

The full version of recursion formulae (17) in the article "Electrophoresis of a colloidal sphere parallel to a dielectric plane" by H. J. Keh & S. B. Chen, published in J. Fluid Mech. 194, 377-390 (1988).

$$A_n = 0 \quad (a)$$

$$\begin{aligned} & \frac{(n+2)(n+3)}{2n+3} G_{n+1} + \frac{1}{2n+3} E_{n+1} - \frac{(n-1)(n-2)}{2n-1} G_{n-1} - \frac{1}{2n-1} E_{n-1} \\ &= \frac{\epsilon \zeta_w E_\infty}{4 \pi \eta} \left[\frac{2n+4}{2n+3} S_{n+1} - 2 S_n + \frac{2n-2}{2n-1} S_{n-1} \right] \end{aligned} \quad (b)$$

$$\begin{aligned} & \frac{(n+2)(n+3)}{2n+3} \left\{ G_{n+1} \cosh\left(n + \frac{3}{2}\right) \xi_p + H_{n+1} \sinh\left(n + \frac{3}{2}\right) \xi_p \right\} \\ & - \frac{(n-2)(n-1)}{2n-1} \left\{ G_{n-1} \cosh\left(n - \frac{1}{2}\right) \xi_p + H_{n-1} \sinh\left(n - \frac{1}{2}\right) \xi_p \right\} \\ & - \frac{1}{2n-1} \left\{ E_{n-1} \cosh\left(n - \frac{1}{2}\right) \xi_p + F_{n-1} \sinh\left(n - \frac{1}{2}\right) \xi_p \right\} \\ & + \frac{1}{2n+3} \left\{ E_{n+1} \cosh\left(n + \frac{3}{2}\right) \xi_p + F_{n+1} \sinh\left(n + \frac{3}{2}\right) \xi_p \right\} \\ &= \frac{\epsilon \zeta_p E_\infty}{4 \pi \eta} \left\{ \frac{2n+4}{2n+3} S_{n+1} \cosh\left(n + \frac{3}{2}\right) \xi_p - 2 S_n \cosh \xi_p \cosh\left(n + \frac{1}{2}\right) \xi_p \right. \\ & + \frac{2n-2}{2n-1} S_{n-1} \cosh\left(n - \frac{1}{2}\right) \xi_p + \frac{2\sqrt{2}}{2n-1} \exp\left[-\left(n - \frac{1}{2}\right) \xi_p\right] \\ & \left. - \frac{2\sqrt{2}}{2n+1} \exp\left[-\left(n + \frac{3}{2}\right) \xi_p\right] \right\} \end{aligned} \quad (c)$$

$$\begin{aligned} & C_n + \frac{n-1}{2n-1} C_{n-1} + \frac{n+2}{2n+3} C_{n+1} + \frac{1}{2n-1} E_{n-1} - \frac{1}{2n+3} E_{n+1} + \\ & \frac{(n+2)(n+3)}{2n+3} G_{n+1} - \frac{(n-1)(n-2)}{2n-1} G_{n-1} \\ &= \frac{\epsilon \zeta_w E_\infty}{4 \pi \eta} \left\{ - \frac{4n^2 + 4n - 6}{(2n-1)(2n+3)} S_n + \frac{(n-1)(n-2)}{2n-1} S_{n-2} - \frac{2(n-1)^2}{2n-1} S_{n-1} \right. \\ & \left. + \frac{2(n+2)^2}{2n+3} S_{n+1} - \frac{(n+2)(n+3)}{2n+3} S_{n+2} \right\} \end{aligned} \quad (d)$$

$$C_n \cosh\left(n + \frac{1}{2}\right) \xi_p + D_n \sinh\left(n + \frac{1}{2}\right) \xi_p + \frac{2 \cosh \xi_p}{\sinh \xi_p} B_n \sinh\left(n + \frac{1}{2}\right) \xi_p$$

