FM 6390 To be held in Editorial files

~~Appendices~~ Appendix to the paper "Motion of a spherical particle in a rarefied gas. I. ~~Motion of a spherical~~ A liquid particle in its saturated vapour" ~~at arbitrary Knudsen numbers"~~ by S.A.Beresnev, V.G.Chernyak, P.E.Suetin.

Coefficients α_{ij}, α_i from eq. (17) have the following form (designations are taken from the work by Cercignani, Pagani and Bassanini 1968):

$$\alpha_0 = 0,$$

$$\alpha_1 = 4\sqrt{\pi}\, U_\infty^2 R \int_R^\infty \frac{dr}{r^2} B_1(r,R),$$

$$\alpha_2 = 8\sqrt{\pi}\, U_\infty^2 R \int_R^\infty \frac{dr}{r}\left[\left(1-\frac{R^2}{r^2}\right)B_2(r,R) + \frac{1}{2}\left(1+\frac{R^2}{r^2}\right)B_3(R,r)\right],$$

$$\alpha_3 = -2\pi U_\infty^2 R^2 (1-\alpha_m),$$

$$\alpha_4 = 4\sqrt{\pi}\, U_\infty^2 R^2 (1-\alpha_m),$$

$$\alpha_5 = 8\sqrt{\pi}\, U_\infty^2 R \int_R^\infty \frac{dr}{r}\left[B_2(r,R) + \frac{1}{2} B_3(R,r)\right],$$

$$\alpha_{11} = 4\pi U_\infty^2 R^2 \int_R^\infty \frac{dr}{r^2} + 4\sqrt{\pi}\, U_\infty^2 R^2 \iint_R^\infty \frac{dr\,dr'}{r^2 r'^2} A_{11}(r,r'),$$

$$\alpha_{12} = \alpha_{21} = 8\sqrt{\pi}\, U_\infty^2 R^2 \iint_R^\infty \frac{dr\,dr'}{r^2 r'}\left[\left(1-\frac{R^2}{r^2}\right)A_{12}(r,r') + \frac{1}{2}\left(1+\frac{R^2}{r^2}\right)A_{13}(r,r')\right],$$

$$\alpha_{13} = \alpha_{31} = -4\sqrt{\pi}\, U_\infty^2 R (1-\alpha_m)\int_R^\infty \frac{dr}{r^2} A_{12}(r,R),$$

$$\alpha_{14} = \alpha_{41} = -4\sqrt{\pi}\, U_\infty^2 \ (1-\alpha_m)\int_R^\infty \frac{dr}{r} A_{23}(R,r),$$

$$\alpha_{15} = \alpha_{51} = 8\sqrt{\pi}\, U_\infty^2 R^2 \iint_R^\infty \frac{dr\,dr'}{r^2 r'}\left[A_{12}(r,r') + \frac{1}{2} A_{13}(r,r')\right],$$

$$\alpha_{22} = -4\pi U_\infty^2 R^2 \int_R^\infty \left(3 - 2\frac{R^2}{r^2} + 3\frac{R^4}{r^4}\right)dr +$$
$$+ 16\sqrt{\pi}\, U_\infty^2 R^2 \iint_R^\infty \frac{dr\,dr'}{r r'}\left[\left(1-\frac{R^2}{r^2}\right)\left(1-\frac{R^2}{r'^2}\right)A_{22}(r,r') +\right.$$
$$\left. + \left(1-\frac{R^2}{r^2}\right)\left(1+\frac{R^2}{r'^2}\right)A_{23}(r,r') + \frac{1}{4}\left(1+\frac{R^2}{r^2}\right)\left(1+\frac{R^2}{r'^2}\right)A_{33}(r,r')\right],$$

$$\alpha_{23} = \alpha_{32} = -8\sqrt{\pi}\, U_\infty^2 R (1-\alpha_m)\int_R^\infty \frac{dr}{r}\left[\left(1-\frac{R^2}{r^2}\right)A_{22}(r,R) + \frac{1}{2}\left(1+\frac{R^2}{r^2}\right)A_{23}(R,r)\right],$$

$$\alpha_{24} = \alpha_{42} = 8\sqrt{\pi}\, U_\infty^2 R (1-\alpha_m)\int_R dr\left[\left(1-\frac{R^2}{r^2}\right)A_{41}(r,R) + \frac{1}{2}\left(1+\frac{R^2}{r^2}\right)A_{42}(r,R)\right],$$

