial y} + \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}\frac{\partial\bar{\bar{V}}}{\partial y}$$

$$+ \frac{4}{3}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}^2 - \frac{4}{3}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial y}^2$$

$$\mathcal{L}_2(2,3) \;=\; (-(\frac{1}{k_1}\frac{1}{R_o}\mu))\frac{\partial}{\partial y} + (\frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\mu)\frac{\partial}{\partial y}$$

$$\mathcal{L}_2(3,2) \;=\; (-(\frac{1}{k_1}\frac{1}{R_o}\mu))\frac{\partial}{\partial y} + (\frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\mu)\frac{\partial}{\partial y}$$

$$\mathcal{L}_3(2,2) \;=\; -(\frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\mu)$$

$$\mathcal{L}_3(3,3) \;=\; -(i\frac{1}{k_1^2}\frac{1}{R_o}\mu)$$

$$\mathcal{L}_3(4,4) \;=\; -(i\frac{1}{k_1^2}\frac{1}{R_o}\mu)$$

$$\mathcal{L}_3(5,5) \;=\; -(i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\kappa)$$

$$
\begin{aligned}
\hat{\mathcal{N}}^{(\text{quad})}(1) \;=\;& \frac{1}{k_1}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)} + R_o\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}. \\
&+ i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} + \frac{1}{k_1}\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{u}_{(n_3,k_3)} \\
&+ i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + R_o\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)} \\
&+ k_2 R_o\bar{\rho}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + i\beta_o R_o k_2\tilde{\rho}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} \\
&+ i\beta_o R_o k_2\tilde{\rho}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathcal{N}}^{(\text{quad})}(2) \;=\;& -2\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} \\
&- \frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial y} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}} \\
&+ \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} + \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)} \\
&+ \frac{1}{k_1}U\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)} + \bar{\bar{V}}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)} \\
&- \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_2,k_2)} + \frac{2}{3}i\frac{1}{k_1}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_2,k_2)} \\
&- i\frac{1}{k_1}\frac{1}{R_o}\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}}k_2\tilde{T}_{(n_2,k_2)} + \frac{1}{\gamma_r}i\frac{1}{k_1}\frac{1}{M^2}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}
\end{aligned}
$$

27

$$-2\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}^2}\tilde{T}_{(n_3,k_3)} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}^2}\tilde{T}_{(n_3,k_3)}$$

$$-\frac{d\mu}{dT}\frac{\partial^2 \tilde{u}_{(n_2,k_2)}}{\partial y^2}\tilde{T}_{(n_3,k_3)} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2 \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+\frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2 \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}\partial y}\tilde{T}_{(n_3,k_3)} + \frac{1}{\gamma_r}\frac{1}{k_1}\frac{1}{M^2}\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+2\frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - \frac{2}{3}\frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$-\frac{8}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - 2\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+\frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - k_2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$-\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} - 3\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+\frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - \frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$-(i\frac{1}{k_1}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)})/3 + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$-(i\frac{1}{k_1}\frac{1}{R_o}\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}k_2\tilde{T}_{(n_3,k_3)})/3 + \frac{1}{\gamma_r}i\frac{1}{k_1}\frac{1}{M^2}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_2,k_2)}\tilde{T}_{(n_3,k_3)}$$

$$+i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}U\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} + i\beta_o R_o W k_2\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}$$

$$-i\omega_o R_o n_2\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} + \frac{4}{3}i\frac{1}{k_1^3}\frac{1}{R_o}k_4\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}$$

$$+2\frac{1}{k_1^2}\alpha_{(n_2,k_2)}^2\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} - \frac{2}{3}\frac{1}{k_1^2}\alpha_{(n_2,k_2)}^2\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}$$

$$-\frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}d[\alpha_{(n_2,k_2)},x2]\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} - \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}$$

$$+(\beta_o)^2\frac{d\mu}{dT}k_2^2\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} - \frac{4}{3}i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{u}_{(n_3,k_3)}$$

$$+k_2\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{u}_{(n_3,k_3)} + \frac{1}{k_1}\rho\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{u}_{(n_3,k_3)}$$

$$+k_2\bar{\bar{V}}\tilde{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + \frac{1}{k_1}\frac{\partial U}{\partial \bar{x}}\tilde{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+ k_2{}^2 \frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + 2\frac{1}{k_1^2}\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- \frac{2}{3}\frac{1}{k_1^2}\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + k_2\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+ (\beta_o)^2\frac{d\mu}{dT}k_2k_3\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\rho\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- frac73 i\frac{1}{k_1}k_2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} - i\frac{1}{k_1}\alpha_{(n_2,k_2)}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$- 2\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{v}_{(n_3,k_3)} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{v}_{(n_3,k_3)}$$

$$- i\frac{1}{k_1}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)} + R_o\rho\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)}$$