$$\begin{aligned}
& - \frac{2}{\sinh \xi_p} \frac{n-1}{2n-1} B_{n-1} \sinh(n - \frac{1}{2}) \xi_p - \frac{2}{\sinh \xi_p} \frac{n+2}{2n+3} B_{n+1} \sinh(n + \frac{3}{2}) \xi_p \\
& = \frac{\epsilon \zeta_p^E}{4 \pi \eta} \left\{ - \frac{4n^2 + 4n - 6}{(2n-1)(2n+3)} S_n \cosh(n + \frac{1}{2}) \xi_p \right. \\
& \quad + \frac{(n-2)(n-1)}{2n-1} S_{n-2} \cosh(n - \frac{3}{2}) \xi_p - 2 \cosh \xi_p \frac{(n-1)^2}{2n-1} S_{n-1} \cosh(n - \frac{1}{2}) \xi_p \\
& \quad + 2 \cosh \xi_p \frac{(n+2)^2}{2n+3} S_{n+1} \cosh(n + \frac{3}{2}) \xi_p - \frac{(n+2)(n+3)}{2n+3} S_{n+2} \cosh(n + \frac{5}{2}) \xi_p \\
& \quad \left. + 2^{5/2} \cosh \xi_p \left[\frac{n-1}{2n-1} \exp[-(n - \frac{1}{2}) \xi_p] + \frac{n+2}{2n+3} \exp[-(n + \frac{3}{2}) \xi_p] \right] \right. \\
& \quad \left. - 2^{5/2} \exp[-(n + \frac{1}{2}) \xi_p] \right\} \tag{e}
\end{aligned}$$

