WATERHOUSE 281, pp 313+318

Moiseyev Solution - when things break down ($h \simeq h_0$)

Section 1
Equations expanded in powers of ε

Near $h = h_0 = 1.058092234$ ($H(h_0) = 0$) write

$$h = h_0 + \varepsilon^{2/5} h_1 \qquad h_1 \sim O(1)$$

and $\lambda = \delta \varepsilon^{-4/5}$ $\quad \lambda \sim O(1)$ $\Big\}$ ⊛ and leading order response is $O(\varepsilon^{-4/5})$

Recall 2-d model:

$$\nabla^2 \phi = 0 \qquad (1)$$

$$\phi_x = \sin t \qquad x = -\varepsilon \cos t,\ \pi - \varepsilon \cos t \qquad (2)$$

$$\phi_z = 0 \qquad z = -h \qquad (3)$$

$z = \varepsilon\eta$
$$\phi_z = \eta_t + \varepsilon \phi_x \eta_x \qquad (4)$$

$$\eta + (1+\delta)\tanh h \left[\phi_t + \tfrac{1}{2}\varepsilon(\phi_x^2 + \phi_z^2)\right] = 0 \qquad (5)$$

$\nabla\phi$ 2π periodic (η zero mean) (6)

Correct expansions are then

$$\phi = \varepsilon^{-4/5}\phi_0 + \varepsilon^{-3/5}\phi_1 + \varepsilon^{-2/5}\phi_2 + \varepsilon^{-1/5}\phi_3 + \phi_4 + \cdots$$

$$\eta = \varepsilon^{-4/5}\eta_0 + \varepsilon^{-3/5}\eta_1 + \varepsilon^{-2/5}\eta_2 + \varepsilon^{-1/5}\eta_3 + \eta_4 + \cdots$$

Substitute ~~these~~, together with ⊛, into (1) - (6), correct to $O(1)$ terms.

①: $\varepsilon^{-4/5}\nabla^2\phi_0 + \varepsilon^{-3/5}\nabla^2\phi_1 + \varepsilon^{-2/5}\nabla^2\phi_2 + \varepsilon^{-1/5}\nabla^2\phi_3 + \nabla^2\phi_4 = 0$

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②: $\varepsilon^{-4/5}\phi_{0x} + \varepsilon^{-3/5}\phi_{1x} + \varepsilon^{-2/5}\phi_{2x} + \varepsilon^{-1/5}\phi_{3x} + \phi_{4x} = \sin t \qquad x = 0, \pi$

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③: $\varepsilon^{-4/5}\phi_{0z} + \varepsilon^{-3/5}\phi_{1z} + \varepsilon^{-2/5}(\phi_{2z} - h_1\phi_{0zz}) + \varepsilon^{-1/5}(\phi_{3z} - h_1\phi_{1zz})$

$+ \phi_{4z} + \frac{h_1^2}{2}\phi_{0zzz} - h_1\phi_{2zz} = 0 \qquad z = -h_0$

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④: $\varepsilon^{-4/5}\phi_{0z} + \varepsilon^{-3/5}\eta_0\phi_{0zz} + \varepsilon^{-2/5}\eta_1\phi_{0zz} + \varepsilon^{-1/5}\eta_2\phi_{0zz} + \eta_3\phi_{0zz}$

$+ \varepsilon^{-2/5}\frac{\eta_0^2}{2}\phi_{0zzz} + \varepsilon^{-1/5}\eta_0\eta_1\phi_{0zzz} + \frac{1}{2}(2\eta_0\eta_1 + \eta_1^2)\phi_{0zzz}$

$+ \frac{1}{6}\varepsilon^{-1/5}\eta_0^3\phi_{0zzzz} + \frac{1}{2}\eta_0^2\eta_1\phi_{0zzzz} + \frac{1}{24}\eta_0^4\phi_{0zzzzz} + \varepsilon^{-3/5}\phi_{1z}$

$+ \varepsilon^{-2/5}\eta_0\phi_{1zz} + \varepsilon^{-1/5}\eta_1\phi_{1zz} + \eta_2\phi_{1zz} + \varepsilon^{-1/5}\frac{\eta_0^2}{2}\phi_{1zzz} + \eta_0\eta_1\phi_{1zzz}$

$+ \frac{\eta_0^3}{6}\phi_{1zzzz} + \varepsilon^{-2/5}\phi_{2z} + \varepsilon^{-1/5}\eta_0\phi_{2zz} + \eta_1\phi_{2zz} + \frac{\eta_0^2}{2}\phi_{2zzz}$

$+ \varepsilon^{-1/5}\phi_{3z} + \eta_0\phi_{3zz} + \phi_{4z} = \varepsilon^{-4/5}\eta_{0t} + \varepsilon^{-3/5}\eta_{1t} + \varepsilon^{-2/5}\eta_{2t} + \varepsilon^{-1/5}\eta_{3t}$

$+ \eta_{4t} + \varepsilon^{-3/5}\phi_{0x}\eta_{0x} + (\varepsilon^{-2/5}\eta_0 + \varepsilon^{-1/5}\eta_1 + \eta_2)\phi_{0xz}\eta_{0x} + (\varepsilon^{-1/5}\frac{\eta_0^2}{2} + \eta_0\eta_1)\phi_{0xzz}\eta_{0x}$

$+ \frac{\eta_0^3}{6}\phi_{0xzzz}\eta_{0x} + \varepsilon^{-2/5}\phi_{0x}\eta_{1x} + \varepsilon^{-2/5}\phi_{1x}\eta_{0x} + (\varepsilon^{-1/5}\eta_0 + \eta_1)(\phi_{0xz}\eta_{1x} + \phi_{1xz}\eta_{0x})$

$+ \frac{\eta_0^2}{2}(\phi_{0xzz}\eta_{1x} + \phi_{1xzz}\eta_{0x}) + \varepsilon^{-1/5}(\phi_{0x}\eta_{2x} + \phi_{2x}\eta_{0x} + \phi_{1x}\eta_{1x}) + \eta_0\phi_{0xz}\eta_{2x}$

$+ \eta_0\phi_{2xz}\eta_{0x} + \eta_0\phi_{1xz}\eta_{1x} + \phi_{3x}\eta_{0x} + \phi_{0x}\eta_{3x} + \phi_{1x}\eta_{2x} + \phi_{2x}\eta_{1x} \qquad z = 0.$

⑤: First note,

$$\tanh h = \tanh(h_0 + \varepsilon^{2/5} h_1) = \tanh h_0 + \varepsilon^{2/5} h_1 (1 - \tanh^2 h_0)$$
$$+ \varepsilon^{4/5} h_1^2 \tanh h_0 (\tanh^2 h_0 - 1)$$

