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SLOW MOTION OF TWO SPHERES IN A SHEAR FIELD,

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Procedure for Solution of Stokes Equation

1. We redefine the coefficients A^i_{mn} and B^i_{mn} as follows:

$$A^0_{mn} = A^m_n \qquad \bar{A}^0_{mn} = C^m_n$$

$$B^0_{mn} = B^m_n \qquad \bar{B}^0_{mn} = D^m_n$$

$$A^{-1}_{mn} = E^m_n \qquad A^{+1}_{mn} = G^m_n$$

$$B^{-1}_{mn} = F^m_n \qquad B^{+1}_{mn} = H^m_n$$

Substitution of eq (4) into the continuity equation gives

$$\left(3+\rho\frac{\partial}{\partial\rho}+z\frac{\partial}{\partial z}\right)\overline{W}^0_m + c\left[\left(\frac{\partial}{\partial\rho}+\frac{|m|+1}{\rho}\right)W^{+1}_m + \left(\frac{\partial}{\partial\rho}-\frac{|m|-1}{\rho}\right)W^{-1}_m + 2\frac{\partial}{\partial z}W^0_m\right] = 0,$$

from which are found two sets of relations among the constants. One set is

$$5C^m_n - (n-|m|)C^m_{n-1} + (n+|m|+1)C^m_{n+1} + 2E^m_n - E^m_{n-1} - E^m_{n+1}$$
$$-2(n-|m|)(n+|m|+1)G^m_n + (n-|m|-1)(n-|m|)G^m_{n-1}$$
$$+(n+|m|+1)(n+|m|+2)G^m_{n+1} + 2(2n+1)B^m_n - 2(n-|m|)B^m_{n-1}$$

n+|m|+1 $-2(n+|m|\ 1)B^m_{n+1} = 0,$ (A1)

and the other is of the same form with B^m_n, C^m_n, E^m_n and G^m_n replaced by A^m_n, D^m_n, F^m_n and H^m_n, respectively.

The boundary value functions $\underset{\sim}{V}^{I}$ and $\underset{\sim}{V}^{II}$ can be written in the form

$$\underset{\sim}{V}^N(\eta,\phi) = \sum_{k=0}^{\infty}\left[\underset{\sim}{X}^N_k(\eta)\sin k\phi + \underset{\sim}{X}^N_{-k}(\eta)\cos k\phi\right] \tag{A2}$$

in which the $\underset{\sim}{X}^N_m$ are arbitrary functions of η. If we denote the ρ, ϕ and z components of $\underset{\sim}{X}^N_m$ by X^N_m, Y^N_m and Z^N_m, respectively, then the boundary conditions (3) require that the functions W^i_m of (4) satisfy

$$\overline{W}^{\circ N}_m = \frac{2c}{z}\left(Z^N_m - W^{\circ N}_m\right) \tag{A3a}$$

$$W^{-1N}_m = \left(X^N_m + Y^N_m - \frac{\rho_N}{z_N} Z^N_m\right) + \frac{\rho_N}{z_N} W^{\circ N}_m \tag{A3b}$$

and
$$W^{+1N}_m = \left(X^N_m - Y^N_m - \frac{\rho_N}{z_N} Z^N_m\right) + \frac{\rho_N}{z_N} W^{\circ N}_m \tag{A3c}$$

for $N = I$ and II. Here the superscript N on W^i_m and the subscript N on ρ and z indicate that these functions are to be evaluated at $\xi = \alpha_I$ for $N = I$ and $\xi = -\alpha_{II}$ for $N = II$. Now substitute the general solutions (5) for W°_m and $\overline{W}^\circ_m$ into (A3a), multiply each term by $P^{|m|}_n(t)\cdot(\cosh\alpha_N - t)^{-1/2}$ and integrate over $t = \cos\eta$ from -1 to 1.