$$+ k_2 R_o U\tilde{\rho}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + R_o\frac{\partial U}{\partial y}\tilde{\rho}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$- 2\frac{1}{k_1^2}\frac{1}{R_o}k_3\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_3\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$+ 2\frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} - \frac{2}{3}\frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$- \frac{4}{3}i\frac{1}{k_1}k_2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} - 2\frac{1}{k_1}\frac{1}{R_o}k_2\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$+ \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + k_2 R_o\rho\tilde{u}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1}\beta_o\alpha_{(n_2,k_2)}\frac{d\mu}{dT}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} - \frac{2}{3}\frac{1}{k_1}\beta_o\alpha_{(n_2,k_2)}\frac{d\mu}{dT}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}$$

$$+ \frac{2}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} + \frac{2}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}k_3\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1}\beta_o\alpha_{(n_3,k_3)}\frac{d\mu}{dT}k_2\tilde{T}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} - \frac{2}{3}\frac{1}{k_1}\beta_o\alpha_{(n_2,k_2)}\frac{d\mu}{dT}k_3\tilde{T}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+ i\beta_o R_o\rho k_2\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$\hat{\mathcal{N}}^{(\text{quad})}(3) = \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial y}$$

$$- \frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}} - 2\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y}$$

29

$$+ \frac{2}{3}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} + \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1}U\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)} + \bar{\bar{V}}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}$$

$$- i\frac{1}{k_1}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_2,k_2)} - i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_2,k_2)}$$

$$- i\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial y}k_2\tilde{T}_{(n_2,k_2)} - \frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2 \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{d\mu}{dT}\frac{\partial^2 \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}\partial y}\tilde{T}_{(n_3,k_3)} - \frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial^2 \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}^2}\tilde{T}_{(n_3,k_3)}$$

$$- 2\frac{d\mu}{dT}\frac{\partial^2 \tilde{v}_{(n_2,k_2)}}{\partial y^2}\tilde{T}_{(n_3,k_3)} + \frac{2}{3}\frac{d\mu}{dT}\frac{\partial^2 \tilde{v}_{(n_2,k_2)}}{\partial y^2}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r}\frac{1}{M^2}R_o\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} + 3\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} + \frac{2}{3}\frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial \check{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- (i\frac{1}{k_1}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)})/3 - \frac{1}{k_1}\frac{1}{R_o}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1^3}\frac{1}{R_o^2}k_4\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - 2i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{1}{k_1^2}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - 2k_2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} - 2\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{2}{3}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} - (i\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial y}k_2\tilde{T}_{(n_3,k_3)})/3$$

$$+ frac73i\frac{1}{k_1}k_2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)} + \frac{2}{3}i\frac{1}{k_1}\alpha_{(n_2,k_2)}\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}$$

$$+ \frac{1}{k_1}\frac{1}{R_o}k_2\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{u}_{(n_3,k_3)} + \frac{2}{3}i\frac{1}{k_1}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{u}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1}\rho\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{u}_{(n_3,k_3)} - 2k_2R_oU\tilde{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+\frac{1}{k_1}\frac{1}{R_o}\frac{\partial \bar{\bar{V}}}{\partial \bar{x}}\tilde{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + \frac{1}{k_1^2}\frac{1}{R_o}k_3\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$-\frac{1}{k_1^2}\frac{1}{R_o}k_2k_4\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + i\frac{1}{k_1}k_2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+\frac{1}{k_1}\frac{1}{R_o}k_2\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} - k_2R_o\rho\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}U\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} + i\beta_oR_oWk_2\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$-i\omega_oR_on_2\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} + i\frac{1}{k_1^3}\frac{1}{R_o}k_4\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$+\frac{1}{k_1^2}\alpha_{(n_2,k_2)}{}^2\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} - i\frac{1}{k_1^2}\frac{1}{R_o}d[\alpha_{(n_2,k_2)},x2]\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$-i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{\partial T}{\partial \bar{x}}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} + (\beta_o)^2\frac{d\mu}{dT}k_2^2\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$+i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\rho\tilde{u}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} - i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{v}_{(n_3,k_3)}$$

$$+\frac{2}{3}k_2\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)} + R_o\rho\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)}$$

$$+\frac{\partial \bar{\bar{V}}}{\partial y}\tilde{\rho}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + 2k_2{}^2\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$-\frac{2}{3}k_2{}^2\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + \frac{1}{k_1^2}\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$+\frac{2}{3}k_2\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)} + (\beta_o)^2\frac{d\mu}{dT}k_2k_3\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$+\frac{2}{3}i\beta_o\frac{\partial T}{\partial y}\frac{d^2\mu}{dT^2}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} + \frac{2}{3}i\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}k_3\tilde{w}_{(n_3,k_3)}$$

$$+i\beta_oR_o\rho k_2\tilde{v}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$\hat{\mathcal{N}}^{(\text{quad})}(4) = -(\frac{1}{k_1^2}\frac{1}{R_o^2}\frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}}) - \frac{d\mu}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial y}$$

$$+\frac{1}{k_1}U\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)} + \bar{\bar{V}}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}$$

$$-i\frac{1}{k_1^2}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_2,k_2)} + \frac{2}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}}k_2\tilde{T}_{(n_2,k_2)}$$