So that,

$$\varepsilon^{-4/5}\eta_0 + \varepsilon^{-3/5}\eta_1 + \varepsilon^{-2/5}\eta_2 + \varepsilon^{-1/5}\eta_3 + \eta_4 + \tanh h_0 \Big\{ \varepsilon^{-4/5}\phi_{0t} + \varepsilon^{-3/5}\eta_0\phi_{0tz}$$
$$+ \varepsilon^{-2/5}\eta_1\phi_{0tz} + \varepsilon^{-1/5}\eta_2\phi_{0tz} + \eta_3\phi_{0tz} + \varepsilon^{-2/5}\frac{\eta_0^2}{2}\phi_{0tzz} + \varepsilon^{-1/5}\eta_0\eta_1\phi_{0tzz}$$
$$+ \eta_0\eta_2\phi_{0tzz} + \frac{1}{2}\eta_1^2\phi_{0tzz} + \frac{1}{6}\varepsilon^{-1/5}\eta_0^3\phi_{0tzzz} + \frac{1}{2}\eta_0^2\eta_1\phi_{0tzzz}$$
$$+ \frac{1}{24}\eta_0^4\phi_{0tzzzz} + \varepsilon^{-3/5}\phi_{1t} + \varepsilon^{-2/5}\eta_0\phi_{1tz} + \varepsilon^{-1/5}\eta_1\phi_{1tz} + \eta_2\phi_{1tz}$$
$$+ \varepsilon^{-1/5}\frac{\eta_0^2}{2}\phi_{1tzz} + \eta_0\eta_1\phi_{1tzz} + \frac{\eta_0^3}{6}\phi_{1tzzz} + \varepsilon^{-2/5}\phi_{2t} + \varepsilon^{-1/5}\eta_0\phi_{2tz}$$
$$+ \eta_1\phi_{2tz} + \frac{1}{2}\eta_0^2\phi_{1tzz} + \varepsilon^{-1/5}\phi_{3t} + \eta_0\phi_{3tz} + \phi_{4t} + \frac{1}{2}\varepsilon^{-3/5}\phi_{0x}^2$$
$$+ \varepsilon^{-2/5}\eta_0\phi_{0x}\phi_{0xz} + \varepsilon^{-1/5}\eta_1\phi_{0x}\phi_{0xz} + \eta_2\phi_{0x}\phi_{0xz} + \frac{1}{2}\varepsilon^{-1/5}\eta_0^2\phi_{0xz}^2$$
$$+ \frac{1}{2}\varepsilon^{-1/5}\eta_0^2\phi_{0x}\phi_{0xzz} + \eta_0\eta_1\phi_{0xz}^2 + \eta_0\eta_1\phi_{0x}\phi_{0xzz} + \frac{1}{2}\eta_0^3\phi_{0xz}\phi_{0xzz}$$
$$+ \frac{1}{6}\eta_0^3\phi_{0x}\phi_{0xzzz} + \varepsilon^{-2/5}\phi_{0x}\phi_{1x} + \varepsilon^{-1/5}\eta_0\phi_{0xz}\phi_{1x} + \varepsilon^{-1/5}\eta_0\phi_{0x}\phi_{1xz}$$
$$+ \eta_1\phi_{0xz}\phi_{1x} + \eta_1\phi_{0x}\phi_{1xz} + \frac{\eta_0^2}{2}\phi_{0xzz}\phi_{1x} + \eta_0^2\phi_{0xz}\phi_{1xz} + \frac{\eta_0^2}{2}\phi_{0x}\phi_{1xzz}$$
$$+ \frac{1}{2}\varepsilon^{-1/5}\phi_{1x}^2 + \varepsilon^{-1/5}\phi_{0x}\phi_{2x} + \eta_0\phi_{1x}\phi_{1xz} + \eta_0\phi_{0xz}\phi_{2x} + \eta_0\phi_{0x}\phi_{2xz}$$
$$+ \phi_{0x}\phi_{3x} + \phi_{1x}\phi_{2x} + \frac{1}{2}\varepsilon^{-3/5}\phi_{0z}^2 + \varepsilon^{-2/5}\eta_0\phi_{0z}\phi_{0zz} + \varepsilon^{-1/5}\eta_1\phi_{0z}\phi_{0zz}$$
$$+ \eta_2\phi_{0z}\phi_{0zz} + \frac{1}{2}\varepsilon^{-1/5}\eta_0^2\phi_{0zz}^2 + \frac{1}{2}\varepsilon^{-1/5}\eta_0^2\phi_{0z}\phi_{0zzz} + \eta_0\eta_1\phi_{0zz}^2$$
$$+ \eta_0\eta_1\phi_{0z}\phi_{0zzz} + \frac{\eta_0^3}{2}\phi_{0zz}\phi_{0zzz} + \frac{\eta_0^3}{6}\phi_{0z}\phi_{0zzzz} + \varepsilon^{-2/5}\phi_{0z}\phi_{1z} +$$

$$+ \varepsilon^{-1/5}\eta_0 \phi_{0zz}\phi_{1z} + \varepsilon^{-1/5}\eta_0\phi_{0z}\phi_{1zz} + \eta_1\phi_{0zz}\phi_{1z} + \eta_1\phi_{0z}\phi_{1zz} + \frac{1}{2}\eta_0^2\phi_{0zzz}\phi_{1z}$$

$$+ \eta_0^2\phi_{0zz}\phi_{1zz} + \frac{1}{2}\eta_0^2\phi_{0z}\phi_{1zzz} + \frac{1}{2}\varepsilon^{-1/5}\phi_{1z}^2 + \varepsilon^{-1/5}\phi_{0z}\phi_{2z} + \eta_0\phi_{1z}\phi_{1zz}$$

$$+ \eta_0\phi_{0zz}\phi_{2z} + \eta_0\phi_{0z}\phi_{2zz} + \phi_{0z}\phi_{3z} + \phi_{1z}\phi_{2z}\Big\}$$

$$+ h_1(1-\tanh^2 h_0)\Big\{\varepsilon^{-2/5}\phi_{0t} + \varepsilon^{-1/5}\eta_0\phi_{0tz} + \eta_1\phi_{0tz} + \frac{\eta_0^2}{2}\phi_{0tzz} + \varepsilon^{-1/5}\phi_{1t}$$

$$+ \eta_0\phi_{1tz} + \phi_{2t} + \frac{1}{2}\varepsilon^{-1/5}\phi_{0x}^2 + \eta_0\phi_{0x}\phi_{0xz} + \phi_{0x}\phi_{1x} + \frac{1}{2}\varepsilon^{-1/5}\phi_{0z}^2$$

$$+ \eta_0\phi_{0z}\phi_{0zz} + \phi_{0z}\phi_{1z}\Big\}$$

$$-\left[h_1^2\tanh h_0(\tanh^2 h_0 - 1) + \lambda\tanh h_0\right]\phi_{0t} = 0 \qquad z=0$$

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⑥ $\nabla\phi_i$, 2π periodic

η_i zero mean $\qquad \forall i = 0, \dots 4.$

[Useful bits & bobs :

$$\varepsilon\eta = \varepsilon^{1/5}\eta_0 + \varepsilon^{2/5}\eta_1 + \varepsilon^{3/5}\eta_2 + \varepsilon^{4/5}\eta_3$$

$$(\varepsilon\eta)^2 = \varepsilon^{2/5}\eta_0^2 + 2\varepsilon^{3/5}\eta_0\eta_1 + \varepsilon^{4/5}(\eta_1^2 + 2\eta_0\eta_2)$$

$$(\varepsilon\eta)^3 = \varepsilon^{3/5}\eta_0^3 + 3\varepsilon^{4/5}\eta_0^2\eta_1$$

$$(\varepsilon\eta)^4 = \varepsilon^{4/5}\eta_0^4$$

correct to $O(\varepsilon^{4/5})$]

[

$$\tanh(h_0 + \varepsilon^{2/5}h_1) = \frac{\tanh h_0 + \tanh \varepsilon^{2/5}h_1}{1 + \tanh h_0 \tanh \varepsilon^{2/5}h_1}$$

$$= (\tanh h_0 + \varepsilon^{2/5}h_1)(1 + \varepsilon^{2/5}h_1 \tanh h_0)^{-1}$$

$$= (\tanh h_0 + \varepsilon^{2/5}h_1)(1 - \varepsilon^{2/5}h_1 \tanh h_0 + \varepsilon^{4/5}h_1^2 \tanh^2 h_0)$$]