When the resulting two equations obtained with $N = I$ and $N = II$ are substracted from one another or are added, expressions relating the coefficients C^m_n and D^m_n to A^m_n's and B^m_n's are obtained:

$$C^m_n = \mathcal{C}^m_n + 2(n-|m|)\left\{\left[h^I_{n-1} + h^{II}_{n+1}\right]B^m_{n-1} + \left[k^I_{n-1} - k^{II}_{n+1}\right]A^m_{n-1}\right\}$$
$$-2(2n+1)\left\{\left[h^I_n\cosh\alpha_I + h^{II}_n\cosh\alpha_{II}\right]B^m_n + \left[k^I_n\cosh\alpha_I - \right.\right.$$

$$-k_n^{II}\cosh\alpha_{II}]A_n^m\} + 2(n+|m|+1)\{[h_{n+1}^{I}+h_{n+1}^{II}]B_{n+1}^m$$
$$+[k_{n+1}^{I}-k_{n+1}^{II}]\,A_{n+1}^m\} \tag{A4a}$$

and

$$D_n^m = \mathcal{D}_n^m + 2(n-|m|)\{[\bar{h}_{n-1}^{I}-\bar{h}_{n-1}^{II}]B_{n-1}^m + [\bar{k}_{n-1}^{I}+\bar{k}_{n-1}^{II}]A_{n-1}^m$$
$$-2(2n+1)\{[\bar{h}_n^{I}\cosh\alpha_I-\bar{h}_n^{II}\cosh\alpha_{II}]B_n^m + [\bar{k}_n^{I}\cosh\alpha_I +$$
$$+\bar{k}_n^{II}\cosh\alpha_{II}]A_n^m\} + 2(n+|m|+1)\{[\bar{h}_{n+1}^{I}-\bar{h}_{n+1}^{II}]B_{n+1}^m$$
$$+[\bar{k}_{n+1}^{I}+\bar{k}_{n+1}^{II}]A_{n+1}^m\}\,. \tag{A4b}$$

Here

$$\mathcal{C}_n^m = \frac{2(n-|m|)!\,(n+1/2)}{(n+|m|)!}\int_{-1}^{1}\left\{\frac{S_n^{II}}{\sinh\alpha_I}R^{I}(t)\,Z_m^{I}(t) - \frac{S_n^{I}}{\sinh\alpha_{II}}R^{II}(t)\,Z_m^{II}(t)\right\}P_n^{|m|}(t)\,dt\,, \tag{A5a}$$

$$\mathcal{D}_n^m = \frac{2(n-|m|)!\,(n+1/2)}{(n+|m|)!}\int_{-1}^{1}\left\{\frac{T_n^{II}}{\sinh\alpha_I}R^{I}(t)\,Z_m^{I}(t) + \frac{T_n^{I}}{\sinh\alpha_{II}}R^{II}(t)\,Z_m^{II}(t)\right\}P_n^{|m|}(t)\,dt\,, \tag{A5b}$$

$$h_r^{I} = S_n^{II}H_r^{I} \qquad \bar{h}_r^{I} = T_n^{II}H_r^{I}$$
$$h_r^{II} = S_n^{I}H_r^{II} \qquad \bar{h}_r^{II} = T_n^{I}H_r^{II}$$
$$k_r^{I} = S_n^{II}K_r^{I} \qquad \bar{k}_r^{I} = T_n^{II}K_r^{I}$$
$$k_r^{II} = S_n^{I}K_r^{II} \quad \text{and} \quad \bar{k}_r^{II} = T_n^{I}K_r^{II}\,,$$

where

$$H_r^N = \frac{\sinh(r+1/2)\alpha_N}{(2r+1)\sinh\alpha_N}\,, \qquad K_r^N = \frac{\cosh(r+1/2)\alpha_N}{(2r+1)\sinh\alpha_N}$$

$$M_n = \sinh[(n+1/2)(\alpha_I+\alpha_{II})]\,, \qquad R^N = (\cosh\alpha_N - t)^{1/2}$$

$$S_n^N = (M_n)^{-1}\sinh(n+1/2)\,\alpha_N\,, \qquad T_n^N = (M_n)^{-1}\cosh(n+1/2)\,\alpha_N\,.$$