$$\alpha_{25} = \alpha_{52} = -4\pi u_\infty^2 R^2 \int_R^\infty dr \left(3 - \frac{R^2}{r^2}\right) + 16\sqrt{\pi}\, u_\infty^2 R^2 \iint_R^\infty \frac{dr\,dr'}{r r'} \times$$
$$\times \left[\left(1 - \frac{R^2}{r^2}\right) A_{22}(r, r') + A_{23}(r, r') + \frac{1}{4}\left(1 + \frac{R^2}{r^2}\right) A_{33}(r, r')\right],$$

$$\alpha_{33} = 2\sqrt{\pi}\, R^2 u_\infty^2 (1 - \alpha_m),$$

$$\alpha_{34} = \alpha_{43} = 0,$$

$$\alpha_{35} = \alpha_{53} = -8\sqrt{\pi}\, u_\infty^2 R \int_R^\infty \frac{dr}{r} \left[A_{22}(r, R) + \frac{1}{2} A_{23}(R, r)\right],$$

$$\alpha_{44} = -2\sqrt{\pi}\, u_\infty^2 R^2 \frac{1 - \alpha_m}{1 - \alpha_\tau},$$

$$\alpha_{45} = \alpha_{54} = 8\sqrt{\pi}\, u_\infty^2 R (1 - \alpha_m) \int_R^\infty dr \left[A_{41}(r, R) + \frac{1}{2} A_{42}(r, R)\right],$$

$$\alpha_{55} = -12\pi u_\infty^2 R^2 \int_R^\infty dr + 16\sqrt{\pi}\, u_\infty^2 R^2 \iint_R^\infty \frac{dr\,dr'}{r r'} \times$$
$$\times \left[A_{22}(r, r') + A_{23}(r, r') + \frac{1}{4} A_{33}(r, r')\right].$$

Some of the given coefficients comprise diverging integrals; these divergences disappear in complete expressions of coefficients. A_{ij} and B_i have the following form:

$$A_{11}(r,r') = \int \left[\rho^2-(r^2+r'^2)\right]\frac{\mathcal{J}_1(\rho)}{\rho}d\rho,$$

$$A_{12}(r,r') = \frac{1}{2r'}\int\left[\rho^4-2r^2\rho^2+(r^4-r'^4)\right]\frac{\mathcal{J}_2(\rho)}{\rho^2}d\rho,$$

$$A_{13}(r,r') = -\frac{1}{2r'}\int\left[\rho^4-2(r^2+r'^2)\rho^2+(r^2-r'^2)^2\right]\frac{\mathcal{J}_2(\rho)}{\rho^2}d\rho,$$

$$A_{22}(r,r') = \frac{1}{4rr'}\int\left[\rho^6-(r^2+r'^2)\rho^4-(r^2-r'^2)^2\rho^2+(r^4-r'^4)(r^2-r'^2)\right]\frac{\mathcal{J}_3(\rho)}{\rho^3}d\rho,$$

$$A_{23}(r,r') = -\frac{1}{4rr'}\int\left[\rho^6-(r^2+3r'^2)\rho^4+(r'^2-r^2)(r^2+3r'^2)\rho^2-\right.$$
$$\left.-(r'^2-r^2)^3\right]\frac{\mathcal{J}_3(\rho)}{\rho^3}d\rho,$$

$$A_{33}(r,r') = \frac{1}{4rr'}\int\left[\rho^6-3(r^2+r'^2)\rho^4+\left[2(r^2+r'^2)^2+(r^2-r'^2)^2\right]\rho^2-\right.$$
$$\left.-(r^2-r'^2)^2(r^2+r'^2)\right]\frac{\mathcal{J}_3(\rho)}{\rho^3}d\rho.$$

The region of integration extends from $|\underline{r}-\underline{r}'|$ to $\sqrt{r^2-R^2}+\sqrt{r'^2-R^2}$.

$A_{12}(r,R)$, $A_{23}(R,r)$, $A_{22}(r,R)$ are derived from the previous expressions by substituting $\underline{r}'$ for $\underline{r}$ and r for R.

$$A_{41}(r,R) = \frac{1}{8r^2R^2}\int\left[\rho^8-2(r^2+R^2)\rho^6+2(r^4-R^4)(r^2-R^2)\rho^2-\right.$$
$$\left.-(r^2-R^2)^4\right]\frac{\mathcal{J}_4(\rho)}{\rho^4}d\rho,$$

$$A_{42}(r,R) = -\frac{1}{8r^2R^2}\int\left[\rho^8 - 2(2r^2+R^2)\rho^6 + 2r^2(3r^2+R^2)\rho^4 - \right.$$
$$\left. - 2(2r^6 - r^4R^2 - R^6)\rho^2 + (r^2-R^2)^2(r^4-R^4)\right] \frac{J_4(\rho)}{\rho^4}\, d\rho,$$

in which the integration is carried out from $r-R$ to $\sqrt{r^2-R^2}$.

