

"Particle Motion in Stokes Flow near a Plane Fluid-Fluid Interface.
Part 2. Linear Shear and Axisymmetric Straining Flows", S. M. Yang
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APPENDIX

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In this appendix, we give detailed forms of the functions defined in Section 4 in terms of $g(x;\theta,d)$, $h(x;\theta,d)$, $k(x;\theta,d)$, $y(x;\theta,d)$, $z(x;\theta,d)$, $A(x;\lambda,\theta,d)$, $C(x;\lambda,\theta,d)$, $D(x;\lambda,\theta,d)$ and $E(x;\lambda,\theta,d)$ for which definitions are given in the appendix of Part 1 of this series.

$$\begin{aligned}
 U(x;\lambda,\theta,d) = & A(x;\lambda,\theta,d) + \frac{1-\lambda}{1+\lambda} \frac{1}{x} \left[\{(1-x\cos 2\theta - 2d\sin \theta)^2 \right. \\
 & \left. + (2\cos \theta(d - x\sin \theta))^2\}^{1/2} - \{(1+x\cos 2\theta + 2d\sin \theta)^2 + (2\cos \theta(d + x\sin \theta))^2\}^{1/2} \right] \\
 & + \frac{1}{x} (d - x\sin \theta) \left[\frac{\sin \theta (3\sin^2 2\theta + 8 - 3\lambda(\sin^2 2\theta + 4\cos^2 \theta))}{2(1+\lambda)(1+\sin^2 \theta)} g(x;\theta,d) \right. \\
 & \left. + \frac{2\cos^2 \theta ((4\cos^2 \theta - 3)(1+3\sin^2 \theta) + 3\lambda \sin^2 \theta (3\cos^2 \theta - 2))}{(1+\lambda)(1+\sin^2 \theta)} k(x;\theta,d) \right. \\
 & \left. - \frac{\lambda (21\sin^2 2\theta - 4\sin^2 \theta - 8)}{2(1+\lambda)} k(x;\theta,d) \right] \\
 - \sin \theta \left[& \frac{2(4\cos^2 \theta - 1)(1+3\sin^2 \theta) + 9\lambda \sin^2 \theta \cos 2\theta}{2(1+\lambda)(1+\sin^2 \theta)} - \frac{\lambda (6\cos^4 \theta + 7\cos^2 \theta - 4)}{2(1+\lambda)\cos^2 \theta} \right] h(x;\theta,d) \\
 & + \frac{4\lambda \cos^2 \theta (1+3\sin^2 \theta) (16\sin^4 \theta - 12\sin^2 \theta + 1)}{(1+\lambda)(1+\sin^2 \theta)} y(x;\theta,d) \\
 & + \frac{\lambda \sin \theta (1+3\sin^2 \theta) (16\cos^4 \theta - 12\cos^2 \theta + 1)}{2(1+\lambda)\cos^2 \theta (1+\sin^2 \theta)} z(x;\theta,d) \Big] \\
 V(x;\lambda,\theta,d) = & E(x;\lambda,\theta,d) + \frac{1-\lambda}{1+\lambda} \frac{1}{x} \left[\{(1-x\cos 2\theta - 2d\sin \theta)^2 \right. \\
 & \left. + (2\cos \theta(d - x\sin \theta))^2\}^{1/2} - \{(1+x\cos 2\theta + 2d\sin \theta)^2 + (2\cos \theta(d + x\sin \theta))^2\}^{1/2} \right] \\
 & + \frac{(d - x\sin \theta)}{x} \left[- \frac{4(3\cos^4 \theta - 2) + 3\lambda \sin^2 2\theta}{2(1+\lambda)\sin \theta} g(x;\theta,d) \right. \\
 & \left. + \frac{2\sin \theta ((12\cos^4 \theta - \cos^2 \theta - 2) - \lambda (30\cos^4 \theta - 19\cos^2 \theta - 3))}{(1+\lambda)} k(x;\theta,d) \right]
 \end{aligned}$$

$$\begin{aligned}
& - \left\{ \frac{3\sin^2 2\theta - 5\cos^2 \theta + 3}{(1 + \lambda)\sin \theta} + \frac{\lambda \sin \theta (12\cos^4 \theta + 2\cos^2 \theta - 5)}{2(1 + \lambda)\cos^2 \theta} \right\} h(x; \theta, d) \\
& + \frac{8\lambda(16\cos^6 \theta - 4\cos^4 \theta - 7\cos^2 \theta + 1)}{(1 + \lambda)} y(x; \theta, d) \\
& + \frac{\lambda(16\sin^6 \theta - 36\sin^4 \theta + 17\sin^2 \theta - 1)}{(1 + \lambda)\sin \theta \cdot \cos^2 \theta} z(x; \theta, d)
\end{aligned}$$