Applying the same operations to the other two pairs of boundary conditions (A3b) and (A3c) results in the following expressions for E_n^m, F_n^m, G_n^m and H_n^m:

$$E_n^m = \mathcal{E}_n^m - (n-|m|)(n-|m|+1)\left\{(h^{I}_{n-1}+h^{II}_{n-1})B^m_{n-1} + (k^{I}_{n-1}-k^{II}_{n-1})A^m_{n-1}\right\}$$
$$+ (n+|m|)(n+|m|+1)\left\{(h^{I}_{n+1}+h^{II}_{n+1})B^m_{n+1} + (k^{I}_{n+1}-k^{II}_{n+1})A^m_{n+1}\right\} \qquad \text{(A4c)}$$

$$F_n^m = \mathcal{F}_n^m - (n-|m|)(n-|m|+1)\left\{(\bar{h}^{I}_{n-1}-\bar{h}^{II}_{n-1})B^m_{n-1} + (\bar{k}^{I}_{n-1}+\bar{k}^{II}_{n-1})A^m_{n-1}\right\}$$
$$+ (n+|m|)(n+|m|+1)\left\{(\bar{h}^{I}_{n+1}-\bar{h}^{II}_{n+1})B^m_{n+1} + (\bar{k}^{I}_{n+1}+\bar{k}^{II}_{n+1})A^m_{n+1}\right\} \qquad \text{(A4d)}$$

$$G_n^m = \mathcal{G}_n^m + \left\{(h^{I}_{n-1}+h^{II}_{n-1})B^m_{n-1} + (k^{I}_{n-1}-k^{II}_{n-1})A^m_{n-1}\right\}$$
$$- \left\{(h^{I}_{n+1}+h^{II}_{n+1})B^m_{n+1} + (k^{I}_{n+1}-k^{II}_{n+1})A^m_{n+1}\right\} \qquad \text{(A4e)}$$

and

$$H_n^m = \mathcal{H}_n^m + \left\{(\bar{h}^{I}_{n-1}-\bar{h}^{II}_{n-1})B^m_{n-1} + (\bar{k}^{I}_{n-1}+\bar{k}^{II}_{n-1})A^m_{n-1}\right\}$$
$$- \left\{(\bar{h}^{I}_{n+1}-\bar{h}^{II}_{n+1})B^m_{n+1} + (\bar{k}^{I}_{n+1}+\bar{k}^{II}_{n+1})A^m_{n+1}\right\}, \qquad \text{(A4f)}$$

where

$$\mathcal{E}_n^m = \frac{(n-|m|+1)!\,(n+1/2)}{(n+|m|-1)!}\int_{-1}^{1}\left\{\frac{S_n^{II}}{R^{I}}\left[X_m^{I}+Y_m^{I}-\frac{\sin\eta}{\sinh\alpha_{I}}Z_m^{I}\right]\right.$$
$$\left. + \frac{S_n^{I}}{R^{II}}\left[X_m^{II}+Y_m^{II}+\frac{\sin\eta}{\sinh\alpha_{II}}Z_m^{II}\right]\right\}P_n^{|m|-1}\,dt \qquad \text{(A5c)}$$

$$\mathcal{F}_n^m = \frac{(n-|m|+1)!\,(n+1/2)}{(n+|m|-1)!}\int_{-1}^{1}\left\{\frac{T_n^{II}}{R^{I}}\left[X_m^{I}+Y_m^{I}-\frac{\sin\eta}{\sinh\alpha_{I}}Z_m^{I}\right]\right.$$
$$\left. - \frac{T_n^{I}}{R^{II}}\left[X_m^{II}+Y_m^{II}+\frac{\sin\eta}{\sinh\alpha_{II}}Z_m^{II}\right]\right\}P_n^{|m|-1}\,dt \qquad \text{(A5d)}$$

