FM 8744 Feng & Wu: Electrophoretic motion of an arbitrary prolate body of revolution towards an infinite conducting wall

the influence of the particle shape are studied and the results are significantly differ ent from ~~that~~ ose in the unbounded case or ~~that~~ of a spherical particle. The most important result in this work is that the wall effect on electrophoretic mobility will reduce with the increase of slenderness ratio of the particle. It is shown that the present method is particularly ~~available~~ suitable for non-slender bodies and needs a relatively small amount of computation compared with other methods such as the boundary integral method. ~~etc.~~ Moreover, this method can be easily extended to more complex particle–particle or particle–boundary interaction problems.

Appendix

The key step in keeping computation time ~~at~~ to a minimum while maintaining high accuracy is to evaluate $TE^{(i)}_{njk}$ $(i=1,\dots,8, k=1,2,3)$ analytically by recurrence formulae. Substituting $SE^{(i)}_{njk}$ $(i=1,2,3)$ and $S^{(i)}_{n}(R,Z-\xi)$ $(i=1,\dots,8)$ into (3.20), (3.27) respectively, we obtain

$$TE^{(1)}_{njk}=G^{(1)}_{njk}(R,Z)-G^{(1)}_{njk}(R,2d-Z), \tag{A1}$$

$$TE^{(2)}_{njk}=(n+1)[G^{(1)}_{n+1,jk}(R,Z)-G^{(1)}_{n+1,jk}(R,2d-Z)], \tag{A2}$$

$$TE^{(3)}_{njk}=(n+1)[G^{(3)}_{n+1,jk}(R,Z)-G^{(3)}_{n+1,jk}(R,2d-Z)], \tag{A3}$$

$$T^{(1)}_{njk}=G^{(1)}_{njk}(R,Z)-G^{(1)}_{njk}(R,2d-Z)+2(n+1)(Z-d)G^{(1)}_{n+1,jk}(R,2d-Z), \tag{A4}$$

$$T^{(2)}_{njk}=G^{(2)}_{njk}(R,Z)-G^{(2)}_{njk}(R,2d-Z)-2(n-2)(Z-d)G^{(1)}_{n-1,jk}(R,2d-Z)$$
$$+2(2n-3)(Z-d)[dG^{(1)}_{njk}(R,2d-Z)-G^{(1)}_{n,j,k+1}(R,2d-Z)], \tag{A5}$$

$$T^{(3)}_{njk}=G^{(3)}_{njk}(R,Z)-G^{(3)}_{njk}(R,2d-Z)-2(n+1)(Z-d)G^{(3)}_{n+1,jk}(R,2d-Z), \tag{A6}$$

$$T^{(4)}_{njk}=G^{(4)}_{njk}(R,Z)-G^{(4)}_{njk}(R,2d-Z)+\frac{2(n-3)(n-1)(Z-d)}{n}G^{(3)}_{n-1,jk}(R,2d-Z)$$
$$-2(2n-3)(Z-d)[dG^{(3)}_{njk}(R,2d-Z)-G^{(3)}_{n,j,k+1}(R,2d-Z)], \tag{A7}$$

$$T^{(5)}_{njk}=G^{(5)}_{njk}(R,Z)-G^{(5)}_{njk}(R,2d-Z)+2(n+1)(Z-d)G^{(5)}_{n+1}(R,2d-Z), \tag{A8}$$

$$T^{(6)}_{njk}=G^{(6)}_{njk}(R,Z)-G^{(6)}_{njk}(R,2d-Z)-2(n-2)(Z-d)G^{(5)}_{n-1}(R,2d-Z)$$

$$+2(2n-3)(Z-d)[dG^{(5)}_{njk}(R,2d-Z)-G^{(5)}_{n,j,k+1}(R,2d-Z)], \qquad \text{(A9)}$$

