DETAILS OF SOLUTIONS

(From 'The effect of Langmuir Circulation on the Distribution of Submerged Bubbles caused by Breaking Wind Waves" by S.A. Thorpe)

Editorial file.

SOLUTION OF (1); VALUES OF COEFFICIENTS.

(i) Coefficients in equation (17)

$$a = \frac{\theta q_1 L A}{4(\phi^2+q^2)^{1/2}} \qquad b = \frac{\theta\left[A(q_1\phi - 2 - q_1^2) + B(2\phi - q_1)\right]}{8(1+\phi^2+q^2)}$$

$$c = \frac{\theta\left[A(q_1 - 2\phi) + B(q_1\phi - q_1^2 - 2)\right]}{8\,(1+\phi^2+q^2)}$$

$$d = \frac{\theta}{8\zeta}\,(Aq_1 + B)$$

· SEE NOTE! →

$$f = \frac{q_1\theta\left[B(1+\zeta) - A(\phi^2+q^2)^{1/2}\right]}{8\left[\phi^2+q^2+(1+\zeta)^2\right]}$$

$$g = \frac{-q_1\theta\left[B(\phi^2+q^2)^{1/2} + A(1+\zeta)\right]}{8\left[\phi^2+q^2+(1+\zeta)^2\right]}$$

$$h = \frac{-C\theta\left[1+\zeta+2(\phi^2+q^2)+2\phi(\phi^2+q^2+\zeta)^{1/2}\right]}{\left[4(\phi^2+q^2+\zeta)+(1-\zeta)^2\right]}$$

The apparent singularity at small ζ in the coefficient d (proportional to ζ^{-1}) in N_1 vanishes, cancelling with a contribution from the coefficient h (for both boundary conditions) when ζ is small.

$$j = \frac{c\theta\,[(1-J)\phi - (1+J)(\phi^2+q^2+J)^{1/2}]}{[4(\phi^2+q^2+J) + (1-J)^2]}$$

$$p = \frac{c\theta\,[\phi(1+3J) + 3(1+J)(\phi^2+q^2+J)^{1/2}]}{[4(\phi^2+q^2+J) + (1+3J)^2]}$$

$$m = \frac{c\theta\,[1+J - 2(\phi^2+q^2) - 2\phi(\phi^2+q^2+J)^{1/2}]}{[4(\phi^2+q^2+J) + (1+3J)^2]}$$

$$r = \begin{cases} -(c+h) & \text{, boundary condition (10)} \\ (a+2lb-\alpha c-\beta h+lj)/\alpha & \text{, boundary condition (11)} \end{cases}$$

$$t = \begin{cases} -(d+g+m) & \text{, boundary condition (10)} \\ -(\alpha d-2lf+\alpha g+\beta m-lp)/\gamma & \text{, boundary condition (11)} \end{cases}$$

(ii) Solution of (1) at order ϵ^3 for $\phi = 0$, $J = 1$

$$N_3 = n_0 \cos ky \{ e^{-\alpha z}[z(A_1 s_1 + B_1 c_1) + C_1 s_1 + D_1 c_1 + E_1 s_3 + F_1 c_3]$$

$$+ e^{-\beta z}(G_1 z + H_1 s_2 + J_1 c_2) + e^{-\gamma z}(L_1 s_1 + M_1 c_1) + L_3 e^{-\beta z}\}$$

$$+ n_0 \cos 3ky \{ e^{-\alpha z}(A_4 s_1 + B_4 c_1 + C_4 s_3 + D_4 c_3) + e^{-\beta z}(E_4 +$$

$$F_4 s_2 + G_4 c_2) + e^{-\gamma z}(H_4 s_1 + J_4 c_1) + K_4 e^{-\gamma_1 z}\},$$

where $s_n = \sin nLz$, $c_n = \cos nLz$; $n = 1, 2, 3 \ldots,$

$$\gamma_1 = L(9+q^2)^{1/2}$$ and

$$A_1 = \frac{2\theta q p L}{1+q^2} \quad , \quad B_1 = -q A_1 \quad ,$$

$$C_1 = \frac{\theta[8p(q^2-1) + 2b(q^2+2) + 2cq - 2dq + q^2 f - qg + 4qr]}{4(1+q^2)}$$

$$D_1 = \frac{\theta[16pq - 2qb + 2c(2+q^2) - 2d(2+q^2) + gq^2 + qf - 4q^2 r]}{4(1+q^2)}$$

$$E_1 = -\frac{\theta[2b(10+3q^2) + 2qc + f(20+3q^2) + 7qg]}{4(9q^2+25)}$$

$$F_1 = \frac{\theta[2qb - 2c(10+3q^2) + 7qf - g(20+3q^2)]}{4(9q^2+25)}$$

$$G_1 = \frac{\theta L[(1+q^2)^{1/2}(p+2j) + 2h - m]}{4(1+q^2)^{1/2}}$$

$$J_1 = \frac{\theta[2p(1+q^2)^{1/2} - 2h(2+q^2) - m(4+q^2)]}{8(2+q^2)}$$

$$H_1 = \frac{-\theta[p(4+q^2) + 2m(1+q^2)^{1/2} + 2j(2+q^2)]}{8(2+q^2)}$$

$$M_1 = \frac{-\theta t(2+q^2)}{2(5+q^2)}$$

$$L_1 = \frac{-3\theta t(4+q^2)^{1/2}}{2(5+q^2)}$$

$$L_3 = \begin{cases} -(D_1 + F_1 + J_1 + M_1), & \text{boundary condition (10)} \\ (B_1 - \alpha D_1 + \ell C_1 - \alpha F_1 + 3\ell E_1 + G_1 - \beta J_1 + 2\ell H_1 - \gamma M_1 + \ell L_1)/\beta, & \text{boundary condition (11)} \end{cases}$$

