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the coordinate system fixed in the space is replaced with that fixed on the cylinder. In particular, the force is obtained from the linearized theory is identical to that obtained from the solution satisfying the exact body-surface condition, which only contains the term $F_j(1)$ in equation (24). All other terms in equation (24) are due to the free-surface effect. As it is assumed that the disturbance on the free surface is small and linearization can still be applied, the results in the tables are therefore not too surprising.

For the vertical motion when $\eta_3/a>1$, the nonlinear contribution increases rapidly and begins to show its presence. This is mainly because the motion in this case alters submergence substantially during the oscillation.

(a) Table 1. Purely vertical motion with $h=3a$. (a) $\nu a=0.1$, (b) $\nu a=1.0$.

η_3/a	$c_3(0)$	$c_3(1)$	$c_3(2)$	$c_3(3)$	$c_3(4)$	$c_3(5)$	$c_3(6)$	$c_3(7)$
0.00	0.0000	1.1114	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0019	1.1116	0.0086	0.0003	0.0000	0.0000	0.0000	0.0000
0.20	0.0039	1.1122	0.0173	0.0011	0.0001	0.0000	0.0000	0.0000
0.30	0.0059	1.1133	0.0262	0.0025	0.0002	0.0000	0.0000	0.0000
0.40	0.0081	1.1148	0.0353	0.0044	0.0005	0.0000	0.0000	0.0000
0.50	0.0103	1.1168	0.0447	0.0070	0.0009	0.0001	0.0000	0.0000
0.60	0.0127	1.1193	0.0545	0.0103	0.0016	0.0002	0.0000	0.0000
0.70	0.0152	1.1224	0.0650	0.0143	0.0026	0.0004	0.0001	0.0000
0.80	0.0181	1.1262	0.0761	0.0192	0.0040	0.0007	0.0001	0.0000
0.90	0.0212	1.1307	0.0881	0.0250	0.0059	0.0011	0.0002	0.0000
1.00	0.0247	1.1361	0.1011	0.0320	0.0084	0.0018	0.0003	0.0001
1.25	0.0360	1.1545	0.1399	0.0554	0.0182	0.0049	0.0012	0.0003
1.50	0.0531	1.1838	0.1927	0.0913	0.0356	0.0114	0.0034	0.0012
1.75	0.0838	1.2359	0.2709	0.1464	0.0640	0.0224	0.0080	0.0049

(b) ~~Table 1 Purely vertical motion with h=3a. (b) $\nu a=1.0$~~

η_3/a	$c_3(0)$	$c_3(1)$	$c_3(2)$	$c_3(3)$	$c_3(4)$	$c_3(5)$	$c_3(6)$	$c_3(7)$
0.00	0.0000	0.8781	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	-0.0024	0.8780	0.0050	0.0001	0.0000	0.0000	0.0000	0.0000
0.20	-0.0049	0.8774	0.0100	0.0005	0.0000	0.0000	0.0000	0.0000
0.30	-0.0074	0.8766	0.0151	0.0012	0.0001	0.0000	0.0000	0.0000
0.40	-0.0099	0.8754	0.0203	0.0021	0.0002	0.0000	0.0000	0.0000
0.50	-0.0124	0.8739	0.0256	0.0033	0.0004	0.0000	0.0000	0.0000
0.60	-0.0151	0.8720	0.0311	0.0048	0.0006	0.0001	0.0000	0.0000
0.70	-0.0177	0.8698	0.0368	0.0067	0.0010	0.0001	0.0000	0.0000
0.80	-0.0205	0.8672	0.0427	0.0088	0.0015	0.0003	0.0000	0.0000
0.90	-0.0233	0.8642	0.0489	0.0114	0.0022	0.0004	0.0001	0.0000
1.00	-0.0262	0.8608	0.0554	0.0143	0.0031	0.0006	0.0001	0.0000
1.25	-0.0340	0.8505	0.0734	0.0237	0.0065	0.0017	0.0004	0.0001
1.50	-0.0424	0.8378	0.0943	0.0365	0.0121	0.0038	0.0012	0.0004

1.75	-0.0510	0.8231	0.1192	0.0541	0.0214	0.0082	0.0032	0.0012

(a) Table 2. Purely horizontal motion with h=2a. (a) νa=0.1, (b) $\nu a = 1.0$.