$$B_1(r,R) = \int\left[\rho^4 - (r^2-R^2)^2\right] \frac{J_3(\rho)}{\rho^3}\, d\rho,$$
$$B_2(r,R) = \frac{1}{2r}\int\left[\rho^6 + (r^2-R^2)\rho^4 - (r^2-R^2)^2\rho^2 - (r^2-R^2)^3\right] \frac{J_4(\rho)}{\rho^4}\, d\rho,$$
$$B_3(R,r) = -\frac{1}{2r}\int\left[\rho^6 - (3r^2+R^2)\rho^4 - (3r^2+R^2)(R^2-r^2)\rho^2 + \right.$$
$$\left. + (R^2-r^2)^3\right] \frac{J_4(\rho)}{\rho^4}\, d\rho,$$

in which the integration is carried out from $r-R$ to $\sqrt{r^2-R^2}$.

Asymptotic expressions for coefficients α_{ij}, α_i from eq. (17) at $R \gg 1 \; (Kn \ll 1)$ have the following form (multiplier u_∞^2 is omitted):

$$\alpha_0 = 0,$$
$$\alpha_1 \simeq -2\pi R + 4\sqrt{\pi} - 24\sqrt{\pi}R^{-2},$$
$$\alpha_2 \simeq -8\sqrt{\pi}R^2 + 4\pi R - 24\sqrt{\pi} + 912\sqrt{\pi}R^{-2},$$
$$\alpha_3 = -2\pi R^2(1-\alpha_m),$$
$$\alpha_4 = 4\sqrt{\pi}R^2(1-\alpha_m),$$
$$\alpha_5 \simeq -12\sqrt{\pi}R^2 + 4\pi R - 168\sqrt{\pi}R^{-2},$$
$$\alpha_{11} \simeq 2\sqrt{\pi} + 2\sqrt{\pi}R^{-2},$$
$$\alpha_{12} = \alpha_{21} \simeq -48\sqrt{\pi}R^{-2},$$
$$\alpha_{13} = \alpha_{31} \simeq (1-\alpha_m)\left[-2\sqrt{\pi}R + 2\pi - 6\sqrt{\pi}R^{-1} + O(R^{-2})\right],$$
$$\alpha_{14} = \alpha_{41} \simeq (1-\alpha_m)\left[2\sqrt{\pi} - 6\pi R^{-1} + 36\sqrt{\pi}R^{-2}\right],$$
$$\alpha_{15} = \alpha_{51} \simeq -2\pi R + 4\sqrt{\pi} + O(R^{-1}) + O(R^{-2}),$$
$$\alpha_{22} \simeq -8\sqrt{\pi}R^2 - 24\sqrt{\pi} + 2256\sqrt{\pi}R^{-2},$$
$$\alpha_{23} = \alpha_{32} \simeq (1-\alpha_m)\left[-4\sqrt{\pi}R + 2\pi + 36\sqrt{\pi}R^{-1} + O(R^{-2})\right],$$
$$\alpha_{24} = \alpha_{42} \simeq (1-\alpha_m)\left[4\sqrt{\pi}R^2 - 6\pi R + 36\sqrt{\pi} - 18\pi R^{-1} - 648\sqrt{\pi}R^{-2}\right],$$
$$\alpha_{25} = \alpha_{52} \simeq -4\sqrt{\pi}R^2 - 2\pi R - 12\sqrt{\pi} + 72\sqrt{\pi}R^{-2},$$
$$\alpha_{33} = 2\sqrt{\pi}R^2(1-\alpha_m), \quad \alpha_{34} = \alpha_{43} = 0, \quad \alpha_{44} = -2\sqrt{\pi}R^2\frac{1-\alpha_m}{1-\alpha_\tau},$$
$$\alpha_{35} = \alpha_{53} \simeq (1-\alpha_m)\left[-2\pi R^2 + 2\sqrt{\pi}R + 2\pi - 24\sqrt{\pi}R^{-1} + O(R^{-2})\right],$$
$$\alpha_{45} = \alpha_{54} \simeq (1-\alpha_m)\left[2\sqrt{\pi}R^2 - 18\pi R^{-1} + 252\sqrt{\pi}R^{-2}\right],$$
$$\alpha_{55} \simeq -10\sqrt{\pi}R^2 - 4\pi R + 12\sqrt{\pi} - 60\sqrt{\pi}R^{-2}.$$