$$\mathcal{G}_n^m = \frac{(n-|m|-1)!\,(n+1/2)}{(n+|m|+1)!}\int_{-1}^{1}\Big\{\frac{S_n^{II}}{R^{I}}\Big(X_m^{I}-Y_m^{I}-\frac{\sin\eta}{\sinh\alpha_I}Z_m^{I}\Big) + \frac{S_n^{I}}{R^{II}}\Big(X_m^{II}-Y_m^{II}+\frac{\sin\eta}{\sinh\alpha_{II}}Z_m^{II}\Big)\Big\}P_n^{|m|+1}\,dt \tag{A5e}$$

and

$$\mathcal{H}_n^m = \frac{(n-|m|-1)!\,(n+1/2)}{(n+|m|+1)!}\int_{-1}^{1}\Big\{\frac{T_n^{II}}{R^{I}}\Big(X_m^{I}-Y_m^{I}-\frac{\sin\eta}{\sinh\alpha_I}Z_m^{I}\Big) - \frac{T_n^{I}}{R^{II}}\Big(X_m^{II}-Y_m^{II}+\frac{\sin\eta}{\sinh\alpha_{II}}Z_m^{II}\Big)\Big\}P_n^{|m|+1}\,dt\ . \tag{A5f}$$

Finally, substitution of the above expressions for C_n^m, D_n^m, E_n^m, F_n^m, G_n^m and H_n^m into the two relations (A1) obtained from the continuity equation gives two sets of recursion relations for the A_n^m and B_n^m. One set is

$$-(n-|m|)\{[2(h_{n-1}^{I}+h_{n-1}^{II})-1]B_{n-1}^m+2(k_{n-1}^{I}-k_{n-1}^{II})A_{n-1}^m\}+(2n+1)\cdot$$
$$\cdot\{[2(h_n^{I}\cosh\alpha_I+h_n^{II}\cosh\alpha_{II})-1]B_n^m+2(k_n^{I}\cosh\alpha_I-k_n^{II}\cosh\alpha_{II})A_n^m\}$$
$$-(n+|m|+1)\{[2(h_{n+1}^{I}+h_{n+1}^{II})-1]B_{n+1}^m+2(k_{n+1}^{I}-k_{n+1}^{II})A_{n+1}^m\}=\mathcal{B}_n^m$$
$$\equiv\frac{1}{2}\Big\{5\mathcal{C}_n^m-(n-|m|)\mathcal{C}_{n-1}^m+(n+|m|+1)\mathcal{C}_{n+1}^m+2\mathcal{E}_n^m-\mathcal{E}_{n-1}^m-\mathcal{E}_{n+1}^m$$
$$-2(n-|m|)(n+|m|+1)\mathcal{G}_n^m+(n-|m|-1)(n-|m|)\mathcal{G}_{n-1}^m+(n+|m|+1)(n+|m|+2)\mathcal{G}_{n+1}^m\Big\} \tag{A6}$$

The other set has the same form, but with h_n, k_n, A_n^m, B_n^m, $\mathcal{C}_n^m$, $\mathcal{E}_n^m$ and $\mathcal{G}_n^m$ replaced, respectively, by $\bar{k}_n$, $\bar{h}_n$, B_n^m, A_n^m, $\mathcal{D}_n^m$, $\mathcal{F}_n^m$ and $\mathcal{H}_n^m$.

2. LIMITING CASES

(a) When the two spheres are same size the expressions (A4) reduce to

$$C_n^m = \mathcal{C}_n^m + \frac{2(n-|m|)}{2n-1}(h_n'-1)B_{n-1}^m - 2h_n' B_n^m + \frac{2(n+|m|+1)}{2n+3}(h_n'+1)B_{n+1}^m$$

$$D_n^m = \mathcal{D}_n^m + \frac{2(n-|m|)}{2n-1}(k_n'-1)A_{n-1}^m - 2k_n' A_n^m + \frac{2(n+|m|+1)}{2n+3}(k_n'+1)A_{n+1}^m$$

$$E_n^m = \mathcal{E}_n^m - \frac{(n-|m|+1)(n-|m|)}{2n-1}(h_n'-1)B_{n-1}^m + \frac{(n+|m|)(n+|m|+1)}{2n+3}(h_n'+1)B_{n+1}^m$$

