

"Particle motion in Stokes flow near a plane fluid-fluid interface. Part I. Slender body in a quiescent fluid"

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## APPENDIX

In this appendix, we give detail forms of functions which are defined in sections III and IV that represent the interfacial effects on the motion of slender body near a plane fluid-fluid interface. For convenience, we first define some functions as follows:

$$g(x;\theta,d) = \sinh^{-1} \left| \frac{l - x \cos 2\theta - 2d \sin \theta}{2 \cos \theta (d - x \sin \theta)} \right| + \sinh^{-1} \left| \frac{l + x \cos 2\theta + 2d \sin \theta}{2 \cos \theta (d - x \sin \theta)} \right|$$

$$h(x;\theta,d) = \frac{[l - x \cos 2\theta - 2d \sin \theta]}{\left[ (l - x \cos 2\theta - 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{1/2}}$$

$$+ \frac{[l + x \cos 2\theta + 2d \sin \theta]}{\left[ (l + x \cos 2\theta + 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{1/2}}$$

$$k(x;\theta,d) = (d - x \sin \theta) \cdot \left[ \frac{1}{\left[ (l - x \cos 2\theta - 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{1/2}} \right. \\ \left. - \frac{1}{\left[ (l + x \cos 2\theta + 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{1/2}} \right]$$

$$y(x;\theta,d) = (d - x \sin \theta)^3 \cdot \left[ \frac{1}{\left[ (l - x \cos 2\theta - 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{3/2}} \right. \\ \left. - \frac{1}{\left[ (l + x \cos 2\theta + 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{3/2}} \right]$$

$$z(x;\theta,d) = \frac{[l - x \cos 2\theta - 2d \sin \theta]^3}{\left[ (l - x \cos 2\theta - 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{3/2}} \\ + \frac{[l + x \cos 2\theta + 2d \sin \theta]^3}{\left[ (l + x \cos 2\theta + 2d \sin \theta)^2 + [2 \cos \theta (d - x \sin \theta)]^2 \right]^{3/2}}$$

Now the specific formulae for the functions representing the "interfacial effects" can be expressed in terms of  $g(x;\theta,d)$ ,  $h(x;\theta,d)$ ,  $k(x;\theta,d)$ ,  $y(x;\theta,d)$ , and  $z(x;\theta,d)$ .

$$\begin{aligned}
 P(x; \lambda, \theta, d) &= \frac{1-\lambda}{1+\lambda} (1 + \cos^2 \theta) \cdot g(x; \theta, d) \\
 - \frac{2[2\cos^2 \theta + (1 - 5\cos^2 \theta)\lambda]}{1+\lambda} \sin \theta k(x; \theta, d) &+ \frac{\cos 2\theta((2 - \cos^2 \theta)\lambda - 2\cos^2 \theta)}{2\cos^2 \theta(1+\lambda)} h(x; \theta, d) \\
 - \frac{16\lambda \sin \theta \cos 2\theta \cos^2 \theta}{(1+\lambda)} y(x; \theta, d) &+ \frac{\lambda \cos 4\theta}{2(1+\lambda) \cos^2 \theta} z(x; \theta, d)
 \end{aligned} \tag{A1}$$

$$\begin{aligned}
 Q(x; \lambda, \theta, d) &= \frac{1-\lambda}{1+\lambda} \cos \theta \sin \theta \cdot g(x; \theta, d) \\
 + \frac{2\cos \theta}{1+\lambda} (\cos 2\theta + 5\lambda \sin^2 \theta) \cdot k(x; \theta, d) &- \frac{\sin \theta(\lambda \cos 2\theta + 4\cos^2 \theta)}{2(1+\lambda) \cos \theta} h(x; \theta, d) \\
 + \frac{4\lambda \cos \theta \cos 4\theta}{(1+\lambda)} y(x; \theta, d) &+ \frac{2\lambda \cos 2\theta \sin \theta}{(1+\lambda) \cos \theta} z(x; \theta, d)
 \end{aligned} \tag{A2}$$

$$\begin{aligned}
 R(x; \lambda, \theta, d) &= -\cos \theta \sin \theta \cdot g(x; \theta, d) - \frac{2\cos \theta}{1+\lambda} (\cos 2\theta + \lambda \sin^2 \theta) \cdot k(x; \theta, d) \\
 + \frac{\sin \theta(4\cos^2 \theta + 5\lambda \cos 2\theta)}{2(1+\lambda) \cos \theta} h(x; \theta, d) & \\
 - \frac{2\lambda \cos 2\theta \sin \theta}{(1+\lambda) \cos \theta} z(x; \theta, d) &- \frac{4\lambda \cos \theta \cos 4\theta}{1+\lambda} \cdot y(x; \theta, d)
 \end{aligned} \tag{A3}$$