31

$$- i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\kappa}{dT}\frac{\partial \tilde{T}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_2,k_2)} - i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}U\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}$$

$$+ \frac{1}{\gamma_r}i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}U\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)} + i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}c_pU\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}$$

$$- i\beta_oR_oWk_2\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)} + \frac{1}{\gamma_r}i\beta_oR_oWk_2\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}$$

$$+ i\beta_oR_oc_pWk_2\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)} + i\omega_oR_on_2\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}$$

$$- \frac{1}{\gamma_r}i\omega_oR_on_2\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)} - i\omega_oR_oc_pn_2\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}$$

$$- \frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o^2}\frac{d\kappa}{dT}\frac{\partial^2 \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}^2}\tilde{T}_{(n_3,k_3)} - \frac{1}{Pr}\frac{d\kappa}{dT}\frac{\partial^2 \tilde{T}_{(n_2,k_2)}}{\partial y^2}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{1}{k_1}U\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} + \frac{1}{\gamma_r}\frac{1}{k_1}U\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \bar{\bar{V}}\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} + \frac{1}{\gamma_r}\bar{\bar{V}}\frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1^3}\frac{1}{Pr}\frac{1}{R_o^2}k_4\frac{d\kappa}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - 2i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\alpha_{(n_2,k_2)}\frac{d\kappa}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- i\frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o}\alpha_{(n_3,k_3)}\frac{d\kappa}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} + \frac{1}{k_1}\rho U\frac{dc_p}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{1}{k_1^2}\frac{1}{Pr}\frac{1}{R_o^2}\frac{\partial T}{\partial \bar{x}}\frac{d^2\kappa}{dT^2}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - \frac{1}{Pr}k_2\frac{d\kappa}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \rho\bar{\bar{V}}\frac{dc_p}{dT}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} - \frac{1}{Pr}\frac{\partial T}{\partial y}\frac{d^2\kappa}{dT^2}\frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o^2}k_2M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - \frac{8}{3}\frac{1}{k_1}\frac{1}{R_o^2}\gamma_rk_2M^2\bar{\bar{V}}\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{8}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} - \frac{8}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_rM^2\frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)} + \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o^2}\gamma_rM^2\frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- 2k_2M^2U\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} + 2\gamma_rk_2M^2U\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ 2M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)} - 2\gamma_rM^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{1}{Pr}(\beta_o)^2 \frac{d\kappa}{dT} k_2 k_3 \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_3,k_3)} - i\omega_o R_o \rho \frac{dc_p}{dT} n_2 \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_3,k_3)}$$

$$+ \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_2,k_2)} \mu \frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} \tilde{u}_{(n_2,k_2)} - \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_2,k_2)} \mu \frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} \tilde{u}_{(n_2,k_2)}$$

$$- \frac{4}{3} i \frac{1}{k_1} M^2 \alpha_{(n_2,k_2)} \mu \frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} \tilde{u}_{(n_2,k_2)} + \frac{4}{3} i \frac{1}{k_1} \gamma_r M^2 \alpha_{(n_2,k_2)} \mu \frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} \tilde{u}_{(n_2,k_2)}$$

$$+ 2i \frac{1}{k_1} \frac{1}{R_o} \beta_o M^2 \mu \frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}} k_2 \tilde{u}_{(n_2,k_2)} - 2i \frac{1}{k_1} \frac{1}{R_o} \beta_o \gamma_r M^2 \mu \frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}} k_2 \tilde{u}_{(n_2,k_2)}$$

$$+ \frac{8}{3} i \frac{1}{k_1} \frac{1}{R_o} k_2 M^2 \alpha_{(n_2,k_2)} \bar{\bar{V}} \frac{d\mu}{dT} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} - \frac{8}{3} i \frac{1}{k_1} \frac{1}{R_o} \gamma_r k_2 M^2 \alpha_{(n_2,k_2)} \bar{\bar{V}} \frac{d\mu}{dT} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)}$$

$$+ \frac{8}{3} i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial U}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} - \frac{8}{3} i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial U}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)}$$

$$- \frac{4}{3} i \frac{1}{k_1} \frac{1}{R_o} M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} + \frac{4}{3} i \frac{1}{k_1} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)}$$

$$+ 2i \frac{1}{k_1} \frac{1}{R_o} \beta_o M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial \bar{x}} k_2 \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} - 2i \frac{1}{k_1} \frac{1}{R_o} \beta_o \gamma_r M^2 \frac{d\mu}{dT} \frac{\partial W}{\partial \bar{x}} k_2 \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)}$$

$$- \frac{1}{k_1} T \frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)} + \frac{1}{\gamma_r} \frac{1}{k_1} T \frac{\partial \tilde{\rho}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)}$$

$$- \frac{1}{k_1} \rho \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)} + \frac{1}{\gamma_r} \frac{1}{k_1} \rho \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1} c_p \rho \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)} + \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_3,k_3)} \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)}$$

$$- \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_3,k_3)} \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)} - 2k_2 M^2 \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y} \tilde{u}_{(n_3,k_3)}$$

$$+ 2\gamma_r k_2 M^2 \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y} \tilde{u}_{(n_3,k_3)} - 2 \frac{1}{k_1} \frac{1}{R_o} k_2 M^2 \mu \frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)}$$