Appendix

Tables of the calculated drag values as α_m & α_τ function at intermediate Kn values

TABLE 1

Drag D , x 10^{-1} as function of α_m and α_τ , R = 4(Kn $\simeq$ 0.22)

α_τ \ α_m	0.0	0.I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	I.0
0.0	2.005	2.I2I	2.22I	2.308	2.385	2.452	2.5II	2.563	2.608	2.646	2.680
0.I	2.I44	2.233	2.3II	2.380	2.44I	2.495	2.543	2.585	2.62I	2.653	2.680
0.2	2.267	2.333	2.393	2.446	2.493	2.536	2.573	2.606	2.635	2.659	2.680
0.3	2.375	2.424	2.468	2.507	2.543	2.574	2.602	2.627	2.648	2.665	2.680
0.4	2.472	2.506	2.537	2.564	2.589	2.6II	2.630	2.647	2.66I	2.672	2.680
0.5	2.558	2.580	2.600	2.6I8	2.633	2.646	2.658	2.667	2.673	2.678	2.680
0.6	2.636	2.648	2.659	2.667	2.675	2.680	2.684	2.686	2.686	2.684	2.680
0.7	2.707	2.7II	2.7I3	2.7I4	2.7I4	2.7I2	2.709	2.704	2.698	2.690	2.680
0.8	2.772	2.768	2.763	2.758	2.75I	2.743	2.733	2.723	2.7I0	2.696	2.680
0.9	2.83I	2.82I	2.8II	2.799	2.786	2.772	2.757	2.740	2.722	2.702	2.680

TABLE 2

Appendix

Drag D, $\times 10^{-1}$ as function of α_m and α_τ, R = 2(Kn $\simeq$ 0.44)

α_τ \ α_m	0.0	0.I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	I.0
0.0	3.347	3.444	3.535	3.62I	3.702	3.778	3.849	3.9I6	3.979	4.037	4.09I
0.I	3.499	3.576	3.649	3.7I8	3.782	3.843	3.953	3.999	4.003	4.049	4.09I
0.2	3.643	3.702	3.758	3.8I0	3.860	3.906	3.949	3.989	4.026	4.060	4.09I
0.3	3.779	3.822	3.862	3.900	3.935	3.968	3.998	4.025	4.050	4.072	4.09I
0.4	3.907	3.936	3.962	3.986	4.008	4.028	4.045	4.060	4.073	4.083	4.09I
0.5	4.030	4.045	4.058	4.069	4.079	4.086	4.092	4.095	4.096	4.095	4.09I
0.6	4.I46	4.I49	4.I50	4.I49	4.I47	4.I43	4.I37	4.I29	4.II9	4.I06	4.09I
0.7	4.256	4.248	4.238	4.227	4.2I4	4.I99	4.I82	4.I63	4.I42	4.II8	4.09I
0.8	4.36I	4.343	4.323	4.302	4.279	4.253	4.226	4.I96	4.I64	4.I29	4.09I
0.9	4.462	4.434	4.405	4.374	4.34I	4.306	4.269	4.229	4.I86	4.I40	4.09I

TABLE 3

Appendix

Drag D , x IO^{-I} as function of α_m and α_τ , R = 1(Kn $\simeq$ 0.89)

α_τ \ α_m	0.0	0.I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	I.0
0.0	4.875	4.94I	5.008	5.075	5.I44	5.2I3	5.283	5.354	5.426	5.498	5.572
0.I	5.062	5.II0	5.I58	5.207	5.257	5.308	5.359	5.4II	5.464	5.5I7	5.572
0.2	5.244	5.275	5.306	5.337	5.369	5.402	5.434	5.468	5.502	5.537	5.572
0.3	5.42I	5.436	5.450	5.464	5.479	5.494	5.509	5.524	5.540	5.556	5.572
0.4	5.594	5.593	5.59I	5.589	5.587	5.585	5.583	5.580	5.577	5.575	5.572
0.5	5.763	5.746	5.729	5.7I2	5.693	5.675	5.655	5.635	5.6I5	5.594	5.572
0.6	5.927	5.896	5.865	5.832	5.798	5.763	5.727	5.690	5.652	5.6I3	5.572
0.7	6.087	6.043	5.997	5.950	5.90I	5.85I	5.799	5.745	5.689	5.63I	5.572
0.8	6.244	6.I87	6.I27	6.066	6.003	5.937	5.869	5.799	5.726	5.650	5.572
0.9	6.397	6.327	6.255	6.I80	6.I02	6.022	5.939	5.852	5.762	5.669	55.572

