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The magnetic boundary conditions

The linearised form of the pre-Maxwell equations and Ohm's law is (see Chandrasekhar, 1961, Chapter 4)

$$\text{Curl } \underset{\sim}{b} = \mu \underset{\sim}{J} , \quad (1)$$

$$\text{Curl } \underset{\sim}{E} = -\frac{\partial \underset{\sim}{b}}{\partial t} , \quad (2)$$

$$\nabla . \underset{\sim}{E} = \nabla . \underset{\sim}{b} = 0 , \quad (3)$$

$$\underset{\sim}{J} = \sigma_e (\underset{\sim}{E} + \underset{\sim}{u} \wedge \underset{\sim}{B}_o), \quad (4)$$

where $\underset{\sim}{B}_o$ is the prevalent magnetic field while $\underset{\sim}{b}$, $\underset{\sim}{J}$, $\underset{\sim}{u}$ and $\underset{\sim}{E}$ the perturbations in $\underset{\sim}{B}_o$, the electric current, the velocity and the electric field , respectively.

We shall work in dimensional quantities here because the material quantities, which are used in transforming the equations into dimensionless form, are not continuous across the boundaries.

If we assume that the variables are proportional to $\exp i(kx + \ell y + \sigma t)$, equations (1), (2) and (4) show that $\underset{\sim}{b}$ and $\underset{\sim}{E}$ outside the layer ($\underset{\sim}{u} = 0$) obey

$$(D^2 - \gamma^2)\underset{\sim}{S} = 0 , \quad (5)$$

where

$$\gamma^2 = i\sigma\mu_o\sigma'_e + a^2 . \quad (6)$$

Here μ_o and σ'_e are the magnetic permeability and electric conductivity of the medium outside the layer. If there are no sources outside the layer, the solution of (5) for the electric field is

$$\underset{\sim}{E} = \underset{\sim}{F} \quad e^{+\gamma z} \qquad \text{for } z \gtrless \pm \frac{d}{2}$$

where $\underset{\sim}{F}$ is uniform and x,y and t dependence has been suppressed for convenience. We will consider the boundary at $z = d/2$. The boundary conditions at $z = -d/2$ can be obtained from those at $z = d/2$ by replacing γ by $-\gamma$.

We take $\underset{\sim}{F} = (F_x, F_y, F_z)$, use the first of (3) to express F_z in terms of F_x, F_y and substitute in equation (2) to find

$$+\sigma\gamma b_x = i\{-k\ell F_x + (\gamma^2 - \ell^2)F_y\}e^{-\gamma z},$$

$$+\sigma\gamma b_y = i\{(k^2 - \gamma^2)F_x + k\ell F_y\}e^{-\gamma z},$$

$$+\sigma\gamma b_z = (\ell F_x - k F_y)e^{-\gamma z}, \qquad z \geq \tfrac{1}{2}d \tag{8}$$

If $X(\tfrac{1}{2})$ denotes the value of the quantity X at $z = d/2$ inside the layer, the continuity of E_x and b_z give

$$F_x = E_x(\tfrac{1}{2})e^{\gamma d/2}, \quad kF_y = \{\ell E_x(\tfrac{1}{2}) + \sigma\gamma b_z(\tfrac{1}{2})\}e^{\gamma d/2} \tag{9}$$

When we substitute these expressions in the first two equations (8) and use the continuity of b_x/μ and b_y/μ, we obtain

$$+(\gamma^2 - a^2)E_x(\tfrac{1}{2}) = -\sigma\gamma\ell b_z(\tfrac{1}{2}) + i\sigma\gamma\delta b_y(\tfrac{1}{2})$$
$$= (\gamma^2 - \ell^2)\sigma\gamma b_z(\tfrac{1}{2}) - i\sigma\gamma\delta k\, b_x(\tfrac{1}{2}) \tag{10}$$

where

$$\delta = \mu_o/\mu\ . \tag{11}$$

If the direction of the prevalent magnetic field is $\hat{\underset{\sim}{B}} = (f,g,h_1)$, the x-component of (4) gives, after using the condition $u_z(\tfrac{1}{2}) = 0$

$$E_x(\tfrac{1}{2}) = \eta\{i\ell b_z(\tfrac{1}{2}) + \gamma b_y(\tfrac{1}{2})\} - B_o h_1 u_y(\tfrac{1}{2}). \tag{12}$$

The boundary conditions are then obtained by eliminating E_x from equations (10) and (12). If we now employ the non-dimensionalization adopted in the paper, express the horizontal components of $\underset{\sim}{b}$ and $\underset{\sim}{u}$

in terms of b, W, ξ and ζ (see Chandrasekhar, 1961 Chpt. 2,4), the boundary conditions become

$$\delta Db + \gamma b = \ell\{a^2 + i\sigma p_m - D^2)b - h_1 DW\}$$

$$+ k\{D\xi + h_1\zeta + p_m\gamma\delta\xi/\bar{p}\} = 0 \qquad \text{at } z = \tfrac{1}{2} \qquad (13)$$

If we use equation (2.4) in the paper and note that $W = 0$ on the boundary, equation (13) reduces to

$$\delta Db + \gamma b \;=\; (D\xi + h_1\zeta) + p_m\gamma\delta k\xi = 0 \qquad \text{at } z = \tfrac{1}{2} \qquad (14)$$

It should be remarked here that if $k = 0$, we can use E_y instead of E_x in equations (9) to arrive at the same result (14). It may also be noted that the boundary conditions (14) hold even if $\sigma=0$ and $\underset{\sim}{E}$ is derivable from a potential as equation (2) indicates.