$$+ 2 \frac{1}{k_1} \frac{1}{R_o} \gamma_r k_2 M^2 \mu \frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{u}_{(n_3,k_3)} - i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} T \tilde{\rho}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r} i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} T \tilde{\rho}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)} - \frac{1}{k_1} \frac{\partial T}{\partial \bar{x}} \tilde{\rho}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r} \frac{1}{k_1} \frac{\partial T}{\partial \bar{x}} \tilde{\rho}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)} + \frac{1}{k_1} c_p \frac{\partial T}{\partial \bar{x}} \tilde{\rho}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)}$$

$$- i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} \rho \tilde{T}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)} + \frac{1}{\gamma_r} i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} \rho \tilde{T}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)}$$

$$+ i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}c_p\rho\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + 2{k_2}^2 M^2 U\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- 2\gamma_r {k_2}^2 M^2 U\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} - \frac{1}{k_1}\frac{\partial\rho}{\partial\bar{x}}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r}\frac{1}{k_1}\frac{\partial\rho}{\partial\bar{x}}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + \frac{1}{k_1}\rho\frac{dc_p}{dT}\frac{\partial T}{\partial\bar{x}}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- 2k_2 M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + 2\gamma_r k_2 M^2\frac{d\mu}{dT}\frac{\partial U}{\partial y}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- 2\frac{1}{k_1}\frac{1}{R_o^2}k_2 M^2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + 2\frac{1}{k_1}\frac{1}{R_o^2}\gamma_r k_2 M^2\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+ {k_2}^2 M^2\mu\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} - \gamma_r {k_2}^2 M^2\mu\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- \frac{4}{3}\frac{1}{k_1^2}M^2\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\mu\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + \frac{4}{3}\frac{1}{k_1^2}\gamma_r M^2\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\mu\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$- (\beta_o)^2 M^2\mu k_2 k_3\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)} + (\beta_o)^2\gamma_r M^2 k_2 k_3\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}$$

$$+ i\frac{1}{k_1^2}\frac{1}{R_o}M^2\alpha_{(n_2,k_2)}\mu\frac{\partial\tilde{v}_{(n_3,k_3)}}{\partial\bar{x}}\tilde{v}_{(n_2,k_2)} - i\frac{1}{k_1^2}\frac{1}{R_o}\gamma_r M^2\alpha_{(n_2,k_2)}\mu\frac{\partial\tilde{v}_{(n_3,k_3)}}{\partial\bar{x}}\tilde{v}_{(n_2,k_2)}$$

$$+ 2i\beta_o M^2\mu\frac{\partial\tilde{w}_{(n_3,k_3)}}{\partial y}k_2\tilde{v}_{(n_2,k_2)} - 2i\beta_o\gamma_r M^2\mu\frac{\partial\tilde{w}_{(n_3,k_3)}}{\partial y}k_2\tilde{v}_{(n_2,k_2)}$$

$$- 2i\frac{1}{k_1}k_2 M^2\alpha_{(n_2,k_2)}U\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} + 2i\frac{1}{k_1}\gamma_r k_2 M^2\alpha_{(n_2,k_2)}U\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$+ 2i\frac{1}{k_1}M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial U}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} - 2i\frac{1}{k_1}\gamma_r M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial U}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$+ 2i\frac{1}{k_1^2}\frac{1}{R_o^2}M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} - 2i\frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial\bar{\bar{V}}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$+ 2i\beta_o M^2\frac{d\mu}{dT}\frac{\partial W}{\partial y}k_2\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} - 2i\beta_o\gamma_r M^2\frac{d\mu}{dT}\frac{\partial W}{\partial y}k_2\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$- 2i\frac{1}{k_1}k_2 M^2\alpha_{(n_2,k_2)}\mu\tilde{u}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)} + 2i\frac{1}{k_1}\gamma_r k_2 M^2\alpha_{(n_2,k_2)}\mu\tilde{u}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$- R_o T\frac{\partial\tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)} + \frac{1}{\gamma_r}R_o T\frac{\partial\tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)}$$

$$- R_o\rho\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)} + \frac{1}{\gamma_r}R_o\rho\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{v}_{(n_3,k_3)}$$

37

$$+ R_o c_p \rho \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y} \tilde{v}_{(n_3,k_3)} + \frac{8}{3} \frac{1}{k_1} \frac{1}{R_o} k_2 M^2 \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{v}_{(n_3,k_3)}$$

$$- \frac{8}{3} \frac{1}{k_1} \frac{1}{R_o} \gamma_r k_2 M^2 \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{v}_{(n_3,k_3)} + 2i \frac{1}{k_1} M^2 \alpha_{(n_3,k_3)} \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y} \tilde{v}_{(n_3,k_3)}$$

$$- 2i \frac{1}{k_1} \gamma_r M^2 \alpha_{(n_3,k_3)} \mu \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y} \tilde{v}_{(n_3,k_3)} + i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_3,k_3)} \mu \frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{v}_{(n_3,k_3)}$$

$$- i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_3,k_3)} \mu \frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{v}_{(n_3,k_3)} - \frac{4}{3} k_2 M^2 \mu \frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y} \tilde{v}_{(n_3,k_3)}$$