$$X(x; \lambda, \theta, d) = D(x; \lambda, \theta, d) - \frac{1}{x} \left[\{(1 - x \cos 2\theta - 2d \sin \theta)^2 \right.$$

$$+ \{(2 \cos \theta (d - x \sin \theta))^2\}^{1/2} - \{(1 + x \cos 2\theta + 2d \sin \theta)^2 + (2 \cos \theta (d + x \sin \theta))^2\}^{1/2}]$$

$$+ \frac{(d - x \sin \theta)}{x} \left[\frac{3\sin^2 2\theta - 6\cos 2\theta - 2 + 12\lambda \sin^2 \theta (1 + \sin^2 \theta)}{2(1 + \lambda)\sin \theta} g(x; \theta, d) \right.$$

$$+ \frac{2(8\cos^4 \theta - 14\cos^2 \theta + 2 - \lambda(30\cos^4 \theta - 67\cos^2 \theta + 27))}{1 + \lambda} k(x; \theta, d)$$

$$+ \left\{ \frac{2(4\cos^4 \theta - 9\cos^2 \theta + 4)}{(1 + \lambda)\sin \theta} - \frac{\lambda \sin \theta (12\cos^4 \theta + 2\cos^2 \theta + 7)}{2(1 + \lambda)\cos^2 \theta} \right\} h(x; \theta, d)$$

$$+ \frac{8\lambda(16\cos^6 \theta - 36\cos^4 \theta + 17\cos^2 \theta - 1)}{(1 + \lambda)} y(x; \theta, d) + \frac{\lambda(16\sin^6 \theta - 4\sin^4 \theta - 7\sin^2 \theta + 1)}{(1 + \lambda)\sin \theta \cdot \cos^2 \theta} z(x; \theta, d) \left. \right]$$

$$Y(x; \lambda, \theta, d) = C(x; \lambda, \theta, d) - \frac{1}{x} \left[\{(1 - x \cos 2\theta - 2d \sin \theta)^2 \right.$$

$$+ \{(2 \cos \theta (d - x \sin \theta))^2\}^{1/2} - \{(1 + x \cos 2\theta + 2d \sin \theta)^2 + (2 \cos \theta (d + x \sin \theta))^2\}^{1/2}]$$

$$+ \frac{(d - x \sin \theta)}{x} \left[\frac{\sin \theta (4(3\cos^4 \theta - 1) + 3\lambda \sin^2 2\theta)}{(1 + \lambda)(1 + \cos^2 \theta)} g(x; \theta, d) \right.$$

$$+ \frac{2\cos^2 \theta ((4\cos^2 \theta - 3)(1 + 3\cos^2 \theta) + 3\lambda \sin^2 \theta (7\cos^2 \theta - 4))}{(1 + \lambda)(1 + \cos^2 \theta)} k(x; \theta, d)$$

$$- \frac{2\lambda(9\cos^4 \theta - 13\cos^2 \theta + 3)}{(1 + \lambda)} k(x; \theta, d)$$

$$- \sin \theta \left[\frac{2(4\cos^2 \theta - 1)(3\cos^2 \theta + 1) - 3\lambda \sin^2 \theta (2\cos^2 \theta + 1)}{2(1 + \lambda)(1 + \cos^2 \theta)} + \frac{\lambda(18\cos^4 \theta - 7\cos^2 \theta + 1)}{2(1 + \lambda)\cos^2 \theta} \right] h(x; \theta, d)$$

$$+ \frac{4\lambda \cos^2 \theta (1 + 3\cos^2 \theta)(16\sin^4 \theta - 12\sin^2 \theta + 1)}{(1 + \lambda)(1 + \cos^2 \theta)} y(x; \theta, d)$$

$$+ \frac{\lambda \sin \theta (16\cos^4 \theta - 12\cos^2 \theta + 1)(1 + 3\cos^2 \theta)}{2(1 + \lambda)\cos^2 \theta (1 + \cos^2 \theta)} z(x; \theta, d) \Big]$$

For specific formulae of the functions $B(x; \lambda, \theta, d)$ and $K(x; \lambda, \theta, d)$ in Eqns. (31b) and (31d), refer to the Appendix in Yang and Leal (1983), Part 1 of this series.