FM 9864

Cox: Electroviscous forces on a charged particle suspended in a flowing liquid. Appendix B

and

$$\bar{p}_{E0} = \frac{8\lambda A_p^2 e^{-2\bar{z}}}{(1 - A_p^2 e^{-2\bar{z}})^2} \,, \qquad \text{(A4a)}$$

$$\bar{p}_{E1} = \frac{4\lambda\alpha_p A_p^2 e^{-2\bar{z}}\left\{2\bar{z} + A_p^2 (e^{-2\bar{z}} - 1)\right\}(1 + A_p^2 e^{-2\bar{z}})}{(1 - A_p^2 e^{-2\bar{z}})^3} \,, \qquad \text{(A4b)}$$

$$\frac{\partial \bar{p}_{E0}}{\partial \bar{x}} = \frac{\partial \bar{p}_{E0}}{\partial \bar{y}} = 0 \,, \qquad \text{(A4c)}$$

$$\left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}\right) \bar{p}_{E0} = + \alpha_p \frac{16\lambda A_p^2 e^{-2\bar{z}}(1 + A_p^2 e^{-2\bar{z}})}{(1 - A_p^2 e^{-2\bar{z}})^3} \,. \qquad \text{(A4d)}$$

APPENDIX B

On the $\bar{z}$-axis ($\bar{x} = \bar{y} = 0$) in the inner region (at a point Q), the values of $\bar{v}_H$, $\bar{p}_H$ and their $\bar{x}$- and $\bar{y}$-derivatives as expansions in ϵ for the purely hydrodynamic problem considered in §5, are obtained as

$$\bar{v}_{Hx} = \epsilon \left\{ \left(\frac{\partial \tilde{v}_{Hx}}{\partial \tilde{z}}\bigg|_Q - \check{\Omega}_y \right) \bar{z} \right\} + \epsilon^2 \left\{ \frac{1}{2} \frac{\partial^2 \tilde{v}_{Hx}}{\partial \tilde{z}^2}\bigg|_Q \bar{z}^2 \right\} + \ldots \,,$$

$$\frac{\partial \bar{v}_{Hx}}{\partial \bar{x}} = \epsilon^{3/2} \left\{ \left. \frac{\partial^2 \tilde{v}_{Hx}}{\partial \tilde{x} \partial \tilde{z}} \right|_Q \bar{z} \right\} + \epsilon^{5/2} \left\{ \frac{1}{2} \left. \frac{\partial^3 \tilde{v}_{Hx}}{\partial \tilde{x} \partial \tilde{z}^2} \right|_Q \bar{z}^2 \right\} + \ldots$$

$$\frac{\partial \bar{v}_{Hx}}{\partial \bar{y}} = \epsilon^{3/2} \left\{ \left. \frac{\partial^2 \tilde{v}_{Hx}}{\partial \tilde{y} \partial \tilde{z}} \right|_Q \bar{z} \right\} + \epsilon^{5/2} \left\{ \frac{1}{2} \left. \frac{\partial^3 \tilde{v}_{Hx}}{\partial \tilde{y} \partial \tilde{z}^2} \right|_Q \bar{z}^2 \right\} + \ldots$$

$$\frac{\partial^2 \bar{v}_{Hx}}{\partial \bar{x}^2} = \epsilon \left\{ \left. \frac{\partial^2 \tilde{v}_{Hx}}{\partial \tilde{x}^2} \right|_Q \right\} + \ldots = \epsilon \left\{ -\, 2a_{11} \left(\left. \frac{\partial \tilde{v}_{Hx}}{\partial \tilde{z}} \right|_Q - \tilde{\Omega}_y \right) \right\} + \ldots$$

$$\frac{\partial^2 \bar{v}_{Hx}}{\partial \bar{y}^2} = \epsilon \left\{ \left. \frac{\partial^2 \tilde{v}_{Hx}}{\partial \tilde{y}^2} \right|_Q \right\} + \ldots = \epsilon \left\{ -\, 2a_{22} \left(\left. \frac{\partial \tilde{v}_{Hx}}{\partial \tilde{z}} \right|_Q - \tilde{\Omega}_y \right) \right\} + \ldots$$

$$\frac{\partial^2 \bar{v}_{Hx}}{\partial \bar{x} \partial \bar{y}} = \epsilon \left\{ \left. \frac{\partial^2 \tilde{v}_{Hx}}{\partial \tilde{x} \partial \tilde{y}} \right|_Q \right\} + \ldots = \epsilon \left\{ -\, 2a_{12} \left(\left. \frac{\partial \tilde{v}_{Hx}}{\partial \tilde{z}} \right|_Q - \tilde{\Omega}_y \right) \right\} + \ldots$$

