Sen, Venkateswarlu & Maji : Stability of Pipe-Poiseuille flow... (1985)

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Editor's file

EXPRESSION FOR $N^{(1)}_{nm}$

$$
\begin{aligned}
N^{(1)}_{nm} = \; & in\alpha R \sum_{p=1}^{m} \Big[K_p \big(g''_{n,m-p} + \frac{3}{r} g'_{n,m-p} - \frac{n^2-1}{r^2} g_{n,m-p} \\
& + n\alpha f'_{n,m-p} + \frac{2n\alpha}{r} f_{n,m-p} \big) \Big] \\
& + in\alpha R \sum_{p=1}^{m} \Big[f^{o}_{p} \big(g''_{n,m-p} + \frac{3}{r} g'_{n,m-p} - \frac{n^2-1}{r^2} g_{n,m-p} \\
& + n\alpha f'_{n,m-p} + \frac{2n\alpha}{r} f_{n,m-p} \big) + f^{o'}_{p} \big(g'_{n,m-p} + \frac{1}{r} g_{n,m-p} + n\alpha f_{n,m-p} \big) \Big] \\
& + \frac{inR}{r^2} \Bigg[- \sum_{p=1}^{n-1} \sum_{d=o}^{m} \Big[(n-p)\alpha \; f_{pd} h_{n-p,m-d} + g_{pd} h'_{n-p,m-d} \\
& + \frac{1}{r} \Big\{ (n-p) h_{pd} h_{n-p,m-d} + h_{pd} g_{n-p,m-d} \Big\} \Big] \\
& \qquad p+d \ngtr m \\
& - \sum_{p=1}^{m} \sum_{d=o}^{m-1} \Big[(n+p)\alpha \; \tilde{f}_{pd} \; h_{n+p,m-p-d} - p\alpha f_{n+p,m-p-d} \; \tilde{h}_{pd} \\
& - \tilde{g}_{pd} \; h'_{n+p,m-p-d} + g_{n+p,m-p-d} \; \tilde{h}'_{pd} + \frac{n}{r} \tilde{h}_{pd} h_{n+p,m-p-d} \\
& + \frac{1}{r} \tilde{h}_{pd} \; g_{n+p,m-p-d} - \frac{1}{r} h_{n+p,m-p-d} \; \tilde{g}_{pd} \Big] \\
& - \sum_{p=1}^{n-1} \sum_{d=o}^{m} \Big[(n-p)\alpha r \; f'_{pd} h_{n-p,m-d} + (n-p)\alpha r f_{pd} h'_{n-p,m-d} \\
& - (n-p)\alpha r h'_{pd} \; f_{n-p,m-d} - (n-p)\alpha r \; h_{pd} f'_{n-p,m-d} \\
& + r g'_{pd} h'_{n-p,m-d} + r g_{pd} h''_{n-p,m-d} - r h'_{pd} \; g'_{n-p,m-d} \\
& - r h_{pd} \; g''_{n-p,m-d} \Big]
\end{aligned}
$$

.....Contd.)

$$- \sum_{\substack{p=1 \\ p+d \geq m}}^{m} \sum_{d=o}^{m-1} \Big[(n+2p)\alpha r \tilde{f}'_{pd}\, h_{n+p,m-p-d} + (n+2p)\alpha r \tilde{f}_{pd} h'_{n+p,m-p-d}$$

$$- (n+2p)\alpha r \tilde{h}'_{pd}\, f_{n+p,m-p-d} - (n+2p)\alpha r \tilde{h}_{pd} f'_{n+p,m-p-d}$$

$$- r \tilde{g}_{pd} h''_{n+p,m-p-d} + r\, g_{n+p,m-p-d} \tilde{h}''_{pd} - r \tilde{h}_{pd} g''_{n+p,m-p-d}$$