$$\begin{aligned}
 W(x; \lambda, \theta, d) &= -(1 + \sin \theta) \cdot g(x; \theta, d) - \frac{2\sin \theta((1 + \sin^2 \theta)\lambda - 2\cos^2 \theta)}{1+\lambda} k(x; \theta, d) \\
 - \frac{\cos 2\theta(2\cos^2 \theta + \lambda(5\cos^2 \theta - 1))}{2(1+\lambda) \cos^2 \theta} h(x; \theta, d) & \\
 - \frac{16\lambda \cos 2\theta \cos^2 \theta \sin \theta}{(1+\lambda)} y(x; \theta, d) &+ \frac{\lambda \cos 4\theta}{2(1+\lambda) \cos^2 \theta} z(x; \theta, d) \\
 - \left[ \frac{2\sin \theta [\cos^2 \theta (3\sin^2 \theta + 1) + \lambda(1 + \sin^2 \theta)(1 - 3\cos^2 \theta)]}{(1+\lambda) \cdot (1 + \sin^2 \theta)} \right] \cdot k(x; \theta, d)
 \end{aligned} \tag{A4}$$

$$\begin{aligned}
 A(x; \lambda, \theta, d) &= \frac{1-\lambda}{1+\lambda} \cdot g(x; \theta, d) - \left[ \frac{8\lambda \cos 2\theta \cos^2 \theta \sin \theta (1 + 3\sin^2 \theta)}{(1+\lambda) \cdot (1 + \sin^2 \theta)} \right] \cdot y(x; \theta, d) \\
 + \left[ \frac{\lambda \cos 4\theta (1 + 3\sin^2 \theta)}{4(1+\lambda) \cos^2 \theta (1 + \sin^2 \theta)} \right] \cdot z(x; \theta, d)
 \end{aligned}$$

$$\begin{aligned}
 & - \left[ \frac{\cos 2\theta [2\cos^2\theta (3\sin^2\theta + 1) - \lambda(12\sin^4\theta - 5\sin^2\theta + 1)]}{4(1 + \lambda)\cos^2\theta (1 + \sin^2\theta)} \right] \cdot h(x;\theta,d) \\
 & - \left[ \frac{2\sin\theta [\cos^2\theta (3\sin^2\theta + 1) + \lambda(1 + \sin^2\theta)(1 - 3\cos^2\theta)]}{(1 + \lambda) \cdot (1 + \sin^2\theta)} \right] \cdot k(x;\theta,d)
 \end{aligned} \tag{A5}$$

$$\begin{aligned}
 B(x;\lambda,\theta,d) = & \frac{1 - \lambda}{1 + \lambda} g(x;\theta,d) - \frac{2\lambda}{1 + \lambda} \sin\theta k(x;\theta,d) \\
 & - \frac{\lambda \cos 2\theta}{2(1 + \lambda)\cos^2\theta} h(x;\theta,d)
 \end{aligned} \tag{A6}$$

$$\begin{aligned}
 C(x;\lambda,\theta,d) = & -g(x;\theta,d) - \left[ \frac{2\lambda (1 + 3\cos^2\theta)\sin 4\theta \cos\theta}{(1 + \lambda) \cdot (1 + \cos^2\theta)} \right] y(x;\theta,d) \\
 & + \left[ \frac{\lambda \cos 4\theta (1 + 3\cos^2\theta)}{4(1 + \lambda) \cos^2\theta (1 + \cos^2\theta)} \right] z(x;\theta,d) \\
 & - \left[ \frac{2\sin\theta [\cos^2\theta (3\cos^2\theta + 1) + \lambda(2\sin^2\theta - 3\sin^4\theta + 2)]}{(1 + \lambda) \cdot (1 + \cos^2\theta)} \right] k(x;\theta,d) \\
 & - \left[ \frac{\cos 2\theta [2\cos^2\theta (3\cos^2\theta + 1) + \lambda(12\cos^4\theta + 5\cos^2\theta - 1)]}{4(1 + \lambda) \cos^2\theta (1 + \cos^2\theta)} \right] h(x;\theta,d)
 \end{aligned} \tag{A7}$$

$$\begin{aligned}
 D(x;\lambda,\theta,d) = & -g(x;\theta,d) - \left[ \frac{\lambda (4\cos 4\theta + 3\sin 2\theta \sin 4\theta)}{(1 + \lambda) \sin\theta} \right] y(x;\theta,d) \\
 & + \left[ \frac{\lambda (3\cos 4\theta - 8\cos 2\theta)}{4(1 + \lambda) \cos^2\theta} \right] z(x;\theta,d) \\
 & - \left[ \frac{(3\sin^2(2\theta) + 4\cos 2\theta) + \lambda 4(4 + 3\sin^2\theta) \sin^2\theta}{2(1 + \lambda) \sin\theta} \right] k(x;\theta,d) \\
 & + \left[ \frac{\cos^2\theta (8 - 6\cos 2\theta) + \lambda \cos 2\theta (7 - 12\cos^2\theta)}{4(1 + \lambda) \cos^2\theta} \right] h(x;\theta,d)
 \end{aligned} \tag{A8}$$