$$+ \frac{4}{3} \gamma_r k_2 M^2 \mu \frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y} \tilde{v}_{(n_3,k_3)} - R_o \frac{\partial T}{\partial y} \tilde{\rho}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r} R_o \frac{\partial T}{\partial y} \tilde{\rho}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} + R_o c_p \frac{\partial T}{\partial y} \tilde{\rho}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$+ \frac{8}{3} \frac{1}{R_o} k_2{}^2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} - \frac{8}{3} \frac{1}{R_o} \gamma_r k_2{}^2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$- R_o \frac{\partial \rho}{\partial y} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} + \frac{1}{\gamma_r} R_o \frac{\partial \rho}{\partial y} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$+ R_o \rho \frac{dc_p}{dT} \frac{\partial T}{\partial y} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} + \frac{8}{3} \frac{1}{k_1} \frac{1}{R_o} k_2 M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial \bar{x}} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$- \frac{8}{3} \frac{1}{k_1} \frac{1}{R_o} \gamma_r k_2 M^2 \frac{d\mu}{dT} \frac{\partial U}{\partial \bar{x}} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} - \frac{4}{3} \frac{1}{R_o} k_2 M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$+ \frac{4}{3} \frac{1}{R_o} \gamma_r k_2 M^2 \frac{d\mu}{dT} \frac{\partial \bar{\bar{V}}}{\partial y} \tilde{T}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} + \frac{8}{3} i \frac{1}{k_1} k_2 M^2 \alpha_{(n_2,k_2)} \mu \tilde{u}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$- \frac{8}{3} i \frac{1}{k_1} \gamma_r k_2 M^2 \alpha_{(n_2,k_2)} \mu \tilde{u}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} + \frac{4}{3} k_2{}^2 M^2 \mu \tilde{v}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$- \frac{4}{3} \gamma_r k_2{}^2 M^2 \mu \tilde{v}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} - \frac{1}{k_1^2} M^2 \alpha_{(n_2,k_2)} \alpha_{(n_3,k_3)} \mu \tilde{v}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1^2} \gamma_r M^2 \alpha_{(n_2,k_2)} \alpha_{(n_3,k_3)} \mu \tilde{v}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} - (\beta_o)^2 M^2 \mu k_2 k_3 \tilde{v}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)}$$

$$+ (\beta_o)^2 \gamma_r M^2 \mu k_2 k_3 \tilde{v}_{(n_2,k_2)} \tilde{v}_{(n_3,k_3)} + i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_2,k_2)} \mu \frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}} \tilde{w}_{(n_2,k_2)}$$

$$- i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_2,k_2)} \mu \frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}} \tilde{w}_{(n_2,k_2)} + 2i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial W}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_2,k_2)}$$

$$- 2i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial W}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_2,k_2)} - \frac{4}{3} i \frac{1}{R_o} \beta_o k_2 M^2 \bar{\bar{V}} \frac{d\mu}{dT} k_2 \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_2,k_2)}$$

38

$$+ \frac{4}{3}i\frac{1}{R_o}\beta_o\gamma_r k_2 M^2 \bar{\bar{V}}\frac{d\mu}{dT}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} - \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o M^2 \frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}$$

$$+ \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o\gamma_r M^2 \frac{d\mu}{dT}\frac{\partial U}{\partial \bar{x}}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} - \frac{4}{3}i\frac{1}{R_o}\beta_o M^2 \frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}$$

$$+ \frac{4}{3}i\frac{1}{R_o}\beta_o\gamma_r M^2 \frac{d\mu}{dT}\frac{\partial \bar{\bar{V}}}{\partial y}k_2\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} - \frac{4}{3}i\beta_o k_2 M^2 \mu k_2\tilde{v}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}$$

$$+ \frac{4}{3}i\beta_o\gamma_r k_2 M^2 \mu k_2\tilde{v}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)} + i\frac{1}{k_1^2}\frac{1}{R_o}M^2 \alpha_{(n_3,k_3)}\mu\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{w}_{(n_3,k_3)}$$

$$- i\frac{1}{k_1^2}\frac{1}{R_o}\gamma_r M^2 \alpha_{(n_3,k_3)}\mu\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{w}_{(n_3,k_3)} - \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o M^2 \mu\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}k_3\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o\gamma_r M^2 \mu\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}k_3\tilde{w}_{(n_3,k_3)} - \frac{4}{3}i\beta_o M^2 \mu\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}k_3\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{4}{3}i\beta_o\gamma_r M^2 \mu\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}k_3\tilde{w}_{(n_3,k_3)} - i\beta_o R_o T k_2\tilde{\rho}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r}i\beta_o R_o T k_2\tilde{\rho}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} - i\beta_o R_o\rho k_2\tilde{T}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{1}{\gamma_r}i\beta_o R_o\rho k_2\tilde{T}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} + i\beta_o R_o c_p\rho k_2\tilde{T}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$- 2\frac{1}{k_1}\beta_o M^2 \alpha_{(n_3,k_3)}\mu k_2\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} + 2\frac{1}{k_1}\beta_o\gamma_r M^2 \alpha_{(n_3,k_3)}\mu k_2\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{4}{3}\frac{1}{k_1}\beta_o M^2 \alpha_{(n_2,k_2)}\mu k_3\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} - \frac{4}{3}\frac{1}{k_1}\beta_o\gamma_r M^2 \alpha_{(n_2,k_2)}\mu k_3\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$- \frac{1}{k_1^2}M^2 \alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\mu\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} + \frac{1}{k_1^2}\gamma_r M^2 \alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\mu\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$- \frac{4}{3}(\beta_o)^2 M^2 \mu k_2 k_3\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} + \frac{4}{3}(\beta_o)^2\gamma_r M^2 \mu k_2 k_3\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)} \ .$$