η_1/a	$c_3(0)$	$c_1(1)$	$c_3(2)$	$c_1(3)$	$c_3(4)$	$c_1(5)$	$c_3(6)$	$c_1(7)$
0.00	0.0000	1.1114	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0019	1.1114	0.0071	0.0002	0.0000	0.0000	0.0000	0.0000
0.20	0.0039	1.1115	0.0142	0.0007	0.0000	0.0000	0.0000	0.0000
0.30	0.0059	1.1117	0.0213	0.0016	0.0001	0.0000	0.0000	0.0000
0.40	0.0079	1.1120	0.0285	0.0028	0.0003	0.0000	0.0000	0.0000
0.50	0.0099	1.1123	0.0357	0.0044	0.0005	0.0001	0.0000	0.0000
0.60	0.0120	1.1127	0.0430	0.0064	0.0009	0.0001	0.0000	0.0000
0.70	0.0141	1.1131	0.0503	0.0087	0.0014	0.0002	0.0000	0.0000
0.80	0.0163	1.1136	0.0577	0.0113	0.0021	0.0003	0.0001	0.0000
0.90	0.0185	1.1140	0.0653	0.0143	0.0029	0.0005	0.0001	0.0000
1.00	0.0208	1.1144	0.0728	0.0177	0.0040	0.0008	0.0002	0.0000
1.25	0.0269	1.1150	0.0922	0.0275	0.0076	0.0019	0.0005	0.0001
1.50	0.0333	1.1147	0.1118	0.0392	0.0129	0.0039	0.0011	0.0003
1.75	0.0400	1.1130	0.1314	0.0526	0.0198	0.0675	0.0022	0.0007

(b) ~~Table 2 Purely horizontal motion with h=2a. (b) νa=1.0~~

η_1/a	$c_3(0)$	$c_1(1)$	$c_3(2)$	$c_1(3)$	$c_3(4)$	$c_1(5)$	$c_3(6)$	$c_1(7)$
0.00	0.0000	0.8781	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	-0.0024	0.8782	0.0015	0.0000	0.0000	0.0000	0.0000	0.0000
0.20	-0.0049	0.8784	0.0031	0.0001	0.0000	0.0000	0.0000	0.0000
0.30	-0.0073	0.8788	0.0045	0.0003	0.0000	0.0000	0.0000	0.0000
0.40	-0.0098	0.8793	0.0060	0.0006	0.0000	0.0000	0.0000	0.0000
0.50	-0.0122	0.8799	0.0073	0.0009	0.0001	0.0000	0.0000	0.0000
0.60	-0.0146	0.8807	0.0086	0.0012	0.0001	0.0000	0.0000	0.0000
0.70	-0.0170	0.8817	0.0098	0.0017	0.0002	0.0000	0.0000	0.0000
0.80	-0.0193	0.8828	0.0108	0.0021	0.0003	0.0000	0.0000	0.0000
0.90	-0.0216	0.8841	0.0118	0.0026	0.0004	0.0000	0.0000	0.0000
1.00	-0.0239	0.8856	0.0125	0.0031	0.0005	0.0001	0.0000	0.0000
1.25	-0.0293	0.8899	0.0138	0.0045	0.0010	0.0001	0.0000	0.0000
1.50	-0.0343	0.8952	0.0141	0.0058	0.0015	0.0003	0.0000	0.0000
1.75	-0.0387	0.9011	0.0134	0.0070	0.0020	0.0004	0.0001	0.0000

table 3 see fig

Table 3 gives the results for ~~the~~ clockwise circular motion. One notable feature in this case is that there is a steady horizontal force. But the force is still dominated by that obtained from the linearized theory and the other components only begin to show their presence at ~~the~~ large amplitude.

Table 3. Circular motion with h=3a and νa=0.5, (a) Horizontal force, (b) vertical force.

