FM 8317

X. W. The nonlinear evolution of high-frequency resonant triad waves in an oscillatory Stokes' layer at high Reynolds number

D Appendix D

$$
\begin{aligned}
N^{(r)} &= -i\alpha\bar{U}_y^{-1}\hat{U}^*_{1,YY}[\hat{A}\hat{U}^{(2,0)}_{2,Y}+\frac{1}{2}\hat{A}\hat{U}^{(2,2)}_{2,Y}-3\beta\hat{U}_1\hat{W}^{(2,2)}_2-\bar{U}_y^{-1}\hat{U}_{1,Y}S^{(2,0)}_{11}]\\
&\quad -i\alpha\bar{U}_y^{-1}\hat{U}_{1,YY}[\hat{A}\hat{U}^{*(0,0)}_{2,Y}+\frac{1}{2}\hat{A}\hat{U}^{*(0,2)}_{2,Y}-\beta\hat{W}_1\hat{U}^{*(0,0)}_2+\frac{1}{4}\hat{U}_1\hat{V}^{*(0,2)}_{2,Y}-\bar{U}_y^{-1}\hat{U}^*_{1,Y}S^{(2,0)}_{11}]\\
&\quad -2i\alpha\beta\hat{U}_{1,Y}\hat{W}^{(r)}_3-4i\alpha\beta\hat{U}_1\hat{W}^{(r)}_{3,Y}-3i\alpha\hat{U}_1\hat{V}^{(r)}_{3,YY}-\hat{A}(\hat{V}^{(r)}_{3,YYY}+\beta\hat{W}^{(r)}_{3,YY})\\
&\quad -2i\alpha\beta\hat{U}^*_{1,Y}\tilde{W}^{(r)}_3-\frac{4}{3}i\alpha\beta\hat{U}^*_1[\tilde{W}^{(r)}_{3,Y}+\frac{1}{2}i\bar{S}^{-1}\hat{W}_{1,Y}\hat{V}^{(2,0)}_{2,YY}]-\frac{1}{3}i\alpha\hat{U}^*_1\tilde{V}^{(r)}_{3,YY}\\
&\quad +[-2i\alpha\hat{U}^{*(0,0)}_2\hat{V}^{(2,0)}_{2,YY}-4i\alpha\beta\hat{U}^{*(0,2)}_{2,Y}\hat{W}^{(2,2)}_2\\
&\quad\ -2\beta\hat{V}^{*(0,2)}_{2,Y}\hat{W}^{(2,2)}_{2,Y}-\beta\hat{V}^{*(0,2)}_{2,YY}\hat{W}^{(2,2)}_2-2i\alpha\bar{U}_y^{-1}\hat{U}^{*(0,0)}_{2,YY}S^{(2,0)}_{11}]\\
&\quad -4i\alpha\beta\hat{U}_1[\frac{1}{2}i\bar{S}^{-1}\hat{W}^*_{1,Y}\hat{V}^{(2,0)}_{2,YY}-i\beta^{-1}\bar{S}^3\sin^6\theta\hat{W}^{(1)}_0\Pi_{0,Y}]\\
&\quad -\hat{A}[\frac{1}{2}i\beta\bar{S}^{-1}\hat{W}^*_{1,Y}\hat{V}^{(2,0)}_{2,YYY}-i\bar{S}^3\sin^6\theta\hat{W}^{(1)}_0\Pi_{0,YY}]\ , \qquad (D\text{-}1)
\end{aligned}
$$

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\begin{aligned}
N^{(s)} &= \bar{U}_y^{-1}\hat{U}_{1,YY}\hat{L}^{(1)}_0(\hat{V}_{3,Y}+\beta\hat{W}_3)-2i\alpha\hat{U}_{1,Y}(\hat{V}_{3,Y}+\beta\hat{W}_3)\\
&\quad +\frac{1}{3}\bar{U}_y^{-1}\hat{U}^*_{1,YY}\hat{L}^{(3)}_0(\tilde{V}_{3,Y}+\beta\tilde{W}_3)+\frac{2}{3}i\alpha\hat{U}^*_{1,Y}(\tilde{V}_{3,Y}+\beta\tilde{W}_3)\\
&\quad -i\alpha\bar{U}_y^{-1}\hat{U}_{1,YY}[(2i\alpha\bar{U}_y)^{-1}\hat{U}^*_{1,Y}\hat{L}^{(2)}_0\hat{V}^{(2,0)}_{2,Y}+\frac{1}{2}\hat{U}^*_1\hat{V}^{(2,0)}_{2,Y}]\\
&\quad -i\alpha\bar{U}_y^{-1}\hat{U}^*_{1,YY}[(2i\alpha\bar{U}_y)^{-1}\hat{U}_{1,Y}\hat{L}^{(2)}_0\hat{V}^{(2,0)}_{2,Y}-\frac{1}{2}\hat{U}_1\hat{V}^{(2,0)}_{2,Y}]\\
&\quad +\bar{U}_y^{-1}[\hat{U}^{*(0,0)}_{2,YY}\hat{L}^{(2)}_0\hat{V}^{(2,0)}_{2,Y}-\frac{1}{2}\hat{A}\hat{U}^*_{1,YYY}\hat{V}^{(2,0)}_{2,Y}]\ , \qquad (D\text{-}2)
\end{aligned}
$$

$$
\begin{aligned}
N^{(t)} &= -2i\alpha\beta\hat{U}_{1,Y}\hat{W}^{(t)}_3-2i\alpha\hat{U}_1[\bar{S}^4\sin^6\theta\hat{W}^{(2)}_0\Pi_0]-\hat{A}[-i\bar{S}^5\sin^6\theta\hat{W}^{(3)}_0\Pi_0]\\
&\quad -i\alpha\bar{U}_y^{-1}\hat{U}_{1,YY}[\frac{3}{2}i\alpha\hat{U}_1\hat{U}^{*(0,2)}_2+\frac{1}{2}\hat{U}_{1,Y}\hat{V}^{*(0,2)}_2]\\
&\quad +[-4i\alpha\beta\hat{U}^{*(0,2)}_2\hat{W}^{(2,2)}_{2,Y}-\beta\hat{V}^{*(0,2)}_2\hat{W}^{(2,2)}_{2,YY}]\ , \qquad (D\text{-}3)
\end{aligned}
$$

$$
\begin{aligned}
N^{(b)} &= -2i\alpha\beta\hat{U}_{1,Y}\hat{W}^{(b)}_3-4i\alpha\beta\hat{U}_1\hat{W}^{(b)}_{3,Y}-3i\alpha\hat{U}_1\hat{V}^{(b)}_{3,YY}-\hat{A}(\hat{V}^{(b)}_{3,YYY}+\beta\hat{W}^{(b)}_{3,YY})\\
&\quad -2i\alpha\beta\hat{U}^*_{1,Y}\tilde{W}^{(b)}_3-\frac{4}{3}i\alpha\beta\hat{U}^*_1\tilde{W}^{(b)}_{3,Y}-\frac{1}{3}i\alpha\hat{U}^*_1\tilde{V}^{(b)}_{3,YY}\\
&\quad -i\alpha\bar{U}_y^{-1}\hat{U}_{1,YY}\hat{U}^*_{1,Y}\hat{B}-i\alpha\bar{U}_y^{-1}\hat{U}^*_{1,YY}\hat{U}_{1,Y}\hat{B}+2i\alpha\hat{U}^{*(0,0)}_{2,YY}\hat{B}\ . \qquad (D\text{-}4)
\end{aligned}
$$