$$\bar{v}_{Hy} = \epsilon \left\{ \left(\left. \frac{\partial \tilde{v}_{Hy}}{\partial \tilde{z}} \right|_Q + \tilde{\Omega}_x \right) \tilde{z} \right\} + \epsilon^2 \left\{ \frac{1}{2} \left. \frac{\partial^2 \tilde{v}_{Hy}}{\partial \tilde{z}^2} \right|_Q \bar{z}^2 \right\} + \ldots$$

$$\frac{\partial \overline{v}_{Hy}}{\partial \overline{x}} = \epsilon^{3/2} \left\{ \left.\frac{\partial^2 \tilde{v}_{Hy}}{\partial \tilde{x} \partial \tilde{z}}\right|_Q \overline{z} \right\} + \epsilon^{5/2} \left\{ \frac{1}{2} \left.\frac{\partial^3 v_{Hy}}{\partial \tilde{x} \partial \tilde{z}^2}\right|_Q \overline{z}^2 \right\} + \ldots$$

$$\frac{\partial \overline{v}_{Hy}}{\partial \overline{y}} = \epsilon^{3/2} \left\{ \left.\frac{\partial^2 \tilde{v}_{Hy}}{\partial \tilde{y} \partial \tilde{z}}\right|_Q \overline{z} \right\} + \epsilon^{5/2} \left\{ \frac{1}{2} \left.\frac{\partial^3 v_{Hy}}{\partial \tilde{y} \partial \tilde{z}^2}\right|_Q \overline{z}^2 \right\} + \ldots$$

$$\frac{\partial^2 \overline{v}_{Hy}}{\partial \overline{x}^2} = \epsilon \left[\left.\frac{\partial \tilde{v}_{Hy}}{\partial \tilde{x}^2}\right|_Q \right\} + \ldots = \epsilon \left\{ -2a_{11} \left(\left.\frac{\partial \tilde{v}_{Hy}}{\partial \tilde{z}}\right|_Q + \tilde{\Omega}_x \right) \right\} + \ldots$$

$$\frac{\partial^2 \overline{v}_{Hy}}{\partial \overline{y}^2} = \epsilon \left[\left.\frac{\partial \tilde{v}_{Hy}}{\partial \tilde{y}^2}\right|_Q \right\} + \ldots = \epsilon \left\{ -2a_{22} \left(\left.\frac{\partial \tilde{v}_{Hy}}{\partial \tilde{z}}\right|_Q + \tilde{\Omega}_x \right) \right\} + \ldots$$

$$\frac{\partial^2 \overline{v}_{Hy}}{\partial \overline{x} \partial \overline{y}} = \epsilon \left[\left.\frac{\partial \tilde{v}_{Hy}}{\partial \tilde{x} \partial \tilde{y}}\right|_Q \right\} + \ldots = \epsilon \left\{ -2a_{12} \left(\left.\frac{\partial \tilde{v}_{Hy}}{\partial \tilde{z}}\right|_Q + \tilde{\Omega}_x \right) \right\} + \ldots$$

$$\overline{v}_{Hz} = \epsilon^2 \left\{ \frac{1}{2} \left.\frac{\partial^2 \tilde{v}_{Hz}}{\partial \tilde{z}^2}\right|_Q \overline{z}^2 \right\} + \epsilon^3 \left\{ \frac{1}{6} \left.\frac{\partial^3 \tilde{v}_{Hz}}{\partial \tilde{z}^3}\right|_Q \overline{z}^3 \right\} + \ldots$$

$$\frac{\partial \bar{v}_{Hz}}{\partial \bar{x}} = \epsilon^{3/2}\left\{\left.\frac{\partial^2 \tilde{v}_{Hz}}{\partial\tilde{x}\partial\tilde{z}}\right|_Q \bar{z}\right\} + \epsilon^{5/2}\left\{\frac{1}{2}\left.\frac{\partial^3\tilde{v}_{Hz}}{\partial\tilde{x}\partial\tilde{z}^2}\right|_Q \bar{z}^2\right\} + \ldots$$

$$= \epsilon^{3/2}\left\{\left[-2a_{11}\left(\left.\frac{\partial\tilde{v}_{Hx}}{\partial\tilde{z}}\right|_Q - \tilde{\Omega}_y\right) - 2a_{12}\left(\left.\frac{\partial\tilde{v}_{Hy}}{\partial\tilde{z}}\right|_Q + \tilde{\Omega}_x\right)\right]\bar{z}\right\} + \epsilon^{5/2}\left\{\frac{1}{2}\left.\frac{\partial^3\tilde{v}_{Hz}}{\partial\tilde{x}\partial\tilde{z}^2}\right|_Q \bar{z}^2\right\} + \ldots$$