η/a	$c_1(0)$	$c_1(1)$	$c_1(2)$	$c_1(3)$	$c_1(4)$	$c_1(5)$	$c_1(6)$	$c_1(7)$
0.00	0.0000	0.8988	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	-0.0070	0.8984	0.0061	0.0002	0.0000	0.0000	0.0000	0.0000
0.20	-0.0140	0.8971	0.0123	0.0006	0.0000	0.0000	0.0000	0.0000
0.30	-0.0210	0.8951	0.0187	0.0015	0.0001	0.0000	0.0000	0.0000
0.40	-0.0280	0.8923	0.0252	0.0027	0.0003	0.0000	0.0000	0.0000
0.50	-0.0350	0.8887	0.0321	0.0042	0.0005	0.0001	0.0000	0.0000
0.60	-0.0419	0.8843	0.0392	0.0062	0.0009	0.0001	0.0000	0.0000
0.70	-0.0489	0.8792	0.0468	0.0087	0.0015	0.0002	0.0000	0.0000
0.80	-0.0558	0.8734	0.0548	0.0117	0.0023	0.0004	0.0001	0.0000
0.90	-0.0626	0.8670	0.0633	0.0152	0.0034	0.0007	0.0001	0.0000
1.00	-0.0694	0.8599	0.0724	0.0195	0.0049	0.0011	0.0002	0.0000
1.25	-0.0853	0.8399	0.0978	0.0333	0.0105	0.0030	0.0007	0.0001
1.50	-0.0984	0.8173	0.1270	0.0527	0.0199	0.0066	0.0019	0.0004
1.75	-0.1055	0.7930	0.1593	0.0779	0.0334	0.0119	0.0033	0.0006

~~Table 3. Circular motion with h=3a and νa=0.5, (b) vertical force~~

(b)

η/a	$c_3(0)$	$c_3(1)$	$c_3(2)$	$c_3(3)$	$c_3(4)$	$c_3(5)$	$c_3(6)$	$c_3(7)$
0.00	0.0000	0.8988	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	-0.0006	0.8985	0.0061	0.0002	0.0000	0.0000	0.0000	0.0000
0.20	-0.0013	0.8975	0.0123	0.0006	0.0000	0.0000	0.0000	0.0000
0.30	-0.0022	0.8958	0.0186	0.0015	0.0001	0.0000	0.0000	0.0000
0.40	-0.0034	0.8934	0.0250	0.0026	0.0003	0.0000	0.0000	0.0000
0.50	-0.0049	0.8901	0.0317	0.0042	0.0005	0.0001	0.0000	0.0000
0.60	-0.0070	0.8859	0.0386	0.0062	0.0009	0.0001	0.0000	0.0000
0.70	-0.0096	0.8806	0.0460	0.0086	0.0015	0.0002	0.0000	0.0000
0.80	-0.0130	0.8741	0.0538	0.0116	0.0023	0.0004	0.0000	0.0000
0.90	-0.0173	0.8661	0.0623	0.0151	0.0034	0.0007	0.0001	0.0000
1.00	-0.0226	0.8566	0.0717	0.0194	0.0049	0.0011	0.0002	0.0000
1.25	-0.0413	0.8237	0.0999	0.0341	0.0107	0.0030	0.0007	0.0002
1.50	-0.0698	0.7753	0.1372	0.0560	0.0206	0.0067	0.0019	0.0005
1.75	-0.1073	0.7132	0.1827	0.0845	0.0344	0.0119	0.0035	0.0013

7. CONCLUSIONS

(1) A cylinder oscillating at frequency ω and with large amplitude will in general generate an infinite number of waves with frequencies $n\omega$ (n=1,2...) and propagating in both directions.

(2) The purely horizontal motion of a submerged circular cylinder generates vertical forces with frequencies $2n\omega$ and horizontal forces with frequencies $(2n+1)\omega$ (n=0,1,2...).

(3) The large-amplitude circular motion of a submerged circular cylinder generates waves propagating in both directions, which differs from the well-known conclusions of the small-amplitude circular motion (fully linearized theory).