$O(\varepsilon^{-4/5})$ terms

Section 2

Problems for

$\phi_0, \phi_1, \phi_2, \phi_3, \phi_4$

$$\nabla^2 \phi_0 = 0 \qquad (0.1)$$

$$\phi_{0x} = 0 \qquad x = 0, \pi \qquad (0.2)$$

$$\phi_{0z} = 0 \qquad z = -h_0 \qquad (0.3)$$

$$\left.\begin{aligned} &\phi_{0z} = \eta_{0t} && (0.4) \\ &\eta_0 + \tanh h_0 \, \phi_{0t} = 0 && (0.5) \end{aligned}\right\} \quad z = 0$$

$\nabla \phi_0$ 2π periodic , η_0 zero mean (0.6)

$O(\varepsilon^{3/5})$ terms

$$\nabla^2 \phi_1 = 0 \qquad (1.1)$$

$$\phi_{1x} = 0 \quad x = 0, \pi \qquad (1.2)$$

$$\phi_{1z} = 0 \quad z = -h_0 \qquad (1.3)$$

$$\left.\begin{aligned} &\eta_0 \phi_{0zz} + \phi_{1z} = \eta_{1t} + \phi_{0x}\eta_{0x} && (1.4) \\ &\eta_1 + \tanh h_0 \left(\eta_0 \phi_{0tz} + \phi_{1t} + \tfrac{1}{2}(\phi_{0x}^2 + \phi_{0z}^2)\right) = 0 \end{aligned}\right\} z = 0 \qquad (1.5)$$

$\nabla \phi_1$ 2π-periodic , η_1 zero mean. (1.6)

$O(\varepsilon^{-2/5})$ terms

$$\nabla^2\phi_2 = 0 \qquad (2.1)$$

$$\phi_{2x} = 0 \quad x = 0, \pi \qquad (2.2)$$

$$\phi_{2z} = h_1 \phi_{0zz} \quad z = -h_0 \qquad (2.3)$$

$$\eta_1 \phi_{0zz} + \frac{\eta_0^2}{2}\phi_{0zzz} + \eta_0\phi_{1zz} + \phi_{2z}$$

$$= \eta_{2t} + \eta_0\phi_{0xz}\eta_{0x} + \phi_{0x}\eta_{1x} + \phi_{1x}\eta_{0x} \quad z=0 \qquad (2.4)$$

$$\eta_2 + \tanh h_0 \big(\eta_1\phi_{0tz} + \tfrac{1}{2}\eta_0^2\phi_{0tzz} + \eta_0\phi_{1tz} + \phi_{2t} + \eta_0\phi_{0x}\phi_{0xz}$$

$$+ \phi_{0x}\phi_{1x} + \eta_0\phi_{0z}\phi_{0zz} + \phi_{0z}\phi_{1z}\big) + h_1(1-\tanh^2 h_0)\phi_{0t} = 0 \quad z=0 \qquad (2.5)$$

$\nabla\phi_2$ 2π periodic, η_2 zero mean (2.6)

$O(\varepsilon^{-1/5})$ terms

$$\nabla^2\phi_3 = 0 \tag{3.1}$$

$$\phi_{3x} = 0 \qquad x = 0, \pi \tag{3.2}$$

$$\phi_{3z} = h_1 \phi_{1zz} \qquad z = -h_0 \tag{3.3}$$

$$\eta_2 \phi_{0zz} + \eta_0\eta_1 \phi_{0zzz} + \tfrac{1}{6}\eta_0^3 \phi_{0zzzz} + \eta_1 \phi_{1zz} + \tfrac{1}{2}\eta_0^2 \phi_{1zzz} + \eta_0 \phi_{2zz}$$
$$-\phi_{3z} = \eta_{3t} + \eta_1 \phi_{0xz}\eta_{0x} + \tfrac{1}{2}\eta_0^2 \phi_{0xzz}\eta_{0x} + \eta_0 \phi_{0xz}\eta_{1x} + \eta_0\phi_{1xz}\eta_{0x}$$
$$+ \phi_{0x}\eta_{2x} + \phi_{2x}\eta_{0x} + \phi_{1x}\eta_{1x} \qquad z = 0 \tag{3.4}$$

$$\eta_3 + \tanh h_0 \big(\eta_2 \phi_{0tz} + \eta_0\eta_1 \phi_{0tzz} + \tfrac{1}{6}\eta_0^3 \phi_{0tzzz} + \eta_1 \phi_{1tz} + \tfrac{1}{2}\eta_0^2 \phi_{1tzz}$$
$$+ \eta_0 \phi_{2tz} + \phi_{3t} + \eta_1 \phi_{0x}\phi_{0xz} + \tfrac{1}{2}\eta_0^2 \phi_{0xz}^2 + \tfrac{1}{2}\eta_0^2 \phi_{0x}\phi_{0xzz} + \eta_0\phi_{0xz}\phi_{1x}$$
$$+ \eta_0 \phi_{0x}\phi_{1xz} + \tfrac{1}{2}\phi_{1x}^2 + \phi_{0x}\phi_{2x} + \eta_1 \phi_{0z}\phi_{0zz} + \tfrac{1}{2}\eta_0^2 \phi_{0zz}^2 + \tfrac{1}{2}\eta_0^2 \phi_{0z}\phi_{0zzz}$$
$$+ \eta_0 \phi_{0zz}\phi_{1z} + \eta_0 \phi_{0z}\phi_{1zz} + \tfrac{1}{2}\phi_{1z}^2 + \phi_{0z}\phi_{2z} \big) + h_1 (1 - \tanh^2 h_0)$$
$$\big(\eta_0 \phi_{0tz} + \phi_{1t} + \tfrac{1}{2}(\phi_{0x}^2 + \phi_{0z}^2) \big) = 0 \qquad \text{on } z = 0 \tag{3.5}$$

$$\nabla\phi_3 \ 2\pi \text{ periodic}, \quad \eta_3 \text{ zero mean} \tag{3.6}$$

$O(i)$ terms

$$\nabla^2 \phi_4 = 0 \qquad (4.1)$$

$$\phi_{4x} = \sin t \qquad x = 0, \pi \qquad (4.2)$$

$$\phi_{4z} = h_1 \phi_{1zz} - \frac{h_1^2}{2} \phi_{0zzz} \qquad z = -h_0 \qquad (4.3)$$

$$\eta_3 \phi_{0zz} + \eta_0 \eta_2 \phi_{0zzz} + \frac{1}{2}\eta_1^2 \phi_{0zzz} + \frac{1}{2}\eta_0^2 \eta_1 \phi_{0zzzz} + \frac{1}{24}\eta_0^4 \phi_{0zzzzz}$$
$$+ \eta_2 \phi_{1zz} + \eta_0 \eta_1 \phi_{1zzz} + \frac{\eta_0^3}{6} \phi_{1zzzz} + \eta_1 \phi_{2zz} + \frac{\eta_0^2}{2} \phi_{2zzz} + \eta_0 \phi_{3zz}$$
$$+ \phi_{4z} = \eta_{4t} + \eta_2 \phi_{0xz} \eta_{0x} + \eta_0 \eta_1 \phi_{0xzz} \eta_{0x} + \tfrac{1}{6} \eta_0^3 \phi_{0xzzz} \eta_{0x}$$
$$+ \eta_1 \phi_{0xz} \eta_{1x} + \eta_1 \phi_{1xz} \eta_{0x} + \frac{1}{2}\eta_0^2 \phi_{0xzz} \eta_{1x} + \frac{1}{2}\eta_0^2 \phi_{1xzz} \eta_{0x}$$
$$+ \eta_0 \phi_{0xz} \eta_{2x} + \eta_0 \phi_{2xz} \eta_{0x} + \eta_0 \phi_{1xz} \eta_{1x} + \phi_{3x} \eta_{0x} + \phi_{0x} \eta_{3x}$$
$$+ \phi_{1x} \eta_{2x} + \phi_{2x} \eta_{1x} \qquad z = 0 \qquad (4.4)$$