$$T^{(7)}_{njk}=4(n+1)G^{(1)}_{n+1,jk}(R,2d-Z), \qquad \text{(A10)}$$

$$T^{(8)}_{njk}=\frac{4n-6}{n}G^{(1)}_{n-1,jk}(R,Z)-\frac{2(2n^2-6n+3)}{n}G^{(1)}_{n-1,jk}(R,2d-Z)$$

$$+4(2n-3)[dG^{(1)}_{njk}(R,2d-Z)-G^{(1)}_{n,j,k+1}(R,2d-Z)], \qquad \text{(A11)}$$

where

$$G^{(i)}_{njk}(R,Z)=\int_{d_{j1}}^{d_{j3}}\xi^{k-1}F^{(i)}_{n}(R,Z-\xi)d\xi \quad (i=1 \text{ ~~to~~ } 6), \qquad \text{(A12)}$$

~~And~~ $G^{(i)}_{njk}(R,Z)$ can be evaluated by the following recurrence formulas.

$$G^{(i)}_{njk}(R,Z)=d^{k-1}_{j3}J^{(i)}_{n}(R,Z-d_{j3})-d^{k-1}_{j1}J^{(i)}_{n}(R,Z-d_{j1})$$

$$-\frac{k-1}{n}G^{(i)}_{n-1,jk}(R,Z) \quad \text{for } i=1,3,5, \qquad \text{(A13)}$$

$$G^{(2)}_{njk}(R,Z)=d^{k-1}_{j3}J^{(2)}_{n}(R,Z-d_{j3})-d^{k-1}_{j1}J^{(2)}_{n}(R,Z-d_{j1})$$

$$-\frac{k-1}{n}G^{(2)}_{n-1,j,k-1}(R,Z)+\frac{2(k-1)}{n(n-1)}[G^{(1)}_{n-2,jk}(R,Z)-ZG^{(1)}_{n-2,j,k-1}(R,Z)$$

$$-\frac{1}{n-2}G^{(1)}_{n-3,j,k-1}(R,Z)], \qquad \text{(A14)}$$

$$G^{(4)}_{njk}(R,Z)=d^{k-1}_{j3}J^{(4)}_{n}(R,Z-d_{j3})-d^{k-1}_{j1}J^{(4)}_{n}(R,Z-d_{j1})$$

$$-\frac{k-1}{n}G^{(4)}_{n-1,j,k-1}(R,Z)+\frac{2(k-1)}{n(n-1)}[G^{(3)}_{n-2,jk}(R,Z)$$

$$-ZG^{(3)}_{n-2,j,k-1}(R,Z)], \qquad \text{(A15)}$$

$$G^{(6)}_{njk}(R,Z)=d^{k-1}_{j3}J^{(6)}_{n}(R,Z-d_{j3})-d^{k-1}_{j1}J^{(6)}_{n}(R,Z-d_{j1})$$

$$-\frac{k-1}{n}G^{(6)}_{n-1,j,k-1}(R,Z)+\frac{2(k-1)}{(n-1)}[G^{(5)}_{n-2,jk}(R,Z)$$

$$-ZG^{(5)}_{n-2,j,k-1}(R,Z)-\frac{1}{n-2}G^{(5)}_{n-3,j,k-1}(R,Z)], \qquad \text{(A16)}$$

In which

$$J_n^{(i)}(R,Z)=\frac{1}{n}F_{n-1}^{(i)}(R,Z), \qquad i=1,3,5, \qquad \text{(A17)}$$

$$J_n^{(2)}(R,Z)=\frac{1}{n}F_{n-1}^{(2)}(R,Z)+\frac{2}{n(n-1)}[ZF_{n-2}^{(1)}+\frac{1}{n-2}F_{n-3}^{(1)}(R,2d-Z)], \qquad \text{(A18)}$$

$$J_n^{(4)}(R,Z)=\frac{1}{n}F_{n-1}^{(4)}(R,Z)+\frac{2}{n(n-1)}ZF_{n-2}^{(3)}, \qquad \text{(A19)}$$

$$J_n^{(6)}(R,Z)=\frac{1}{n}F_{n-1}^{(6)}(R,Z)+\frac{2}{n(n-1)}[ZF_{n-2}^{(5)}+\frac{1}{n-2}F_{n-3}^{(5)}(R,2d-Z)]. \qquad \text{(A20)}$$