$$F_n^m = \mathcal{F}_n^m - \frac{(n-|m|+1)(n-|m|)}{2n-1}(k_n'-1)A_{n-1}^m + \frac{(n+|m|)(n+|m|+1)}{2n+3}(k_n'+1)A_{n+1}^m$$

$$G_n^m = \mathcal{G}_n^m + \frac{(h_n'-1)}{2n-1}B_{n-1}^m - \frac{(h_n'+1)}{2n+3}B_{n+1}^m$$

and
$$H_n^m = \mathcal{H}_n^m + \frac{(k_n'-1)}{2n-1}A_{n-1}^m - \frac{(k_n'+1)}{2n+3}A_{n+1}^m$$

where
$$h_n' = \coth\alpha \tanh(n+1/2)\alpha$$
$$k_n' = \coth\alpha \coth(n+1/2)\alpha$$

and
$$\left\{(2n-1)(h_n'-1)-(2n+3)(h_n'-1)\right\}\left\{\frac{n-|m|}{2n-1}B_{n-1}^m - \frac{n}{2n+1}B_n^m\right\} -$$
$$-\left\{(2n+5)(h_n'+1)-(2n+3)(h_n'+1)\right\}\left\{\frac{n+1}{2n+1}B_n^m - \frac{n+|m|+1}{2n+3}B_{n+1}^m\right\} = \mathcal{B}_n^m .$$

The set of equations for A_n^m is the same, but with h_n', B_n^m and $\mathcal{B}_n^m$ replaced by k_n', A_n^m and $\mathcal{A}_n^m$.

Let the undisturbed shear field be arbitrarily oriented and given by $\underline{u}_0 = \underline{\underline{K}} \cdot \underline{r}$, where $\underline{\underline{K}}$ is the velocity gradient tensor and $\underline{r}$ is the position vector. Then the non-zero terms of (A5) become

$$\mathcal{C}_n^{-1} = 2a\,(\Omega_y^{I} - \Omega_y^{II})\,\nu_n(\alpha)\,\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{C}_n^{0} = \left[\,(u_z^{I} - u_z^{II})\,\nu_n(\alpha)/\sinh\alpha - 2a\sinh\alpha\,\lambda_n(\alpha)\,K_{zz}\right]\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{C}_n^{+1} = -2a\,(\Omega_x^{I} - \Omega_x^{II})\,\nu_n(\alpha)\,\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{D}_n^{-1} = 2a\,(\Omega_y^{I} + \Omega_y^{II} + 2K_{zx})\,\nu_n(\alpha)\,\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{D}_n^{0} = (u_z^{I} + u_z^{II})\,\nu_n(\alpha)/\sinh\alpha\cdot\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{D}_n^{+1} = -2a\,(\Omega_x^{I} + \Omega_x^{II} - 2K_{zy})\,\nu_n(\alpha)\cdot\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{E}_n^{-2} = 2a\sinh\alpha\,(K_{yy} - K_{xx})\,\lambda_n(\alpha)\,\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{E}_n^{-1} = \left\{(u_x^{I} + u_x^{II})\,\lambda_n(\alpha) + a\,(\Omega_y^{I} - \Omega_y^{II})\left[-\nu_n(\alpha)/2 + (n+1/2)\,\lambda_n(\alpha)\sinh\alpha\right]\right\}\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{E}_n^{0} = -\left\{a\sinh\alpha\,(\Omega_z^{I} + \Omega_z^{II} + K_{xy} - K_{yx} + 3K_{zz})\,\lambda_n(\alpha) + (u_z^{I} - u_z^{II})\,\nu_n(\alpha)/\sinh\alpha\right\}\cdot$$
$$\cdot\, n(n+1)\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{E}_n^{+1} = \left\{(u_y^{I} + u_y^{II})\,\lambda_n(\alpha) - a\,(\Omega_x^{I} - \Omega_x^{II})\left[-\nu_n(\alpha)/2 + (n+1/2)\,\lambda_n(\alpha)\sinh\alpha\right]\right\}\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{E}_n^{+2} = -2a\sinh\alpha\,(K_{xy} + K_{yx})\,\lambda_n(\alpha)\,\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{F}_n^{-1} = \Big\{(u_x^{I} - u_x^{II})\,\lambda_n(\alpha) + a\,(\Omega_y^{I} + \Omega_y^{II})\left[-\nu_n(\alpha)/2 + (n+1/2)\,\lambda_n(\alpha)\sinh\alpha\right]$$
$$-2a\Big[\sinh\alpha\,(2n+1)\,\lambda_n\,K_{xz} + \left(\frac{n(n-1)}{2n-1}\,\lambda_{n-1} - \frac{(n+1)(n+2)}{2n+3}\right)K_{zx}\Big\}\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{F}_n^{0} = -\left[a\sinh\alpha\,(\Omega_z^{I} - \Omega_z^{II})\,\lambda_n + (u_z^{I} + u_z^{II})\,\nu_n/\sinh\alpha\right]n(n+1)\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{F}_n^{+1} = \Big\{(u_y^{I} - u_y^{II})\,\lambda_n - a\,(\Omega_x^{I} + \Omega_x^{II})\left[-\nu_n/2 + (n+1/2)\,\lambda_n\sinh\alpha\right] - 2a\Big[(2n+1)\cdot$$
$$\sinh\alpha\,\lambda_n\,K_{yz} + \left(\frac{n(n-1)}{2n-1}\,\lambda_{n-1} - \frac{(n+1)(n+2)}{2n+3}\,\lambda_{n+1}\right)K_{zy}\Big]\Big\}\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{G}_n^{-1} = -2a\,(\Omega_y^{I} - \Omega_y^{II})\,\nu_n\,\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{G}_n^{0} = -\left[\alpha \sinh\alpha\ \lambda_n\left(\Omega_z^{I}+\Omega_z^{II}+K_{xy}-K_{yx}-3K_{zz}\right)-\left(u_z^{I}-u_z^{II}\right)\nu_n/\sinh\alpha\right]\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{G}_n^{+1} = 2\alpha\left(\Omega_x^{I}-\Omega_x^{II}\right)\nu_n(\alpha)\operatorname{sech}(n+1/2)\alpha$$