$$\eta_4 + \tanh h_0 \Big\{ \eta_3 \phi_{0tz} + \eta_0 \eta_2 \phi_{0tzz} + \frac{1}{2}\eta_1^2 \phi_{0tzz} + \frac{1}{2}\eta_0^2 \eta_1 \phi_{0tzzz}$$
$$+ \frac{1}{24}\eta_0^4 \phi_{0tzzzz} + \eta_2 \phi_{1tz} + \eta_0 \eta_1 \phi_{1tzz} + \tfrac{1}{6} \eta_0^3 \phi_{1tzzz} + \eta_1 \phi_{2tz}$$
$$+ \frac{1}{2}\eta_0^2 \phi_{2tzz} + \eta_0 \phi_{3tz} + \phi_{4t} + \eta_2 \phi_{0x} \phi_{0xz} + \eta_0 \eta_1 \phi_{0xz}^2 + \eta_0 \eta_1 \phi_{0x} \phi_{0xzz}$$
$$+ \frac{1}{2}\eta_0^3 \phi_{0xz} \phi_{0xzz} + \tfrac{1}{6}\eta_0^3 \phi_{0x} \phi_{0xzzz} + \eta_1 \phi_{0xz} \phi_{1x} + \eta_1 \phi_{0x} \phi_{1xz}$$
$$+ \frac{1}{2}\eta_0^2 \phi_{0xzz} \phi_{1x} + \eta_0^2 \phi_{0xz} \phi_{1xz} + \frac{1}{2}\eta_0^2 \phi_{0x} \phi_{1xzz} + \eta_0 \phi_{1x} \phi_{1xz}$$
$$+ \eta_0 \phi_{0xz} \phi_{2x} + \eta_0 \phi_{0x} \phi_{2xz} + \phi_{0x} \phi_{3x} + \phi_{1x} \phi_{2x} + \eta_2 \phi_{0z} \phi_{0zz} +$$

$$+ \eta_0\eta_1 \phi_{0zz}^2 + \eta_0\eta_1 \phi_{0z}\phi_{0zzz} + \tfrac{1}{2}\eta_0^3 \phi_{0zz}\phi_{0zzz} + \tfrac{1}{6}\eta_0^3 \phi_{0z}\phi_{0zzzz}$$

$$+ \eta_1 \phi_{0zz}\phi_{1z} + \eta_1 \phi_{0z}\phi_{1zz} + \tfrac{1}{2}\eta_0^2 \phi_{0zzz}\phi_{1z} + \eta_0^2 \phi_{0zz}\phi_{1zz}$$

$$+ \tfrac{1}{2}\eta_0^2 \phi_{0z}\phi_{1zzz} + \eta_0 \phi_{1z}\phi_{1zz} + \eta_0 \phi_{0zz}\phi_{2z} + \eta_0 \phi_{0z}\phi_{2zz}$$

$$+ \phi_{0z}\phi_{3z} + \phi_{1z}\phi_{2z} \Big\} + h_1(1-\tanh^2 h_0) \Big\{ \eta_1 \phi_{0tz} + \tfrac{1}{2}\eta_0^2 \phi_{0tzz}$$

$$+ \eta_0 \phi_{1tz} + \phi_{2t} + \eta_0 \phi_{0x}\phi_{0xz} + \phi_{0x}\phi_{1x} + \eta_0 \phi_{0z}\phi_{0zz} + \phi_{0z}\phi_{1z} \Big\}$$

$$+ \tanh h_0 \left(\lambda + h_1^2(\tanh^2 h_0 - 1)\right) \phi_{0t} = 0 \qquad z=0 \qquad (4.5)$$

Section 3

Solution for

$\phi_0, \phi_1, \phi_2, \phi_3$

ϕ_0 problem

Combine (0.4) & (0.5) to get

$$\phi_{0z} + \tanh h_0 \, \phi_{0tt} = 0 \qquad z = 0$$

So solution is

$$\boxed{\phi_0 = A\cos x \cosh(z+h_0)\sin t}$$

$$\boxed{\eta_0 = -A\cos x \sinh h_0 \cos t} \quad \text{from (0.5)}$$

(so for future reference

$$\phi_{0zz} = \phi_0$$
$$\phi_{0zzz} = \phi_{0z}$$
$$\phi_{0zzzz} = \phi_0 \quad \ldots)$$

ϕ_1 problem

(1.4) & (1.5) give

Referring to dissertation P16 solution to this problem is

$$\phi_{1z} + \tanh h\, \phi_{1tt} = -\eta_0 \phi_{0zz} + \phi_{0x}\eta_{0x} - \frac{\partial}{\partial t}\left(\tanh h_0 \left(\eta_0 \phi_{0tz} + \tfrac{1}{2}(\phi_{0x}^2 + \phi_{0z}^2)\right)\right)$$

on $z=0$

$$\Rightarrow \boxed{\begin{aligned}\phi_1 = {} & \beta_0 t + B\cos x \cosh(z+h_0)\sin t \\ & + (A_1 + A_2\cos 2x \cosh 2(z+h_0))\sin 2t\end{aligned}}$$

where

$$A_1 = 0.4637438013\, A^2$$

$$A_2 = -0.1168242347\, A^2$$

$$\beta_0 = -\frac{A^2}{8} \quad \text{- determined from } \int_0^{\pi} \eta_1\, dx = 0$$

and η_1 is given by (1.5), ie.

$$\eta_1 = -\tanh h_0 \left(\eta_0 \phi_{0tz} + \phi_{1t} + \tfrac{1}{2}(\phi_{0x}^2 + \phi_{0z}^2)\right)\Big|_{z=0}.$$

ϕ_2 problem

From (2.3),

$$\phi_{2z} = Ah_1 \cos x \sin t \qquad z = -h_0$$

so put $\phi_2 = Ah_1 \cos x \sin t \sinh(z+h_0) + \tilde{\phi}_2$

and (2.1) – (2.6) become

$$\nabla^2 \tilde{\phi}_2 = 0, \quad \tilde{\phi}_{2x} = 0 \quad x = 0, \pi, \quad \tilde{\phi}_{2z} = 0 \quad z = -h_0$$

$$\sim + \tilde{\phi}_{2z} = \eta_{2t} + \sim - Ah_1 \cosh h_0 \cos x \sin t \qquad z = 0$$

$$\eta_2 + \tanh h_0 \left(\tilde{\phi}_{2t} + Ah_1 \sinh h_0 \cos x \cos t + \sim \right)$$

$$+ h_1 (1 - \tanh^2 h_0) \phi_{0t} = 0 \qquad z = 0$$

Combine these two equations to give

$$\tilde{\phi}_{2z} + \tanh h_0 \, \tilde{\phi}_{2tt} = a_1 \sin t \cos x + a_2 \cos 2x \sin 2t + a_3 \sin 2t$$

$$+ a_4 \cos x \sin 3t + a_5 \cos 3x \sin t + a_6 \cos 3x \sin 3t$$

So

$$\boxed{\begin{aligned}\tilde{\phi}_2 = {} & C \cos x \cosh(z+h_0) \sin t + \sin 2t \left(C_1 + C_2 \cos 2x \cosh 2(z+h_0) \right) \\ & + \sin 3t \left(C_3 \cos x \cosh(z+h_0) + C_4 \cos 3x \cosh 3(z+h_0) \right) \\ & + C_5 \cos 3x \cosh 3(z+h_0) \sin t + \beta_1 t\end{aligned}}$$

where $$C_1 = \frac{-a_3}{4\tanh h_0} = 0.9274876028\, AB$$

$$C_2 = \frac{a_2}{2(\sinh 2h_0 - 2\tanh h_0 \cosh 2h_0)} = -0.2336484693\, AB$$

$$C_3 = \frac{a_4}{8\sinh h_0} = -0.5851819198\, A^3$$

$$C_4 = \frac{a_6}{3(\sinh 3h_0 - 3\tanh h_0 \cosh 3h_0)} = 0.0039125489 16\, A^3$$

$$C_5 = \frac{a_5}{3\ \sinh 3h_0 - \tanh h_0 \cosh 3h_0} = 0.0099523340 82\, A^3$$

and η_2 is determined from (25)