$$E(x;\lambda,\theta,d) = \frac{1 - \lambda}{1 + \lambda} g(x;\theta,d) - \left[ \frac{\lambda (3\sin 4\theta \sin 2\theta - 4\cos 4\theta)}{(1 + \lambda) \sin\theta} \right] y(x;\theta,d)$$

$$\begin{aligned}
 & + \left[ \frac{\lambda(3\cos 4\theta + 8\cos 2\theta)}{4(1+\lambda)\cos^2 \theta} \right] \cdot z(x;\theta,d) \\
 & - \left[ \frac{2[3\cos^2 \theta \sin^2 \theta - \cos 2\theta + \lambda \sin^2 \theta (1 - 3\cos^2 \theta)]}{(1+\lambda)\sin \theta} \right] \cdot k(x;\theta,d) \\
 & - \left[ \frac{\cos^2 \theta (8 + 6\cos 2\theta) + \lambda \cos 2\theta (12\cos^2 \theta - 5)}{4(1+\lambda)\cos^2 \theta} \right] \cdot h(x;\theta,d)
 \end{aligned} \tag{A9}$$

$$H(x;\lambda,\theta,d) = \frac{(1+\sin^2 \theta)}{2} A(x;\lambda,\theta,d) + \frac{\cos^2 \theta}{2} E(x;\lambda,\theta,d) \tag{A10}$$

$$J(x;\lambda,\theta,d) = \frac{(1+\cos^2 \theta)}{2} C(x;\lambda,\theta,d) + \frac{\sin^2 \theta}{2} D(x;\lambda,\theta,d) \tag{A11}$$

$$\begin{aligned}
 K(x;\lambda,\theta,d) = & \frac{1}{x} \cdot \frac{1-\lambda}{1+\lambda} \left[ [(l - x\cos 2\theta - 2d\sin \theta)^2 + [2\cos \theta (d - x\sin \theta)]^2]^{1/2} \right. \\
 & - \left. [(l + x\cos 2\theta + 2d\sin \theta)^2 + [2\cos \theta (d - x\sin \theta)]^2]^{1/2} \right] \\
 & + B(x;\lambda,\theta,d) + \frac{1}{x} (d - x\sin \theta) \left[ \frac{2}{1+\lambda} \sin \theta g(x;\theta,d) \right. \\
 & \left. - \frac{2\lambda}{1+\lambda} (4\sin^2 \theta - 1) k(x;\theta,d) - \frac{\lambda \sin \theta (4\cos^2 \theta - 1)}{(1+\lambda)\cos^2 \theta} \cdot h(x;\theta,d) \right]
 \end{aligned} \tag{A12}$$

$$\begin{aligned}
 G(x;\lambda,\theta,d) = & \frac{\sin^2 \theta - \lambda}{1+\lambda} g(x;\theta,d) + \frac{[2\sin^2 \theta \cos^2 \theta - \lambda \cos 2\theta (3\cos^2 \theta - 4)]}{2(1+\lambda)\cos^2 \theta} \cdot h(x;\theta,d) \\
 & - \frac{[(1+2\sin^2 \theta)\cos^2 \theta + 2\lambda \sin^2 \theta]}{(1+\lambda)\sin \theta} \cdot k(x;\theta,d) \\
 & - \frac{2\lambda(\sin 4\theta \sin 2\theta + \cos^2 \theta \cos 4\theta)}{(1+\lambda)\sin \theta} \cdot y(x;\theta,d) + \frac{\lambda(\cos 4\theta - 2\cos 2\theta \cos^2 \theta)}{2(1+\lambda)\cos^2 \theta} \cdot z(x;\theta,d) \\
 & + \frac{\sin^2 \theta - \lambda}{(1+\lambda)x} \left[ [(l - x\cos 2\theta - 2d\sin \theta)^2 + [2\cos \theta (d - x\sin \theta)]^2]^{1/2} \right. \\
 & \left. - [(l + x\cos 2\theta + 2d\sin \theta)^2 + [2\cos \theta (d - x\sin \theta)]^2]^{1/2} \right] \\
 & + \frac{(d - x\sin \theta)}{x} \left[ \frac{1+\sin^2 \theta}{(1+\lambda)\sin \theta} \cdot g(x;\theta,d) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\cos^2\theta(4\cos^4\theta - 7\cos^2\theta + 2) - \lambda\sin^2\theta(6\cos^4\theta - 5\cos^2\theta + 2)}{(1 + \lambda)\cos^2\theta\sin\theta} \cdot h(x;\theta,d) \\
 & + \frac{2[\cos^2\theta(2\cos 2\theta - 3) - \lambda(6\sin^4\theta + \sin^2\theta - 4)]}{(1 + \lambda)} \cdot k(x;\theta,d) \\
 & + \frac{4\lambda\cos^2\theta(16\sin^4\theta - 4\sin^2\theta - 3)}{(1 + \lambda)} y(x;\theta,d) \\
 & + \left. \frac{\lambda[\sin^2 2\theta(4\cos^2\theta - 5) + \sin^2\theta + 1]}{2(1 + \lambda)\cos^2\theta\sin\theta} z(x;\theta,d) \right] . \quad (A13)
 \end{aligned}$$