$$\mathcal{H}_n^{-1} = -2\alpha\left(\Omega_y^{I}+\Omega_y^{II}+2K_{zx}\right)\nu_n\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{H}_n^{0} = -\left[\alpha\sinh\alpha\left(\Omega_z^{I}-\Omega_z^{II}\right)\lambda_n-\nu_n/\sinh\alpha\cdot\left(u_z^{I}+u_z^{II}\right)\right]\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{H}_n^{+1} = 2\alpha\left(\Omega_x^{I}+\Omega_x^{II}-2K_{zy}\right)\nu_n\operatorname{cosech}(n+1/2)\alpha$$

$$\mathcal{B}_n^{-2} = \alpha\sinh\alpha\left(K_{yy}-K_{xx}\right)\lambda_n\left[e^{\alpha}\operatorname{sech}(n-1/2)\alpha-2\operatorname{sech}(n+1/2)\alpha+\right.$$
$$\left.+\ e^{-\alpha}\operatorname{sech}(n+3/2)\alpha\right]$$

$$\mathcal{B}_n^{-1} = \tfrac{1}{2}\left(u_x^{I}+u_x^{II}\right)\lambda_n\left[e^{\alpha}\operatorname{sech}(n-1/2)\alpha-2\operatorname{sech}(n+1/2)\alpha+e^{-\alpha}\operatorname{sech}(n+3/2)\alpha\right]$$
$$+\tfrac{1}{2}\alpha\left(\Omega_y^{I}-\Omega_y^{II}\right)\lambda_n\left[(2n+1)\left(\frac{e^{\alpha}}{2n-1}+\frac{e^{-\alpha}}{2n+1}\right)\operatorname{sech}(n+1/2)\alpha-\frac{2n-1}{2n+1}\operatorname{sech}(n-1/2)\alpha\right.$$
$$\left.-\frac{2n+3}{2n+1}\operatorname{sech}(n+3/2)\alpha\right]$$