Then $$\int_0^{\pi} \eta_2\, dx = 0 \Rightarrow \beta_1 = -\frac{1}{4} AB$$

Section 4

Solvability condition & response

The ϕ_4 problem boils down to

$$\nabla^2\phi_4 = 0$$

$$\phi_{4x} = \sin t \qquad x = 0, \pi$$

$$\phi_{4z} = h_1 \phi_{2zz} - \cancel{\frac{h_1^2}{2}\phi_{0zzz}} \qquad z = -h_0$$

$$\phi_{4z} + h_{0t}\,\phi_{4tt} = F(x,t) - \lambda h_{0t}\,\phi_{0tt}$$

Solv cond^n comes from considering

$$\int_0^{2\pi}\int_{-h_0}^{0}\int_0^{\pi} \nabla^2\phi_4 \cos x \cosh(z+h_0) \sin t \, dx\,dz\,dt = 0 \qquad (*)$$

Now,

$$\int_0^{\pi} \phi_{4xx}\cos x\,dx = -2\sin t - \int_0^{\pi}\phi_4 \cos x\,dx$$

$$\int_{-h_0}^{0} \phi_{4zz}\cosh(z+h_0)\,dz = \left(F(x,t) - \lambda h_{0t}\,\phi_{0tt}\big|_{z=0} - h_{0t}\,\phi_{4tt}\big|_{z=0}\right)\cosh h_0$$

$$- h_1\phi_{2zz}\big|_{z=-h_0} - \phi_4\big|_{z=0}\sinh h_0 + \int_{-h_0}^{0}\phi_4\cosh(z+h_0)\,dz$$

$$\therefore (*) \Rightarrow -2\pi\sinh h_0 + \cosh h_0 \iint F(x,t)\cos x \sin t\,dx\,dt + \frac{\pi^2}{2}\lambda A \sinh h_0 \cosh h_0$$

$$- h_1 \iint \phi_{2zz}\big|_{z=-h_0}\cos x \sin t\,dx\,dt = 0$$

$$\phi_0 = A\cos(x)\cosh(z+h)\sin(t)$$

$$\eta_0 = -A\sinh(h)\cos(x)\cos(t)$$

$$\phi_1 = b0\,t + B\cos(x)\cosh(z+h)\sin(t) + (A1 + A2\,\cos(2\,x)\cosh(2\,z+2\,h))\sin(2\,t)$$

$$\begin{aligned}\eta_1 \;=\; & \frac{1}{2}\tanh(h)\Big(2\,A^2(\cos(x))^2(\cos(t))^2(\cosh(h))^2 - 3\,A^2(\cos(x))^2(\cos(t))^2 - 2\,b0 - \\ & 2\,B\cos(x)\cosh(h)\cos(t) - 4\,\cos(2\,t)A1 - 4\,\cos(2\,t)A2\,\cos(2\,x)\cosh(2\,h) - A^2(\cosh(h))^2 \\ & + A^2(\cosh(h))^2(\cos(t))^2 + A^2(\cos(x))^2\Big)\end{aligned}$$

$$\begin{aligned}\phi_2 \;=\; & Ah1\,\cos(x)\sinh(z+h)\sin(t) + C\cos(x)\cosh(z+h)\sin(t) \\ & + \sin(2\,t)\,(C1 + C2\,\cos(2\,x)\cosh(2\,z+2\,h)) \\ & + \sin(3\,t)\,(C3\,\cos(x)\cosh(z+h) + C4\,\cos(3\,x)\cosh(3\,z+3\,h)) \\ & + C5\,\cos(3\,x)\cosh(3\,z+3\,h)\sin(t) + b1\,t\end{aligned}$$

$$\begin{aligned}\eta_2 \;=\; & -2\,\sinh(h)C\cos(x)(\cosh(h))^2\cos(t) - A^3(\cos(x))^3\cos(t)\sinh(h) \\ & +2\,\sinh(h)C5\,\cos(3\,x)\cosh(3\,h)\cos(t)\cosh(h) + 4\,\sinh(h)\cos(2\,t)\cosh(h)C1 \\ & +4\,\sinh(h)A\sin(x)(\cosh(h))^2\sin(t)A2\,\sin(2\,x)\cosh(2\,h)\sin(2\,t) \\ & +4\,\sinh(h)\cos(2\,t)\cosh(h)C2\,\cos(2\,x)\cosh(2\,h) + 6\,\sinh(h)\cos(3\,t)(\cosh(h))^2 C3\,\cos(x) \\ & +6\,\sinh(h)\cos(3\,t)\cosh(h)C4\,\cos(3\,x)\cosh(3\,h) + 2\,h1\,A\cos(x)(\cosh(h))^3\cos(t) \\ & +2\,\sinh(h)A(\cosh(h))^3 B - 4\,A(\cos(x))^2(\cosh(h))^3 B\sinh(h)(\cos(t))^2 \\ & -2\,A(\cos(x))^2\cosh(h)B\sinh(h) + 6\,A(\cos(x))^2\cosh(h)B\sinh(h)(\cos(t))^2 \\ & +3\,A^3(\cos(x))^3(\cos(t))^3(\cosh(h))^4\sinh(h) - 6\,A^3(\cos(x))^3(\cos(t))^3(\cosh(h))^2\sinh(h) \\ & -3\,A^3\cos(x)\cos(t)(\cosh(h))^4\sinh(h) + 3\,A^3\cos(x)(\cos(t))^3(\cosh(h))^4\sinh(h) \\ & +3\,A^3\cos(x)\cos(t)(\cosh(h))^2\sinh(h) - 3\,A^3\cos(x)(\cos(t))^3(\cosh(h))^2\sinh(h) \\ & +3\,A^3(\cos(x))^3(\cos(t))^3\sinh(h) - 4\,A\cos(x)\cos(t)\cos(2\,t)A1\,\sinh(h)(\cosh(h))^2 \\ & +4\,A\cos(x)\cos(t)\cos(2\,t)A1\,\sinh(h) + A^3(\cos(x))^3\cos(t)(\cosh(h))^2\sinh(h) \\ & +2\,\sinh(h)b1\,\cosh(h) - 4\,A\cos(x)\cos(t)\cos(2\,t)A2\,\cos(2\,x)\cosh(2\,h)\sinh(h)(\cosh(h))^2 \\ & +4\,A\cos(x)\cos(t)\cos(2\,t)A2\,\cos(2\,x)\cosh(2\,h)\sinh(h) - 2\,A\cos(x)\cos(t)b0\,\sinh(h)(\cosh(h))^2 \\ & +2\,A\cos(x)\cos(t)b0\,\sinh(h) - 8\,A\cos(x)\cos(t)(\cosh(h))^3 A2\,\cos(2\,x)\sinh(2\,h)\cos(2\,t) \\ & +8\,A\cos(x)\cos(t)\cosh(h)A2\,\cos(2\,x)\sinh(2\,h)\cos(2\,t) \\ & +4\,A\cos(x)\sin(t)(\cosh(h))^3 A2\,\cos(2\,x)\sinh(2\,h)\sin(2\,t) \\ & -4\,A\cos(x)\sin(t)\cosh(h)A2\,\cos(2\,x)\sinh(2\,h)\sin(2\,t) \\ & -2\,\sinh(h)A(\cosh(h))^3 B(\cos(t))^2/(2\,(\cosh(h))^2)\end{aligned}$$