$$\hat{\mathcal{N}}^{(\mathrm{cub})}(2) = \frac{1}{k_1}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)} + i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\bar{\rho}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}\tilde{u}_{(n_4,k_4)}$$

$$+ R_o\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)} + k_2 R_o\bar{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+ i\beta_o R_o k_2\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_4,k_4)}$$

39

$$\hat{\mathcal{N}}^{(\text{cub})}(3) = \frac{1}{k_1}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)} - k_2 R_o \tilde{\rho}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$+ i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}\tilde{v}_{(n_2,k_2)} + R_o\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+ i\beta_o R_o k_2 \tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}\tilde{w}_{(n_4,k_4)}$$

$$\hat{\mathcal{N}}^{(\text{cub})}(4) = \frac{1}{k_1}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)} + R_o\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+ i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}\tilde{w}_{(n_2,k_2)} + i\beta_o R_o k_2 \tilde{\rho}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_4,k_4)}$$

$$\hat{\mathcal{N}}^{(\text{cub})}(5) = \frac{4}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{4}{3}\frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+ M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_3,k_3)} - \gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{4}{3}M^2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_3,k_3)} - \frac{4}{3}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{v}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ \frac{1}{k_1^2}\frac{1}{R_o^2}M^2\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$- \frac{1}{k_1^2}\frac{1}{R_o^2}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_3,k_3)}$$

$$+ M^2\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_3,k_3)} - \gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{w}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_3,k_3)}$$

$$+ 2\frac{1}{k_1}\frac{1}{R_o}M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_4,k_4)}$$

$$- 2\frac{1}{k_1}\frac{1}{R_o}\gamma_r M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial y}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial \bar{x}}\tilde{T}_{(n_4,k_4)}$$

$$- \frac{4}{3}\frac{1}{k_1}\frac{1}{R_o}M^2\frac{d\mu}{dT}\frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}}\frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y}\tilde{T}_{(n_4,k_4)}$$

$$+ \frac{4}{3} \frac{1}{k_1} \frac{1}{R_o} \gamma_r M^2 \frac{d\mu}{dT} \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} \tilde{T}_{(n_4,k_4)}$$

$$+ \frac{1}{k_1} U \frac{dc_p}{dT} \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_4,k_4)} + \bar{\bar{V}} \frac{dc_p}{dT} \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_4,k_4)}$$

$$+ i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} U \frac{dc_p}{dT} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_4,k_4)} + i \beta_o R_o W \frac{dc_p}{dT} k_2 \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_4,k_4)}$$

$$- i \omega_o R_o \frac{dc_p}{dT} n_2 \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_4,k_4)}$$

$$+ \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)}$$

$$- \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial \tilde{u}_{(n_3,k_3)}}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)}$$

$$- \frac{4}{3} i \frac{1}{k_1} M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} \tilde{T}_{(n_4,k_4)} \tilde{u}_{(n_2,k_2)}$$

$$+ \frac{4}{3} i \frac{1}{k_1} \gamma_r M^2 \alpha_{(n_2,k_2)} \frac{d\mu}{dT} \frac{\partial \tilde{v}_{(n_3,k_3)}}{\partial y} \tilde{T}_{(n_4,k_4)} \tilde{u}_{(n_2,k_2)}$$

$$+ 2i \frac{1}{k_1} \frac{1}{R_o} \beta_o M^2 \frac{d\mu}{dT} \frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}} k_2 \tilde{T}_{(n_4,k_4)} \tilde{u}_{(n_2,k_2)}$$

$$- 2i \frac{1}{k_1} \frac{1}{R_o} \beta_o \gamma_r M^2 \frac{d\mu}{dT} \frac{\partial \tilde{w}_{(n_3,k_3)}}{\partial \bar{x}} k_2 \tilde{T}_{(n_4,k_4)} \tilde{u}_{(n_2,k_2)}$$

$$+ \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} M^2 \alpha_{(n_3,k_3)} \frac{d\mu}{dT} \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_3,k_3)}$$

$$- \frac{4}{3} i \frac{1}{k_1^2} \frac{1}{R_o} \gamma_r M^2 \alpha_{(n_3,k_3)} \frac{d\mu}{dT} \frac{\partial \tilde{u}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_3,k_3)}$$

$$- \frac{4}{3} \frac{1}{k_1^2} M^2 \alpha_{(n_2,k_2)} \alpha_{(n_3,k_3)} \frac{d\mu}{dT} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)}$$

$$+ \frac{4}{3} \frac{1}{k_1^2} \gamma_r M^2 \alpha_{(n_2,k_2)} \alpha_{(n_3,k_3)} \frac{d\mu}{dT} \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)} - (\beta_o)^2 M^2 \frac{d\mu}{dT} k_2 k_3 \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)}$$