$$\begin{aligned}
& - \frac{2n(n+1)}{(2n-1)(2n+3)} \left\{ C_n \cosh(n + \frac{1}{2}) \xi_p + D_n \sinh(n + \frac{1}{2}) \xi_p \right\} \\
& - \frac{(n+2)(n+3)}{(2n+3)(2n+5)} \left\{ C_{n+2} \cosh(n + \frac{5}{2}) \xi_p + D_{n+2} \sinh(n + \frac{5}{2}) \xi_p \right\} \\
& - \frac{(n-2)(n-1)}{(2n-3)(2n-1)} \left\{ C_{n-2} \cosh(n - \frac{3}{2}) \xi_p + D_{n-2} \sinh(n - \frac{3}{2}) \xi_p \right\} \\
& - \frac{1}{(2n-1)(2n+3)} \left\{ E_n \cosh(n + \frac{1}{2}) \xi_p + F_n \sinh(n + \frac{1}{2}) \xi_p \right\} \\
& + \frac{n+2}{(2n+3)(2n+5)} \left\{ E_{n+2} \cosh(n + \frac{5}{2}) \xi_p + F_{n+2} \sinh(n + \frac{5}{2}) \xi_p \right\} \\
& - \frac{\cosh \xi_p}{2n+3} \left\{ E_{n+1} \cosh(n + \frac{3}{2}) \xi_p + F_{n+1} \sinh(n + \frac{3}{2}) \xi_p \right\} \\
& + \frac{\cosh \xi_p}{2n-1} \left\{ E_{n-1} \cosh(n - \frac{1}{2}) \xi_p + F_{n-1} \sinh(n - \frac{1}{2}) \xi_p \right\} \\
& - \frac{n-1}{(2n-3)(2n-1)} \left\{ E_{n-2} \cosh(n - \frac{3}{2}) \xi_p + F_{n-2} \sinh(n - \frac{3}{2}) \xi_p \right\} \\
& - \frac{3(n-1)(n+2)}{(2n-1)(2n+3)} \left\{ G_n \cosh(n + \frac{1}{2}) \xi_p + H_n \sinh(n + \frac{1}{2}) \xi_p \right\} \\
& - \frac{(n+2)(n+3)(n+4)}{(2n+3)(2n+5)} \left\{ G_{n+2} \cosh(n + \frac{5}{2}) \xi_p + H_{n+2} \sinh(n + \frac{5}{2}) \xi_p \right\} \\
& + \cosh \xi_p \frac{(n+2)(n+3)}{2n+3} \left\{ G_{n+1} \cosh(n + \frac{3}{2}) \xi_p + H_{n+1} \sinh(n + \frac{3}{2}) \xi_p \right\} \\
& - \cosh \xi_p \frac{(n-1)(n-2)}{2n-1} \left\{ G_{n-1} \cosh(n - \frac{1}{2}) \xi_p + H_{n-1} \sinh(n - \frac{1}{2}) \xi_p \right\} \\
& + \frac{(n-3)(n-2)(n-1)}{(2n-3)(2n-1)} \left\{ G_{n-2} \cosh(n - \frac{3}{2}) \xi_p + H_{n-2} \sinh(n - \frac{3}{2}) \xi_p \right\} \\
& = - \frac{\epsilon \zeta_p^E}{4 \pi \eta} \left\{ \cosh^2 \xi_p \frac{2n^2 + 2n - 6}{(2n-1)(2n+3)} S_n \cosh(n + \frac{1}{2}) \xi_p \right. \\
& \quad + \cosh^2 \xi_p \frac{2(n+3)^2(n+2)}{(2n+3)(2n+5)} S_{n+2} \cosh(n + \frac{5}{2}) \xi_p \\
& \quad \left. - \cosh^2 \xi_p \frac{2(n-2)^2(n-1)}{(2n-3)(2n-1)} S_{n-2} \cosh(n - \frac{3}{2}) \xi_p \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{4n^2 + 4n - 6}{(2n-1)(2n+3)} S_n \cosh\left(n + \frac{1}{2}\right) \xi_p \\
& + \frac{(n+3)(n+2)}{2n+3} S_{n+2} \cosh\left(n + \frac{5}{2}\right) \xi_p \\
& - \frac{(n-2)(n-1)}{2n-1} S_{n-2} \cosh\left(n - \frac{3}{2}\right) \xi_p \\
& - \cosh \xi_p \frac{(n+2)(n+3)(n+4)}{(2n+3)(2n+5)} S_{n+3} \cosh\left(n + \frac{7}{2}\right) \xi_p \\
& + \cosh \xi_p \frac{(n-3)(n-2)(n-1)}{(2n-3)(2n-1)} S_{n-3} \cosh\left(n - \frac{5}{2}\right) \xi_p \\
& + \cosh \xi_p \frac{(n-1)(10n^3 - 13n^2 - 24n + 36)}{(2n-3)(2n-1)(2n+3)} S_{n-1} \cosh\left(n - \frac{1}{2}\right) \xi_p \\
& - \cosh \xi_p \frac{(n+2)(20n^4 + 96n^3 + 107n^2 - 42n - 37)}{(2n-1)(2n+1)(2n+3)(2n+5)} S_{n+1} \cosh\left(n + \frac{3}{2}\right) \xi_p \\
& + 2^{5/2} \left[\cosh^2 \xi_p \frac{2n^2 + 2n - 3}{(2n-1)(2n+3)} + 1 \right] \exp\left[-\left(n + \frac{1}{2}\right) \xi_p\right] \\
& + 2^{5/2} \cosh^2 \xi_p \frac{(n+3)(n+2)}{(2n+5)(2n+3)} \exp\left[-\left(n + \frac{5}{2}\right) \xi_p\right] \\
& - 2^{7/2} \cosh \xi_p \left[\frac{n+2}{2n+3} \exp\left[-\left(n + \frac{3}{2}\right) \xi_p\right] + \frac{n-1}{2n-1} \exp\left[-\left(n - \frac{1}{2}\right) \xi_p\right] \right] \\
& + 2^{5/2} \cosh^2 \xi_p \frac{(n-2)(n-1)}{(2n-3)(2n-1)} \exp\left[-\left(n - \frac{3}{2}\right) \xi_p\right] \} \quad (f)
\end{aligned}$$

$$\begin{aligned}
& 5 C_n - (n-1) C_{n-1} + (n+2) C_{n+1} - E_{n-1} + 2 E_n - E_{n+1} \\
& + (n-2)(n-1) G_{n-1} - 2(n-1)(n+2) G_n + (n+2)(n+3) G_{n+1} \\
& - 2(n-1) B_{n-1} + 2(2n+1) B_n - 2(n+2) B_{n+1} = 0 \quad (g)
\end{aligned}$$

$$\begin{aligned}
& 5 D_n - (n-1) D_{n-1} + (n+2) D_{n+1} - F_{n-1} + 2 F_n - F_{n+1} \\
& + (n-2)(n-1) H_{n-1} - 2(n-1)(n+2) H_n + (n+2)(n+3) H_{n+1} \\
& - 2(n-1) A_{n-1} + 2(2n+1) A_n - 2(n+2) A_{n+1} = 0 \quad (h)
\end{aligned}$$