$$\begin{aligned}\phi_3 \;=\; & h1\,(B\cos(x)\sinh(z+h)\sin(t) + 2\,A2\,\cos(2\,x)\sinh(2\,z+2\,h)\sin(2\,t)) \\ & +F\cos(x)\cosh(z+h)\sin(t) + D1\,\cosh(3\,z+3\,h)\cos(3\,x)\sin(t) \\ & +(D2\,\cosh(2\,z+2\,h)\cos(2\,x) + D3\,\cosh(4\,z+4\,h)\cos(4\,x) + D4)\sin(2\,t) \\ & +(D5\,\cosh(z+h)\cos(x) + D6\,\cosh(3\,z+3\,h)\cos(3\,x))\sin(3\,t) \\ & +(D7\,\cosh(4\,z+4\,h)\cos(4\,x) + D8\,\cos(2\,x)\cosh(2\,z+2\,h) + D9)\sin(4\,t) + b2\,t\end{aligned}$$

$$A1 = \frac{A^2\left(4\,(\sinh(h))^2+1\right)}{16}$$

$$A2 = -\frac{3\,A^2}{16\,(\sinh(h))^2}$$

$$b0 = -\frac{A^2}{8}$$

$$b1 = -\frac{AB}{4}$$

$$b2 = -0.25AC - 0.125B^2 - 0.0068239496650A^4$$

$$C1 = \frac{(\cosh(h) - 2\,\cosh(3\,h) + \cosh(5\,h))\,BA}{32\,\cosh(h)\left((\cosh(h))^2 - 1\right)}$$

$$C2 = -\frac{3\,BA}{8\,(\cosh(h))^2 - 8}$$

$$C3 = -\frac{(-12 + 13\,\cosh(2\,h) + 5\,\cosh(6\,h) + 30\,\cosh(4\,h))\,A^3}{1024\,(\cosh(h))^2\left((\cosh(h))^2 - 1\right)}$$

$$C4 = \frac{A^3\,(10 - \cosh(4\,h) + 9\,\cosh(2\,h))}{512\,(\cosh(h))^2\left((\cosh(h))^4 - 2\,(\cosh(h))^2 + 1\right)}$$

$$C5 = \frac{A^3\,(5\,\cosh(2\,h) + \cosh(4\,h))}{512\,(\cosh(h))^4\left((\cosh(h))^2 - 1\right)}$$

$$D1 = 0.029857002290A^2B$$

$$D2 = 0.2586541320A^4 - 0.2336484693AC + 0.2976669197h1\,A^2 - 0.1168242346B^2$$

$$D3 = 0.0037834189780A^4$$

$$D4 = 1.022364981h1\,A^2 + 0.4637438010B^2 + 0.5992508382A^4 + 0.9274876027AC$$

$$D5 = -1.755545759A^2B$$

$$D6 = 0.011737646760A^2B$$

$$D7 = 0.0011230190250A^4$$

$$D8 = 0.1539057777A^4$$

$$D9 = 0.1309421371A^4$$

> read maps;

η_3

$$s := -\frac{1}{6}\Big(6 \sinh(h)\, A \cosh(h)^4\, C - 6 \sinh(h)\, A \cosh(h)^4\, C \cos(t)^2$$

$$- 12 \sinh(h)\, A \cosh(h)^4\, C \cos(x)^2 \cos(t)^2 + 27 \cosh(h)\, A^2 \cos(x)^3 \cos(t)^3\, B \sinh(h)$$

$$- 6\, A \cos(x) \cos(t)\, b1 \cosh(h)^3 \sinh(h) + 6\, A \cos(x) \cos(t)\, b1 \cosh(h) \sinh(h)$$

$$+ 27 \cosh(h)^5 A^2 \cos(t)^3\, B \cos(x) \sinh(h)$$

$$+ 18 \sinh(h)\, A \sin(x) \cosh(h)^3\, C5 \sin(3\,x) \cosh(3\,h)$$

$$- 18 \sinh(h)\, A \sin(x) \cosh(h)^3\, C5 \sin(3\,x) \cosh(3\,h) \cos(t)^2$$

$$+ 27 \cosh(h)^5 A^2 \cos(x)^3 \cos(t)^3\, B \sinh(h) - 54 \cosh(h)^3 A^2 \cos(x)^3 \cos(t)^3\, B \sinh(h)$$

$$- 6\, A \cos(x) \cos(t)^2\, C5 \cos(3\,x) \cosh(3\,h) \cosh(h)^3 \sinh(h)$$

$$+ 12\, A^2 \cos(x)^2 \cos(t)^2 \cos(2\,t)\, A2 \cos(2\,x) \cosh(2\,h) \sinh(h)$$

$$+ 12 \sinh(h) \cosh(h)^2 A2^2 \cosh(2\,h)^2 - 12 \sinh(h) \cosh(h)^2 A2^2 \cosh(2\,h)^2 \cos(2\,t)^2$$

$$- 12 \sinh(h) \cosh(h)^2 A2^2 \cos(2\,x)^2 + 12 \sinh(h) \cosh(h)^2 A2^2 \cos(2\,x)^2 \cos(2\,t)^2$$

$$- 24 \cosh(h)^3\, b0\, A2 \cos(2\,x) \sinh(2\,h) \cos(2\,t)$$

$$+ 48 \cosh(h) \cos(2\,t)^2 A2^2 \cos(2\,x)^2 \cosh(2\,h) \sinh(2\,h)$$

$$- 48\, A^4 \cos(x)^2 \cos(t)^2 \cosh(h)^4 \sinh(h) + 45\, A^4 \cos(x)^2 \cos(t)^4 \cosh(h)^4 \sinh(h)$$

$$- 6\, A^4 \cos(x)^4 \cos(t)^2 \cosh(h)^4 \sinh(h) + 44\, A^4 \cos(x)^4 \cos(t)^4 \cosh(h)^4 \sinh(h)$$

$$+ 24\, A^4 \cos(x)^2 \cos(t)^2 \cosh(h)^2 \sinh(h) - 21\, A^4 \cos(x)^2 \cos(t)^4 \cosh(h)^2 \sinh(h)$$

$$+ 6\, A^2 \cosh(h)^5\, h1 + 3\, A^4 \cosh(h)^4 \sinh(h) + 3 \sinh(h) \cosh(h)^4 B^2 - 3\, A^2 \cosh(h)^3\, h1$$

$$+ 9\, A^4 \cos(x)^4 \cos(t)^2 \cosh(h)^2 \sinh(h) - 37\, A^4 \cos(x)^4 \cos(t)^4 \cosh(h)^2 \sinh(h)$$

$$- 6 \cosh(h)^3\, b0\, B \cos(x) \cos(t) \sinh(h) + 6 \cosh(h)\, b0\, B \cos(x) \cos(t) \sinh(h)$$

$$+ 24\, A^4 \cos(x)^2 \cos(t)^2 \cosh(h)^6 \sinh(h) - 24\, A^4 \cos(x)^2 \cos(t)^4 \cosh(h)^6 \sinh(h)$$

$$- 16\, A^4 \cos(x)^4 \cos(t)^4 \cosh(h)^6 \sinh(h)$$

$$+ 60 \cosh(h)\, A^2 \cos(x)^2 \cos(t)^2\, A2 \cos(2\,x) \sinh(2\,h) \cos(2\,t)$$