$$+ (\beta_o)^2 \gamma_r M^2 \frac{d\mu}{dT} k_2 k_3 \tilde{T}_{(n_3,k_3)} \tilde{u}_{(n_2,k_2)} \tilde{u}_{(n_3,k_3)} - \frac{1}{k_1} \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{\rho}_{(n_3,k_3)} \tilde{u}_{(n_4,k_4)}$$

$$+ \frac{1}{\gamma_r} \frac{1}{k_1} \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{\rho}_{(n_3,k_3)} \tilde{u}_{(n_4,k_4)} + \frac{1}{k_1} c_p \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{\rho}_{(n_3,k_3)} \tilde{u}_{(n_4,k_4)}$$

$$- i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{u}_{(n_4,k_4)} + \frac{1}{\gamma_r} i \frac{1}{k_1} R_o \alpha_{(n_2,k_2)} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{u}_{(n_4,k_4)}$$

41

$$+\,i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}c_p\tilde{\rho}_{(n_3,k_3)}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_4,k_4)}-\frac{1}{k_1}\frac{\partial\tilde{\rho}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$+\,\frac{1}{\gamma_r}\frac{1}{k_1}\frac{\partial\tilde{\rho}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}+\frac{1}{k_1}\rho\frac{dc_p}{dT}\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$-\,2k_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}+2\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$-\,2\frac{1}{k_1}\frac{1}{R_o}k_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}+2\frac{1}{k_1}\frac{1}{R_o}\gamma_r k_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$-\,i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_2,k_2)}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}+\frac{1}{\gamma_r}i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\tilde{\rho}_{(n_2,k_2)}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$+\,\frac{1}{k_1}\frac{dc_p}{dT}\frac{\partial T}{\partial\bar{x}}\tilde{\rho}_{(n_2,k_2)}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}+i\frac{1}{k_1}R_o\alpha_{(n_2,k_2)}\rho\frac{dc_p}{dT}\tilde{T}_{(n_2,k_2)}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$+\,k_2{}^2M^2\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}-\gamma_r k_2{}^2M^2\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{u}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}$$

$$+\,i\frac{1}{k_1^2}\frac{1}{R_o}M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_3,k_3)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}-i\frac{1}{k_1^2}\frac{1}{R_o}\gamma_r M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_3,k_3)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}$$

$$+\,2i\beta_o M^2\frac{d\mu}{dT}\frac{\partial\tilde{w}_{(n_3,k_3)}}{\partial y}k_2\tilde{T}_{(n_4,k_4)}\tilde{v}_{(n_2,k_2)}-2i\beta_o\gamma_r M^2\frac{d\mu}{dT}\frac{\partial\tilde{w}_{(n_3,k_3)}}{\partial y}k_2\tilde{T}_{(n_4,k_4)}\tilde{v}_{(n_2,k_2)}$$

$$-\,2i\frac{1}{k_1}k_2M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}\tilde{v}_{(n_2,k_2)}+2i\frac{1}{k_1}\gamma_r k_2M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_4,k_4)}\tilde{v}_{(n_2,k_2)}$$

$$+\,i\frac{1}{k_1^2}\frac{1}{R_o}M^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_3,k_3)}-i\frac{1}{k_1^2}\frac{1}{R_o}\gamma_r M^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_3,k_3)}$$

$$+\,2i\frac{1}{k_1}M^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_4,k_4)}\tilde{v}_{(n_3,k_3)}-2i\frac{1}{k_1}\gamma_r M^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_4,k_4)}\tilde{v}_{(n_3,k_3)}$$

$$-\,\frac{1}{k_1^2}M^2\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$+\,\frac{1}{k_1^2}\gamma_r M^2\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$-\,(\beta_o)^2M^2\frac{d\mu}{dT}k_2k_3\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}+(\beta_o)^2\gamma_r M^2\frac{d\mu}{dT}k_2k_3\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}$$

$$-\,R_o\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}+\frac{1}{\gamma_r}R_o\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+\,R_o c_p\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{\rho}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}-R_o\frac{\partial\tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+\frac{1}{\gamma_r}R_o\frac{\partial\tilde{\rho}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}+R_o\rho\frac{dc_p}{dT}\frac{\partial\tilde{T}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+\frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}k_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}-\frac{8}{3}\frac{1}{k_1}\frac{1}{R_o}\gamma_rk_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$-\frac{4}{3}k_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}+\frac{4}{3}\gamma_rk_2M^2\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial y}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$+R_o\frac{dc_p}{dT}\frac{\partial T}{\partial y}\tilde{\rho}_{(n_2,k_2)}\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}+\frac{8}{3}i\frac{1}{k_1}k_2M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}\tilde{v}_{(n_4,k_4)}$$

$$-\frac{8}{3}i\frac{1}{k_1}\gamma_rk_2M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{u}_{(n_2,k_2)}\tilde{v}_{(n_4,k_4)}+\frac{4}{3}k_2{}^2M^2\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}$$