$$A_4 = \frac{\theta\,[14dq - qg + f(q^2+20)]}{4(25+q^2)}$$

$$B_4 = \frac{\theta\,[2d(10-q^2) + g(20+q^2) + qf]}{4(25+q^2)}$$

$$C_4 = \frac{\theta q\,(3g - qf)}{12(9+q^2)}$$

$$D_4 = \frac{-\theta q\,(qg + 3f)}{12(9+q^2)}$$

$$E_4 = \frac{\theta}{16}\,[3m + p(1+q^2)^{1/2}]$$

$$F_4 = \frac{\theta\,[4(1+q^2)^{1/2}m + p(2-q^2)]}{8(10+q^2)}$$

$$G_4 = \frac{\theta\,[(2-q^2)m - 4p(1+q^2)^{1/2}]}{8(10+q^2)}$$

$$H_4 = \frac{5\theta t\,(4+q^2)^{1/2}}{2(13+q^2)}$$

$$J_4 = \frac{\theta t\,(2-q^2)}{2(13+q^2)}$$

$$K_4 = \begin{cases} -(B_4 + D_4 + E_4 + G_4 + H_4), & \text{boundary condition (10)} \\ (\ell A_4 - \alpha B_4 + 3\ell C_4 - \alpha D_4 - \beta E_4 + 2\ell F_4 - \beta G_4 + \ell H_4 - \gamma J_4)/\gamma & \text{boundary condition (11)} \end{cases}$$

and $\rho = \dfrac{5q\theta^2}{8(1+q^2)}$.

(iii) Part of N_4 independent of y for $\phi = 0$, $J = 1$

The terms independent of y are $\overline{N_4}$ where

$$\frac{\overline{N_4}}{n} = -e^{-\alpha z}\left[R_2 + z\left(A_2 + B_2 s_2 + C_2 c_2\right) + z^2 A_3 + E_2 s_2 + F_2 c_2\right.$$

$$\left. + E_3 s_4 + F_3 c_4\right] + -e^{-\beta z}\left[z\left(G_2 s_1 + H_2 c_1\right) + J_2 s_1 + K_2 c_1\right.$$

$$\left. + L_2 s_3 + M_2 c_3\right] + e^{-\gamma z}\left(M_3 + P_2 s_2 + Q_2 c_2\right),$$

where

$$A_2 = \frac{\theta}{4q}\left(2qlC_1 - A_1\right)$$

$$A_3 = \frac{5\theta^2 q\rho l^2}{4(1+q^2)}$$

$$B_2 = -\frac{\theta\left[A_1(2+q^2) + B_1 q\right]}{8(1+q^2)}$$

$$D_2 = \frac{\theta\left[A_1 q - B_1(2+q^2)\right]}{8(1+q^2)}.$$

$$E_2 = \frac{\theta}{8(1+q^2)}\left[E_1(2+q^2) + F_1 q - D_1 q - C_1(2+q^2)\right] + \frac{\left[2B_2 q - 2D_2(2+q^2) + \theta A_1\right]}{4l(1+q^2)}$$

$$F_2 = \frac{\theta\left[C_1 q - (2+q^2)D_1 - qE_1 + F_1(2+q^2)\right]}{8(1+q^2)} + \frac{\left[4B_2(2+q^2) + 4qD_2 + \theta(qB_1 + A_1)\right]}{8l(1+q^2)}$$

$$E_3 = \frac{-\theta\left[2qF_1 + (8+q^2)E_1\right]}{16(4+q^2)}$$

$$F_3 = \frac{\theta\,[2qE_1 - (8+q^2)F_1]}{16(4+q^2)}$$

$$G_2 = \frac{-\theta G_1}{2(1+q^2)^{1/2}}$$

$$H_2 = -\frac{\theta G_1}{2}$$

$$J_2 = \frac{\theta\,[(1+q^2)^{1/2}H_1 + J_1 - 2N_1 + 2G_1 l^{-1}]}{4(1+q^2)^{1/2}}$$

$$K_2 = \frac{\theta\,[(1+q^2)^{1/2}J_1 - H_1 - 2(1+q^2)^{1/2}N_1 - 2G_1 l^{-1}]}{4(1+q^2)^{1/2}}$$

$$L_2 = \frac{\theta\,[5(1+q^2)^{1/2}J_1 + 3(5+q^2)H_1}{4(25+9q^2)}$$

$$M_2 = \frac{\theta\,[5(1+q^2)^{1/2}H_1 - 3(5+q^2)J_1]}{4(25+9q^2)}$$

$$P_2 = \frac{-\theta\,[2M_1 + (4+q^2)^{1/2}L_1]}{8(4+q^2)^{1/2}}$$

$$Q_2 = \frac{\theta\,[2L_1 - (4+q^2)^{1/2}M_1]}{8(4+q^2)^{1/2}}$$

$$M_3 = -\frac{\theta}{8}(4+q^2)^{1/2}L_1$$

$$R_2 = \begin{cases} -(F_2+F_3+K_2+M_2+M_3+Q_2)\ , & \text{boundary condition (10)} \\ (A_2+D_2+2lE_2-\alpha F_2+4lE_3-\alpha F_3+H_2+lJ_2-\beta K_2+3lL_2-\beta M_2 & \\ \quad -\gamma M_3+2lP_2-\gamma Q_2)/\alpha\ , & \text{boundary condition (11).} \end{cases}$$