$$\frac{\partial \bar{v}_{Hz}}{\partial \bar{y}} = \epsilon^{3/2}\left\{\left.\frac{\partial^2 \tilde{v}_{Hz}}{\partial\tilde{y}\partial\tilde{z}}\right|_Q \bar{z}\right\} + \epsilon^{5/2}\left\{\frac{1}{2}\left.\frac{\partial^3\tilde{v}_{Hz}}{\partial\tilde{y}\partial\tilde{z}^2}\right|_Q \bar{z}^2\right\} + \ldots$$

$$= \epsilon^{3/2}\left\{\left[-2a_{12}\left(\left.\frac{\partial\tilde{v}_{Hx}}{\partial\tilde{z}}\right|_Q - \Omega_y\right) - 2a_{22}\left(\left.\frac{\partial\tilde{v}_{Hy}}{\partial\tilde{z}}\right|_Q + \tilde{\Omega}_x\right)\right]\bar{z}\right\} + \epsilon^{5/2}\left\{\frac{1}{2}\left.\frac{\partial^3\tilde{v}_{Hz}}{\partial\tilde{y}\partial\tilde{z}^2}\right|_Q \bar{z}^2\right\} + \ldots$$

$$\frac{\partial^2 \bar{v}_{Hz}}{\partial \bar{x}^2} = \epsilon\left\{\left.\frac{\partial^2\tilde{v}_{Hz}}{\partial\tilde{x}^2}\right|_Q\right\} + \ldots = \epsilon\left\{-2a_{11}\left.\frac{\partial\tilde{v}_{Hz}}{\partial\tilde{z}}\right|_Q\right\} + \ldots = 0 + O(\epsilon^2)$$

$$\frac{\partial^2 \bar{v}_{Hz}}{\partial \bar{y}^2} = \epsilon\left\{\left.\frac{\partial^2\tilde{v}_{Hz}}{\partial\tilde{y}^2}\right|_Q\right\} + \ldots = \epsilon\left\{-2a_{22}\left.\frac{\partial\tilde{v}_{Hz}}{\partial\tilde{z}}\right|_Q\right\} + \ldots = 0 + O(\epsilon^2)$$

$$\frac{\partial^2 \bar{v}_{Hz}}{\partial \bar{x}\partial\bar{y}} = \epsilon\left\{\left.\frac{\partial^2\tilde{v}_{Hz}}{\partial\bar{x}\partial\tilde{y}}\right|_Q\right\} + \ldots = \epsilon\left\{-2a_{12}\left.\frac{\partial\tilde{v}_{Hz}}{\partial\tilde{z}}\right|_Q\right\} + \ldots = 0 + O(\epsilon^2)$$

$$\overline{p}_H = \left\{\tilde{p}_H\big|_Q\right\} + \epsilon \left\{\frac{\partial \tilde{p}_H}{\partial \tilde{z}}\bigg|_Q \overline{z}\right\} + \ldots$$

$$\frac{\partial \overline{p}_H}{\partial \overline{x}} = \epsilon^{1/2} \left\{\frac{\partial \tilde{p}_H}{\partial \tilde{x}}\bigg|_Q\right\} + \epsilon^{3/2} \left\{\frac{\partial^2 \tilde{p}_H}{\partial \tilde{x} \partial \tilde{z}}\bigg|_Q \overline{z}\right\} + \ldots$$

$$\frac{\partial \overline{p}_H}{\partial \overline{y}} = \epsilon^{1/2} \left\{\frac{\partial \tilde{p}_H}{\partial \tilde{y}}\bigg|_Q\right\} + \epsilon^{3/2} \left\{\frac{\partial^2 \tilde{p}_H}{\partial \tilde{y} \partial \tilde{z}}\bigg|_Q \overline{z}\right\} + \ldots$$

$$\frac{\partial^2 \overline{p}_H}{\partial \overline{x}^2} = \epsilon \left\{\frac{\partial^2 \tilde{p}_H}{\partial \tilde{x}^2}\bigg|_Q\right\} + \ldots$$

$$\frac{\partial^2 \overline{p}_H}{\partial \overline{y}^2} = \epsilon \left\{\frac{\partial^2 \tilde{p}_H}{\partial \tilde{y}^2}\bigg|_Q\right\} + \ldots$$

$$\frac{\partial^2 \overline{p}_H}{\partial \overline{x} \partial \overline{y}} = \epsilon \left\{\frac{\partial^2 \tilde{p}_H}{\partial \tilde{x} \partial \tilde{y}}\bigg|_Q\right\} + \ldots$$