$$+ 48 \cosh(h)^5 A^2 \cos(x)^2 \cos(t)^2\, A2 \cos(2\,x) \sinh(2\,h) \cos(2\,t)$$

$$- 108 \cosh(h)^3 A^2 \cos(x)^2 \cos(t)^2\, A2 \cos(2\,x) \sinh(2\,h) \cos(2\,t)$$

$$- 6 \cosh(h)^4 B^2 \cos(x)^2 \cos(t)^2 \sinh(h) + 9 \cosh(h)^2 B^2 \cos(x)^2 \cos(t)^2 \sinh(h)$$

$$+ 12\, h1 \cosh(h) \cos(2\,t)\, A1 + 6 \sinh(h)\, A \cosh(h)^4 \sin(t) \sin(3\,t)\, C3$$

$$- 9 \cosh(h)\, A^2 \cos(x)^3\, B \cos(t) \sinh(h) + 9 \cosh(h)^3 A^2 \cos(x)^3\, B \cos(t) \sinh(h)$$

$$+ 12 \cosh(h)^5 A^2 \cos(t)^2 A2 \cos(2 x) \sinh(2 h) \cos(2 t)$$

$$- 27 \cosh(h)^3 A^2 \cos(t)^3 B \cos(x) \sinh(h)$$

$$- 12 \cosh(h)^3 A^2 \cos(t)^2 A2 \cos(2 x) \sinh(2 h) \cos(2 t)$$

$$- 12 \cosh(h)^5 A^2 A2 \cos(2 x) \sinh(2 h) \cos(2 t)$$

$$+ 12 \cosh(h)^3 A^2 A2 \cos(2 x) \sinh(2 h) \cos(2 t)$$

$$- 27 \cosh(h)^5 A^2 B \cos(x) \cos(t) \sinh(h) + 27 \cosh(h)^3 A^2 B \cos(x) \cos(t) \sinh(h)$$

$$- 48 \cosh(h)^3 \cos(2 t)^2 A2^2 \cos(2 x)^2 \cosh(2 h) \sinh(2 h)$$

$$- 12 \cosh(h)^3 \cos(2 t) A2 \cos(2 x) \cosh(2 h) B \cos(x) \cos(t) \sinh(h)$$

$$+ 12 \cosh(h) \cos(2 t) A2 \cos(2 x) \cosh(2 h) B \cos(x) \cos(t) \sinh(h)$$

$$- 48 \cosh(h)^3 \cos(2 t)^2 A1\, A2 \cos(2 x) \sinh(2 h)$$

$$+ 48 \cosh(h) \cos(2 t)^2 A1\, A2 \cos(2 x) \sinh(2 h)$$

$$- 12 \cosh(h)^3 \cos(2 t) A1\, B \cos(x) \cos(t) \sinh(h)$$

$$+ 12 \cosh(h) \cos(2 t) A1\, B \cos(x) \cos(t) \sinh(h)$$

$$- 24 \cosh(h)^4 B \cos(x) \cos(t) A2 \cos(2 x) \sinh(2 h) \cos(2 t)$$

$$+ 24 \cosh(h)^2 B \cos(x) \cos(t) A2 \cos(2 x) \sinh(2 h) \cos(2 t)$$

$$+ 24 \cosh(h)\, b0\, A2 \cos(2 x) \sinh(2 h) \cos(2 t) - 12 A^2 \cos(x)^2 \cos(t)^2 h1 \cosh(h)^5$$

$$+ 6 A^2 \cos(x)^2 \cos(t)^2 h1 \cosh(h)^3$$

$$- 18 A \cos(x) \cos(t) \cos(3 t) \cosh(h)^3 C4 \cos(3 x) \cosh(3 h) \sinh(h)$$

$$+ 18 A \cos(x) \cos(t) \cos(3 t) \cosh(h) C4 \cos(3 x) \cosh(3 h) \sinh(h)$$

$$- 36 A \cos(x)^2 \cos(t) \cos(3 t) \cosh(h)^4 C3 \sinh(h)$$

$$+ 36 A \cos(x)^2 \cos(t) \cos(3 t) \cosh(h)^2 C3 \sinh(h)$$

$$- 24 A^2 \cos(x) \cos(t) \sin(x) \cosh(h)^4 \sin(t) A2 \sin(2 x) \cosh(2 h) \sin(2 t) \sinh(h)$$

$$+ 24 A^2 \cos(x) \cos(t) \sin(x) \cosh(h)^2 \sin(t) A2 \sin(2 x) \cosh(2 h) \sin(2 t) \sinh(h)$$

$$- 12 A \cos(x) \cos(t) \cos(2 t) \cosh(h)^3 C1 \sinh(h)$$

$$+ 12 A \cos(x) \cos(t) \cos(2 t) \cosh(h) C1 \sinh(h)$$

$$+ 6 A \cos(x) \cos(t)^2 C5 \cos(3 x) \cosh(3 h) \cosh(h) \sinh(h)$$

$$- 3 A^4 \cos(x)^4 \cos(t)^2 \sinh(h) + 18 A \cos(x)^2 \cos(t)^2 C \cosh(h)^2 \sinh(h)$$

$$- 12 A \cos(x) \cos(t) \cos(2 t) \cosh(h)^3 C2 \cos(2 x) \cosh(2 h) \sinh(h)$$

$$+ 12 A \cos(x) \cos(t) \cos(2 t) \cosh(h) C2 \cos(2 x) \cosh(2 h) \sinh(h)$$

$+ 12 \cosh(h)^3 A^2 \cos(x)^2 A2 \cos(2 x) \sinh(2 h) \cos(2 t)$

$- 12 \cosh(h) A^2 \cos(x)^2 A2 \cos(2 x) \sinh(2 h) \cos(2 t)$

$- 24 A^2 \cos(x)^2 \cos(t) \sin(t) \cosh(h)^5 A2 \cos(2 x) \sinh(2 h) \sin(2 t)$

$+ 36 A^2 \cos(x)^2 \cos(t) \sin(t) \cosh(h)^3 A2 \cos(2 x) \sinh(2 h) \sin(2 t)$

$+ 6 A^2 \cos(x)^2 \cos(t)^2 b0 \sinh(h) - 18 A^2 \cos(x)^2 \cos(t)^2 b0 \cosh(h)^2 \sinh(h)$

$+ 12 A^2 \cos(x)^2 \cos(t)^2 b0 \cosh(h)^4 \sinh(h)$

$+ 48 A^2 \cos(x)^2 \cos(t)^2 \cos(2 t) A2 \cos(2 x) \cosh(2 h) \cosh(h)^4 \sinh(h)$

$- 60 A^2 \cos(x)^2 \cos(t)^2 \cos(2 t) A2 \cos(2 x) \cosh(2 h) \cosh(h)^2 \sinh(h)$

$+ 12 A^2 \cos(x)^2 \cos(t)^2 \cos(2 t) A1 \sinh(h)$

$+ 24 A^2 \cos(x)^2 \cos(t)^2 \cos(2 t) A1 \cosh(h)^4 \sinh(h) - 3 A^4 \cosh(h)^6 \sinh(h)$

$+ 6 \sinh(h) b2 \cosh(h)^2 + 6 h1 \cosh(h) b0$

$- 36 A^2 \cos(x)^2 \cos(t)^2 \cos(2 t) A1 \cosh(h)^2 \sinh(h) + 9 A^4 \cos(x)^4 \cos(t)^4 \sinh(h)$

