Appendix

Consider the integral (3.12) written in the form.

$$u(x,t) = -\frac{1}{16\pi^3 R i} \int_{C_R} d\ell_\xi \, \frac{1}{r_{x\xi}} \int_{b-i\infty}^{b+i\infty} e^{pt} (p-i\sigma)^{-1} \times$$

$$\times \{(p^2 + d^2(x-\xi))(p^2+N^2)\}^{-1/2} \, dp, \quad b>0,$$

where $r_{x\xi}$ is the distance between x and ξ. Suppose that $x \in K^{\sigma}(0^+)$ or $x \in K^{\sigma}(0^-)$ and $x \neq 0^{\pm}$, $x \notin C_R$ *). Let ξ_0 be the point of C_R for which the distance $r_{x\xi}$ in the integrand is minimal and φ be the polar angle of the polar coordinate system on the plane $\xi_3 = 0$ with center $\hat{x}$. For the sake of clarity we assume that $\varphi \in (-\pi, \pi)$ and φ equals zero at ξ_0. In the vicinity of ξ_0 the curve C_R can be parametrized by φ. We use the designation: $r(\varphi) = r_{x\xi}$, $\rho(\varphi) = r_{\hat{x}\xi}$, where ξ belongs to C_R within this vicinity and the polar angle of ξ is φ. We can rewrite $u(x,t)$ as follows:

$$u(x,t) = A(\delta\varphi, x, t) + I(\delta\varphi, x, t)$$

where

$$A(\delta\varphi, x, t) = \frac{e^{i3\pi/4}}{16\pi^3 R i} \int_{-\delta\varphi}^{\delta\varphi} d\varphi \, \frac{(\rho^2(\varphi) + (\rho'(\varphi))^2)^{1/2}}{(N^2-\sigma^2)^{1/2}(2\sigma)^{1/2}} \, \frac{1}{r(\varphi)} \times$$

$$\times \int_{b-i\infty}^{b+i\infty} e^{pt} (p-i\sigma)^{-1} (p - i d(x-\xi(\varphi)))^{-1/2} \, dp,$$

$$I(\delta\varphi, x, t) = u(x,t) - A(\delta\varphi, x, t)$$

---

*) Hereafter x is asssumed to be fixed and to belong to those halves of $K(0^{\pm})$ which contain C_R.

$\delta\varphi$ is sufficiently small.

One can show, that $|I(\delta\varphi, x, t)| < C(\delta\varphi, x)$.
Let us consider $A(\delta\varphi, x, t)$. Using the convolution property of the Laplace transform we find:

$$A(\delta\varphi, x, t) = \frac{e^{i(\sigma t + 3\pi/4)}}{8\pi^{5/2} R (N^2 - \sigma^2)^{1/2}} (2\sigma)^{-1/2} \times$$

$$\times \int_{-\delta\varphi}^{\delta\varphi} d\varphi \, \frac{1}{r(\varphi)} (\rho^2(\varphi) + (\rho'(\varphi))^2)^{1/2} \int_0^t d\tau \, \tau^{-1/2} e^{i(d(x - \xi(\varphi)) - \sigma)\tau}$$

we designate the integral part of the expression in the right -hand side of this equality as $P(\delta\varphi, x, t)$. Making the change of variable:

$$s(\varphi) = \begin{cases} -(\sigma - d(x - \xi(\varphi)))^{1/2} & \text{if} \quad -\delta\varphi \le \varphi < 0 \\ (\sigma - d(x - \xi(\varphi)))^{1/2} & \text{if} \quad 0 \leqslant \varphi \leqslant \delta\varphi \end{cases}$$

we get:

$$P(\delta\varphi, x, t) = \int_{s_1}^{s_2} ds \, \psi(s) \int_0^t \tau^{-1/2} e^{-is^2\tau} d\tau$$

where $s_1 = s(-\delta\varphi)$, $s_2 = s(\delta\varphi)$ and

$$\psi(s) = \frac{2s}{N|x_3|} \left[(\rho^2(\varphi) + (\rho'(\varphi))^2)^{1/2} \frac{r(\varphi)}{r'(\varphi)} \right] \Big|_{\varphi = \varphi(s)}$$

We note, that $\psi(0) = (2R/\sigma|\hat{x}_1|)^{1/2}$

After that we represent $P(\delta\varphi, x, t)$ in the following way:

$$P(\delta\varphi, x, t) = 2\psi(0)\int_0^{\delta} ds \int_0^{t} \tau^{-1/2} e^{-is^2\tau}\, d\tau + $$
$$+ B_1(\delta\varphi, x, t)$$

where $\delta = \min(-s_1, s_2)$ and $B_1(\delta\varphi, x, t)$ is bounded in time. i.e. $|B_1(\delta\varphi, x, t)| < C_1(\delta\varphi, x)$. The integral on the right we estimate by the chain of transformations:

$$\int_0^{\delta} ds \int_0^{t} \tau^{-1/2} e^{-is^2\tau}\, d\tau = \int_0^{t} d\tau\, \tau^{-1/2} \int_0^{\delta} e^{-is^2\tau}\, ds =$$
$$= \int_0^{t} d\tau\, \tau^{-1} \int_0^{\delta\tau^{1/2}} e^{-i\beta^2}\, d\beta = \frac{\pi}{2}^{1/2} e^{-i\frac{\pi}{4}} \ln t + B_2(\delta\varphi, x, t)$$

where $B_2(\delta\varphi, x, t)$ is bounded in time. Here we took advantage of the asymptotic behaviour of the Fresnel integral at large values of arguments.

Thus we obtain:

$$u(x,t) \sim \frac{\ln t\; e^{i(\sigma t + \pi/2)}}{8\pi^2\sigma\,[(N^2-\sigma^2)R|\hat{x}|]^{1/2}}$$

$\cancel{x \in K^{\sigma}(0^{\pm})}$ ~~and~~ $\cancel{x \neq 0^{\pm},\ x \notin C_R}$.

At points x not belonging to the cones $K^{\sigma}(0^{\pm})$ the limiting amplitude $w(x) = \lim_{t\to\infty}(e^{-i\sigma t}u(x,t))$ exists and has the form:

It can be shown that on the other halves of $K(0^{\pm})$

$$u(x,t) \sim \frac{\ln t\; e^{i(\sigma t + \pi)}}{8\pi^2\sigma\,[(N^2-\sigma^2)R|\hat{x}|]^{1/2}}$$

$$w(x)=\frac{1}{8\pi^2 R}\int\limits_{C_R} dl_\xi \, \frac{e^{i\frac{\pi}{4}\left(3-\operatorname{sgn}\left(\sigma-N\frac{|x_3|}{r_{x\xi}}\right)\right)}}{(N^2-\sigma^2)^{1/2}\,|x_3^2N^2-\sigma^2 r_{x\xi}^2|^{1/2}}$$

This integral can be shown to diverge at points of $K^{\sigma}(0^{\pm})$.