$$+ r\, h_{n+p,m-p-d} \tilde{g}'_{pd} \Big]$$

$$- \sum_{p=1}^{n-1} \sum_{d=o}^{m} \Big[n(n-p)\alpha\, f_{pd} g_{n-p,m-d} + n\, g_{pd}\, g'_{n-p,m-d}$$

$$+ \frac{n(n-p)}{r} h_{pd}\, g_{n-p,m-d} + \frac{n}{r} h_{pd}\, h_{n-p,m-d} \Big]$$

$$- \sum_{\substack{p=1 \\ p+d \geq m}}^{m} \sum_{d=o}^{m-1} \Big[n(n+p)\alpha \tilde{f}_{pd}\, g_{n+p,m-p-d} + np\alpha\, f_{n+p,m-p-d} \tilde{g}_{pd}$$

$$- n \tilde{g}_{pd} g'_{n+p,m-p-d} - n\, g_{n+p,m-p-d} \tilde{g}'_{pd}$$

$$+ \frac{n(n+p)}{r} \tilde{h}_{pd}\, g_{n+p,m-p-d}$$

$$+ \frac{np}{r} h_{n+p,m-p-d} \tilde{g}_{pd}$$

$$+ \frac{2n}{r} \tilde{h}_{pd}\, h_{n+p,m-p-d} \Big] \Big] .$$

EXPRESSION FOR $N_{nm}^{(2)}$

$$N_{nm}^{(2)} = in\alpha R \sum_{p=1}^{m} K_p \left[\alpha r^2 g'_{n,m-p} + \alpha r g_{n,m-p} + n(1+\alpha^2 r^2) f_{n,m-p} \right]$$

$$+ in\alpha R \sum_{p=1}^{m} \left[f_p^o \left\{ \alpha r^2 g'_{n,m-p} + \alpha r g_{n,m-p} + n(1+\alpha^2 r^2) f_{n,m-p} \right\} \right.$$

$$\left. + \frac{f_p^{o'}}{\alpha} g_{n,m-p} \right] - inR \left[-\sum_{p=1}^{n-1} \sum_{d=o}^{m} \left\{ (n-p)\alpha f_{pd} f_{n-p,m-d} \right. \right.$$

$$\left. + g_{pd} f'_{n-p,m-d} + \frac{n-p}{r} h_{pd} f_{n-p,m-d} \right\}$$

$$\underset{p+d \ngtr m}{- \sum_{p=1}^{m} \sum_{d=o}^{m-1}} \left\{ n\alpha \tilde{f}_{pd} f_{n+p,m-p-d} - \tilde{g}_{pd} f'_{n+p,m-p-d} \right.$$

$$\left. + \frac{n+p}{r} \tilde{h}_{pd} f_{n+p,m-p-d} + g_{n+p,m-p-d} \tilde{f}'_{pd} - \frac{p}{r} h_{n+p,m-p-d} \tilde{f}_{pd} \right\}$$

$$+ \sum_{p=1}^{n-1} \sum_{d=o}^{m} \left\{ (n-p)\alpha^2 r f_{pd} h_{n-p,m-d} + \alpha r g_{pd} h'_{n-p,m-d} \right.$$

$$\left. + \alpha h_{pd} g_{n-p,m-d} + (n-p)\alpha h_{pd} h_{n-p,m-d} \right\}$$

$$\underset{p+d \ngtr m}{+ \sum_{p=1}^{m} \sum_{d=o}^{m-1}} \left\{ (n+p)\alpha^2 r \tilde{f}_{pd} h_{n+p,m-p-d} - p\alpha^2 r f_{n+p,m-p-d} \tilde{h}_{pd} \right.$$

$$- \alpha r \tilde{g}_{pd} h'_{n+p,m-p-d} + \alpha r g_{n+p,m-p-d} \tilde{h}'_{pd} + \alpha \tilde{h}_{pd} g_{n+p,m-p-d}$$

$$\left. \left. - \alpha h_{n+p,m-p-d} \tilde{g}_{pd} + n\alpha \tilde{h}_{pd} h_{n+p,m-p-d} \right\} \right] .$$