In order to use these formulae, the following integrals ~~should be known,~~ are needed, They are calculated from direct manipulation by substituting the corresponding expression into (A.12):

$$G_{0j1}^{(1)}=-\sinh^{-1}\frac{Z-\xi}{R}\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{0j2}^{(1)}=ZG_{0j1}+(R^2+(Z-\xi)^2)^{1/2}\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{0j1}^{(3)}=\frac{(R^2+(Z-\xi)^2)^{1/2}}{R}\Bigg|_{d_{j1}}^{d_{j1}},$$

$$G_{0j2}^{(3)}=ZG_{0j1}^{(3)}+\frac{1}{2R}\left[R^2\sinh^{-1}\frac{Z-\xi}{R}-(Z-\xi)(R^2+(Z-\xi)^2)^{1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{0j1}^{(5)}=-\left[\frac{R^2}{2}\sinh^{-1}\frac{Z-\xi}{R}+\frac{R-\xi}{2}(R^2+(Z-\xi)^2)^{1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{0j2}^{(5)}=ZG_{0j1}^{(3)}-\frac{1}{3}(R^2+(Z-\xi)^2)^{1/2}\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{2j1}^{(2)}=-\left[-\sinh^{-1}\frac{Z-\xi}{R}-\frac{1}{2}(Z-\xi)(R^2+(Z-\xi)^2)^{-1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{2j2}^{(2)}=ZG_{2j1}^{(2)}+\left[(R^2+(Z-\xi)^2)^{1/2}+\frac{R^2}{2}(R^2+(Z-\xi)^2)^{-1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{2j3}^{(2)}=-Z^2G_{2j1}^{(2)}+2ZG_{2j2}^{(2)}+\left[R^2\sinh^{-1}\frac{Z-\xi}{R}-\frac{Z-\xi}{2}(R^2+(Z-\xi)^2)^{1/2}\right.,$$

$$\left.-\frac{R^2(Z-\xi)}{2}(R^2+(Z-\xi)^2)^{-1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{2j1}^{(4)}=\left[\frac{R}{2}(R^2+(Z-\xi)^2)^{-1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G_{2j2}^{(4)}=ZG_{2j1}^{(4)}+\frac{R}{2}\left[\sinh^{-1}\frac{Z-\xi}{R}-(Z-\xi)(R^2+(Z-\xi)^2)^{-1/2}\right]\Bigg|_{d_{j1}}^{d_{j3}},$$

$$G^{(4)}_{2j3}=-Z^2G^{(4)}_{2j1}+2ZG^{(4)}_{2j2}-\frac{R}{2}\ [(R^2+(Z-\xi)^2)^{1/2}+R^2(R^2+(Z-\xi)^2)^{-1/2}]\Big|^{d_{j3}}_{d_{j1}},$$

$$G^{(6)}_{2j1}=-\frac{R}{2}\sinh^{-1}\frac{Z-\xi}{R}\Big|^{d_{j3}}_{d_{j1}},$$

$$G^{(6)}_{2j2}=ZG^{(6)}_{2j1}+[\frac{R^2}{2}(R^2+(Z-\xi)^2)^{1/2}]\Big|^{d_{j3}}_{d_{j1}},$$

$$G^{(6)}_{2j3}=-Z^2G^{(6)}_{2j1}+2ZG^{(6)}_{2j2}+\frac{R^2}{2}[\frac{R^2}{2}\sinh^{-1}\frac{Z-\xi}{R}\ -\frac{Z-\xi}{2}(R^2+(Z-\xi)^2)^{1/2}]\Big|^{d_{j3}}_{d_{j1}}.$$

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