$$-\frac{4}{3}\gamma_rk_2{}^2M^2\frac{d\mu}{dT}\tilde{T}_{(n_2,k_2)}\tilde{v}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}+i\frac{1}{k_1^2}\frac{1}{R_o}M^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial\tilde{w}_{(n_3,k_3)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}$$

$$-i\frac{1}{k_1^2}\frac{1}{R_o}\gamma_rM^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}\frac{\partial\tilde{w}_{(n_3,k_3)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}-\frac{4}{3}i\beta_ok_2M^2\frac{d\mu}{dT}k_2\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}\tilde{w}_{(n_2,k_2)}$$

$$+\frac{4}{3}i\beta_o\gamma_rk_2M^2\frac{d\mu}{dT}k_2\tilde{T}_{(n_3,k_3)}\tilde{v}_{(n_4,k_4)}\tilde{w}_{(n_2,k_2)}+i\frac{1}{k_1^2}\frac{1}{R_o}M^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial\tilde{w}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_3,k_3)}$$

$$-i\frac{1}{k_1^2}\frac{1}{R_o}\gamma_rM^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\frac{\partial\tilde{w}_{(n_2,k_2)}}{\partial\bar{x}}\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_3,k_3)}$$

$$-\frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_oM^2\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial\bar{x}}k_3\tilde{T}_{(n_4,k_4)}\tilde{w}_{(n_3,k_3)}$$

$$+\frac{4}{3}i\frac{1}{k_1}\frac{1}{R_o}\beta_o\gamma_rM^2\frac{d\mu}{dT}\frac{\partial\tilde{u}_{(n_2,k_2)}}{\partial\bar{x}}k_3\tilde{T}_{(n_4,k_4)}\tilde{w}_{(n_3,k_3)}-\frac{4}{3}i\beta_oM^2\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial y}k_3\tilde{T}_{(n_4,k_4)}\tilde{w}_{(n_3,k_3)}$$

$$+\frac{4}{3}i\beta_o\gamma_rM^2\frac{d\mu}{dT}\frac{\partial\tilde{v}_{(n_2,k_2)}}{\partial y}k_3\tilde{T}_{(n_4,k_4)}\tilde{w}_{(n_3,k_3)}-2\frac{1}{k_1}\beta_oM^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}k_2\tilde{T}_{(n_4,k_4)}\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+2\frac{1}{k_1}\beta_o\gamma_rM^2\alpha_{(n_3,k_3)}\frac{d\mu}{dT}k_2\tilde{T}_{(n_4,k_4)}\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+\frac{4}{3}\frac{1}{k_1}\beta_oM^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}k_3\tilde{T}_{(n_4,k_4)}\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$-\frac{4}{3}\frac{1}{k_1}\beta_o\gamma_rM^2\alpha_{(n_2,k_2)}\frac{d\mu}{dT}k_3\tilde{T}_{(n_4,k_4)}\tilde{u}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$-\frac{1}{k_1^2}M^2\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+\frac{1}{k_1^2}\gamma_rM^2\alpha_{(n_2,k_2)}\alpha_{(n_3,k_3)}\frac{d\mu}{dT}\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}-\frac{4}{3}(\beta_o)^2M^2\frac{d\mu}{dT}k_2k_3\tilde{T}_{(n_3,k_3)}\tilde{w}_{(n_2,k_2)}\tilde{w}_{(n_3,k_3)}$$

$$+ \frac{4}{3}(\beta_o)^2 \gamma_r M^2 \frac{d\mu}{dT} k_2 k_3 \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_2,k_2)} \tilde{w}_{(n_3,k_3)} - i\beta_o R_o k_2 \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{w}_{(n_4,k_4)}$$

$$+ \frac{1}{\gamma_r} i\beta_o R_o k_2 \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{w}_{(n_4,k_4)} + i\beta_o R_o c_p k_2 \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{w}_{(n_4,k_4)}$$

$$- i\beta_o R_o k_2 \tilde{\rho}_{(n_2,k_2)} \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_4,k_4)} + \frac{1}{\gamma_r} i\beta_o R_o k_2 \tilde{\rho}_{(n_2,k_2)} \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_4,k_4)}$$

$$+ i\beta_o R_o \rho \frac{dc_p}{dT} k_2 \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_3,k_3)} \tilde{w}_{(n_4,k_4)}$$

$$
\begin{aligned}
\hat{\mathcal{N}}^{(\text{quart})}(5) &= \frac{1}{k_1} \frac{dc_p}{dT} \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial \bar{x}} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_4,k_4)} \tilde{u}_{(n_5,k_5)} \\
&\quad + i\frac{1}{k_1} R_o \alpha_{(n_2,k_2)} \frac{dc_p}{dT} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_4,k_4)} \tilde{u}_{(n_5,k_5)} \\
&\quad + R_o \frac{dc_p}{dT} \frac{\partial \tilde{T}_{(n_2,k_2)}}{\partial y} \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_4,k_4)} \tilde{v}_{(n_5,k_5)} \\
&\quad + i\beta_o R_o \frac{dc_p}{dT} k_2 \tilde{\rho}_{(n_3,k_3)} \tilde{T}_{(n_2,k_2)} \tilde{T}_{(n_4,k_4)} \tilde{w}_{(n_5,k_5)}
\end{aligned}
$$