$- 24 A^2 \cos(x) \cos(t) \sin(x) \cosh(h)^5 \sin(t) A2 \sin(2 x) \sinh(2 h) \sin(2 t)$

$+ 24 A^2 \cos(x) \cos(t) \sin(x) \cosh(h)^3 \sin(t) A2 \sin(2 x) \sinh(2 h) \sin(2 t)$

$- 36 A \cos(x) \cos(t)^2 \cosh(h)^4 C5 \cos(3 x) \sinh(3 h)$

$+ 36 A \cos(x) \cos(t)^2 \cosh(h)^2 C5 \cos(3 x) \sinh(3 h)$

$+ 54 A \cos(x) \cos(t) \cosh(h)^2 \cos(3 t) C4 \cos(3 x) \sinh(3 h)$

$+ 24 A \cos(x) \cos(t) \cosh(h)^2 \cos(2 t) C2 \cos(2 x) \sinh(2 h)$

$- 24 A \cos(x) \cos(t) \cosh(h)^4 \cos(2 t) C2 \cos(2 x) \sinh(2 h)$

$- 54 A \cos(x) \cos(t) \cosh(h)^4 \cos(3 t) C4 \cos(3 x) \sinh(3 h)$

$+ 12 \cosh(h)^4 B \cos(x) \sin(t) A2 \cos(2 x) \sinh(2 h) \sin(2 t)$

$- 12 \cosh(h)^2 B \cos(x) \sin(t) A2 \cos(2 x) \sinh(2 h) \sin(2 t)$

$- 12 A^2 \cos(x)^2 \cos(t) \sin(t) \cosh(h) A2 \cos(2 x) \sinh(2 h) \sin(2 t)$

$- 3 \cosh(h)^2 B^2 \cos(x)^2 \sinh(h) - 3 A^4 \cosh(h)^2 \cos(x)^2 \sinh(h)$

$+ 12 A^2 \cosh(h)^4 \cos(2 t) A2 \cos(2 x) \cosh(2 h) \sinh(h) \cos(t)^2$

$+ 12 A^2 \cosh(h)^2 \cos(2 t) A2 \cos(2 x) \cosh(2 h) \sinh(h)$

$- 12 A^2 \cosh(h)^2 \cos(2 t) A2 \cos(2 x) \cosh(2 h) \sinh(h) \cos(t)^2$

$- 12 A^2 \cosh(h)^4 \cos(2 t) A2 \cos(2 x) \cosh(2 h) \sinh(h)$

$$- 12\,A^2\cosh(h)^4\cos(2\,t)\,A1\,\sinh(h) + 12\,A^2\cosh(h)^4\cos(2\,t)\,A1\,\sinh(h)\cos(t)^2$$
$$+ 12\,A^2\cosh(h)^2\cos(2\,t)\,A1\,\sinh(h) - 12\,A^2\cosh(h)^2\cos(2\,t)\,A1\,\sinh(h)\cos(t)^2$$
$$- 18\,A\cos(x)\cosh(h)^2\,C5\cos(3\,x)\sinh(3\,h)$$
$$+ 18\,A\cos(x)\cosh(h)^4\,C5\cos(3\,x)\sinh(3\,h)$$
$$+ 18\,A\cos(x)\sin(t)\cosh(h)^4\sin(3\,t)\,C4\cos(3\,x)\sinh(3\,h)$$
$$- 18\,A\cos(x)\sin(t)\cosh(h)^2\sin(3\,t)\,C4\cos(3\,x)\sinh(3\,h)$$
$$- 6\,A\cos(x)^2\sin(t)\cosh(h)^2\sin(3\,t)\,C3\sinh(h)$$
$$+ 12\,A\cos(x)\sin(t)\cosh(h)^4\sin(2\,t)\,C2\cos(2\,x)\sinh(2\,h)$$
$$- 12\,A\cos(x)\sin(t)\cosh(h)^2\sin(2\,t)\,C2\cos(2\,x)\sinh(2\,h)$$
$$- 6\,A\cos(x)^2\cosh(h)^2\,C\sinh(h)$$
$$- 24\,A^2\cos(x)^2\cos(t)\sin(t)\cosh(h)^4\,A2\cos(2\,x)\cosh(2\,h)\sin(2\,t)\sinh(h)$$
$$+ 24\,A^2\cos(x)^2\cos(t)\sin(t)\cosh(h)^2\,A2\cos(2\,x)\cosh(2\,h)\sin(2\,t)\sinh(h)$$
$$- 6\,A^2\cosh(h)^4\,b0\sinh(h) + 6\,A^2\cosh(h)^4\,b0\sinh(h)\cos(t)^2$$
$$+ 6\,A^2\cosh(h)^2\,b0\sinh(h) - 6\,A^2\cosh(h)^2\,b0\sinh(h)\cos(t)^2$$
$$- 6\,A^2\cosh(h)^5\,h1\cos(t)^2 + 3\,A^2\cosh(h)^3\,h1\cos(t)^2 - 3\,A^4\cosh(h)^6\cos(t)^4\sinh(h)$$
$$+ 3\,A^4\cosh(h)^4\cos(t)^4\sinh(h) + 6\,A^4\cosh(h)^6\sinh(h)\cos(t)^2$$
$$- 6\,A^4\cosh(h)^4\sinh(h)\cos(t)^2 + 3\,A^4\cosh(h)^4\sinh(h)\cos(x)^2$$
$$+ 9\,h1\cosh(h)\,A^2\cos(x)^2\cos(t)^2 - 3\,h1\cosh(h)\,A^2\cos(x)^2$$
$$- 3\sinh(h)\cosh(h)^4\,B^2\cos(t)^2 + 12\,h1\cosh(h)\cos(2\,t)\,A2\cos(2\,x)\cosh(2\,h)$$
$$+ 24\sinh(h)\cos(4\,t)\cosh(h)^2\,D8\cos(2\,x)\cosh(2\,h)$$
$$+ 12\sinh(h)\cos(2\,t)\cosh(h)^2\,D3\cosh(4\,h)\cos(4\,x)$$
$$+ 24\sinh(h)\cos(4\,t)\cosh(h)^2\,D9$$
$$+ 12\sinh(h)\cos(2\,t)\cosh(h)^2\,D2\cosh(2\,h)\cos(2\,x) + 6\,h1\cosh(h)^4\,B\cos(x)\cos(t)$$
$$+ 18\sinh(h)\cos(3\,t)\cosh(h)^2\,D6\cosh(3\,h)\cos(3\,x)$$
$$+ 18\sinh(h)\cos(3\,t)\cosh(h)^3\,D5\cos(x)$$
$$+ 24\sinh(h)\,h1\cosh(h)^2\,A2\cos(2\,x)\sinh(2\,h)\cos(2\,t)$$
$$+ 24\sinh(h)\cos(4\,t)\cosh(h)^2\,D7\cosh(4\,h)\cos(4\,x)$$
$$+ 12\sinh(h)\cos(2\,t)\cosh(h)^2\,D4 + 6\sinh(h)\,F\cos(x)\cosh(h)^3\cos(t)$$

$$+ 6\sinh(h)\, D1 \cosh(3h)\cos(3x)\cos(t)\cosh(h)^2$$
$$+ 12\sinh(h)\cosh(h)^3 B\sin(x)\sin(t)\, A2 \sin(2x)\cosh(2h)\sin(2t)$$
$$+ 12\sinh(h)\, A\sin(x)\cosh(h)^3\sin(t)\sin(2t)\, C2\sin(2x)\cosh(2h)$$
$$+ 18\sinh(h)\, A\sin(x)\cosh(h)^3\sin(t)\sin(3t)\, C4\sin(3x)\cosh(3h)\Big) \Big/ \cosh(h)^3$$

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