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Tables of the calculated drag values as α_m & α_τ function at intermediate Kn values

TABLE 1

Drag D, x 10^{-1} as function of α_m and α_τ, R = 4(Kn $\simeq$ 0.22)

α_τ \ α_m	0.0	0.I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	I.0
0.0	2.005	2.I2I	2.22I	2.308	2.385	2.452	2.5II	2.563	2.608	2.646	2.680
0.I	2.I44	2.233	2.3II	2.380	2.44I	2.495	2.543	2.585	2.62I	2.653	2.680
0.2	2.267	2.333	2.393	2.446	2.493	2.536	2.573	2.606	2.635	2.659	2.680
0.3	2.375	2.424	2.468	2.507	2.543	2.574	2.602	2.627	2.648	2.665	2.680
0.4	2.472	2.506	2.537	2.564	2.589	2.6II	2.630	2.647	2.66I	2.672	2.680
0.5	2.558	2.580	2.600	2.6I8	2.633	2.646	2.658	2.667	2.673	2.678	2.680
0.6	2.636	2.648	2.659	2.667	2.675	2.680	2.684	2.686	2.686	2.684	2.680
0.7	2.707	2.7II	2.7I3	2.7I4	2.7I4	2.7I2	2.709	2.704	2.698	2.690	2.680
0.8	2.772	2.768	2.763	2.758	2.75I	2.743	2.733	2.723	2.7I0	2.696	2.680
0.9	2.83I	2.82I	2.8II	2.799	2.786	2.772	2.757	2.740	2.722	2.702	2.680

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By S.A. Beresnev, V.G. Chernyak & P.E. Suetin
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TABLE 2

Drag D, $\times 10^{-1}$ as function of α_m and α_τ, R = 2(Kn $\simeq$ 0.44)

α_τ \ α_m	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	3.347	3.444	3.535	3.621	3.702	3.778	3.849	3.916	3.979	4.037	4.091
0.1	3.499	3.576	3.649	3.718	3.782	3.843	3.953	3.999	4.003	4.049	4.091
0.2	3.643	3.702	3.758	3.810	3.860	3.906	3.949	3.989	4.026	4.060	4.091
0.3	3.779	3.822	3.862	3.900	3.935	3.968	3.998	4.025	4.050	4.072	4.091
0.4	3.907	3.936	3.962	3.986	4.008	4.028	4.045	4.060	4.073	4.083	4.091
0.5	4.030	4.045	4.058	4.069	4.079	4.086	4.092	4.095	4.096	4.095	4.091
0.6	4.146	4.149	4.150	4.149	4.147	4.143	4.137	4.129	4.119	4.106	4.091
0.7	4.256	4.248	4.238	4.227	4.214	4.199	4.182	4.163	4.142	4.118	4.091
0.8	4.361	4.343	4.323	4.302	4.279	4.253	4.226	4.196	4.164	4.129	4.091
0.9	4.462	4.434	4.405	4.374	4.341	4.306	4.269	4.229	4.186	4.140	4.091

TABLE 3

Drag D, x 10^{-I} as function of α_m and α_τ, $R = 1 (Kn \simeq 0.89)$

α_τ \ α_m	0.0	0.I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	I.0
0.0	4.875	4.94I	5.008	5.075	5.I44	5.2I3	5.283	5.354	5.426	5.498	5.572
0.I	5.062	5.II0	5.I58	5.207	5.257	5.308	5.359	5.4II	5.464	5.5I7	5.572
0.2	5.244	5.275	5.306	5.337	5.369	5.402	5.434	5.468	5.502	5.537	5.572
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0.4	5.594	5.593	5.59I	5.589	5.587	5.585	5.583	5.580	5.577	5.575	5.572
0.5	5.763	5.746	5.729	5.7I2	5.693	5.675	5.655	5.635	5.6I5	5.594	5.572
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0.7	6.087	6.043	5.997	5.950	5.90I	5.85I	5.799	5.745	5.689	5.63I	5.572
0.8	6.244	6.I87	6.I27	6.066	6.003	5.937	5.869	5.799	5.726	5.650	5.572
0.9	6.397	6.327	6.255	6.I80	6.I02	6.022	5.939	5.852	5.762	5.669	5.572