$$\begin{aligned}
 Z(x;\lambda,\theta,d) = & - \frac{\cos^2\theta + \lambda}{(1 + \lambda)} g(x;\theta,d) - \frac{2\cos^4\theta + \lambda\cos 2\theta(3\cos^2\theta + 1)}{2(1 + \lambda)\cos^2\theta} h(x;\theta,d) \\
 & + \frac{\sin\theta(2\lambda - 2\cos^2\theta - 1)}{(1 + \lambda)} k(x;\theta,d) + \frac{2\lambda(\sin\theta\cos 4\theta - 2\cos\theta\sin 4\theta)}{(1 + \lambda)} y(x;\theta,d) \\
 & + \frac{\lambda(2\cos 2\theta\sin^2\theta + \cos 4\theta)}{2(1 + \lambda)\cos^2\theta} z(x;\theta,d) \\
 & - \frac{\cos^2\theta + \lambda}{(1 + \lambda)x} \left[ [(l - x\cos 2\theta - 2d\sin\theta)^2 + [2\cos\theta(d - x\sin\theta)]^2]^{1/2} \right. \\
 & \left. - [(l + x\cos 2\theta + 2d\sin\theta)^2 + [2\cos\theta(d - x\sin\theta)]^2]^{1/2} \right] \\
 & + \frac{(d - x\sin\theta)}{x} \left[ \frac{\sin\theta}{1 + \lambda} g(x;\theta,d) \right. \\
 & - \frac{\sin\theta[\cos^2\theta(4\cos^2\theta + 1) + \lambda(6\cos^4\theta + \cos^2\theta - 1)]}{(1 + \lambda)\cos^2\theta} h(x;\theta,d) \\
 & + \frac{2[(4\cos^4\theta - \cos^2\theta - 1) - \lambda(6\cos^4\theta - 7\cos^2\theta)]}{(1 + \lambda)} k(x;\theta,d) \\
 & + \frac{4\lambda[\sin^2 2\theta(4\sin^2\theta - 5) + \cos^2\theta + 1]}{(1 + \lambda)} y(x;\theta,d) \\
 & \left. + \frac{\lambda\sin\theta(16\sin^4\theta - 28\sin^2\theta + 9)}{2(1 + \lambda)\cos^2\theta} z(x;\theta,d) \right] \quad (A14)
 \end{aligned}$$

$$L(x; \lambda, \theta, d) = \sin^2 \theta \cdot G(x; \lambda, \theta, d) + \cos^2 \theta \cdot Z(x; \lambda, \theta, d) \quad (A15)$$

For the perpendicular orientation (i.e.  $\theta = 90^\circ$ ),

$$a(x; \lambda, d) = \frac{\lambda - 1}{1 + \lambda} l \ln \left[ \frac{2d - l - x}{2d + l - x} \right] + \frac{4\lambda l}{1 + \lambda} (d - x) \cdot \frac{l^2 - d(2d - x)}{(2d - x)^2 - l^2} \quad (A16)$$

$$b(x; \lambda, d) = l \ln \left[ \frac{2d - l - x}{2d + l - x} \right] + \frac{4\lambda l}{1 + \lambda} (d - x) \cdot \frac{l^2 - d(2d - x)}{(2d - x)^2 - l^2} \quad (A17)$$

and

$$\begin{aligned} c(x; \lambda, d) &= a(x; \lambda, d) + 2 \left[ \frac{(\lambda - 1)l}{(1 + \lambda)x} + \frac{1}{1 + \lambda} \frac{(x - d)}{x} \cdot l \ln \left[ \frac{2d - l - x}{2d + l - x} \right] \right] \\ &\quad + \frac{4\lambda l}{(1 + \lambda)x} \cdot \left[ \frac{3l^2 - (2d - x) \cdot (4d - x)}{(2d - x)^2 - l^2} \right] \cdot (x - d)^2 \end{aligned} \quad (A18)$$