$$\mathcal{B}_n^{0} = \sqrt{2}\,\alpha\sinh^2\alpha\operatorname{sech}(n+1/2)\alpha\ K_{zz}\left[(n+1)(3n+5)\operatorname{sech}(n+3/2)\alpha-n(3n-2)\cdot\right.$$
$$\left.\cdot\operatorname{sech}(n-1/2)\alpha\right]+\tfrac{1}{2}\left(u_z^{I}-u_z^{II}\right)/\sinh\alpha\left[(4n^2+4n-5)\nu_n\operatorname{sech}(n+1/2)\alpha-\right.$$
$$\left.-n(2n-3)\nu_{n-1}\operatorname{sech}(n-1/2)\alpha-(n+1)(2n+5)\nu_{n+1}\operatorname{sech}(n+3/2)\alpha\right]$$

$$\mathcal{B}_n^{+1} = \tfrac{1}{2}\left(u_y^{I}+u_y^{II}\right)\lambda_n\left[e^{\alpha}\operatorname{sech}(n-1/2)\alpha-2\operatorname{sech}(n+1/2)\alpha+e^{-\alpha}\operatorname{sech}(n+3/2)\alpha\right]$$
$$-\tfrac{1}{2}\alpha\left(\Omega_x^{I}-\Omega_x^{II}\right)\lambda_n\left[(2n+1)\left(\frac{e^{\alpha}}{2n-1}+\frac{e^{-\alpha}}{2n+1}\right)\operatorname{sech}(n+1/2)\alpha-\frac{2n-1}{2n+1}\operatorname{sech}(n-1/2)\alpha\right.$$
$$\left.-\frac{2n+3}{2n+1}\operatorname{sech}(n+3/2)\alpha\right]$$

$$\mathcal{B}_n^{+2} = -\alpha\sinh\alpha\left(K_{xy}+K_{yx}\right)\lambda_n\left[e^{\alpha}\operatorname{sech}(n-1/2)\alpha-2\operatorname{sech}(n+1/2)\alpha\right.$$
$$\left.+\ e^{-\alpha}\operatorname{sech}(n+3/2)\alpha\right]$$

$$\mathcal{A}_n^{-1} = \tfrac{1}{2}\left(u_x^{I}-u_x^{II}\right)\lambda_n\left[e^{\alpha}\operatorname{cosech}(n-1/2)\alpha-2\operatorname{cosech}(n+1/2)\alpha+e^{-\alpha}\operatorname{cosech}(n+3/2)\alpha\right]$$
$$+\tfrac{1}{2}\alpha\left(\Omega_y^{I}+\Omega_y^{II}\right)\lambda_n\left[(2n+1)\left(\frac{e^{\alpha}}{2n-1}+\frac{e^{-\alpha}}{2n+3}\right)\operatorname{cosech}(n+1/2)\alpha-\frac{2n-1}{2n+1}\operatorname{cosech}(n-1/2)\alpha\right.$$

(b) *For the case of a sphere near a plane wall equation (A4) becomes*

$$C_n^m = \mathcal{C}_n^m + (n-|m|)\, B_{n-1}^m - (2n+1)\, B_n^m + (n+|m|+1)\, B_{n+1}^m$$
$$D_n^m = \mathcal{D}_n^m - 2\, k_n'' \left[\frac{n-|m|}{2n-1}\, B_{n-1}^m - B_n^m + \frac{n+|m|+1}{2n+3}\, B_{n+1}^m \right]$$
$$E_n^m = \mathcal{E}_n^m - (n-|m|)(n-|m|+1)/2\; B_{n-1}^m + (n+|m|)(n+|m|+1)/2\; B_{n+1}^m$$
$$F_n^m = \mathcal{F}_n^m + k_n'' \left[\frac{(n-|m|)(n-|m|+1)}{2n-1}\, B_{n-1}^m - \frac{(n+|m|)(n+|m|+1)}{2n+3}\, B_{n+1}^m \right]$$
$$G_n^m = \mathcal{G}_n^m + 1/2 \left[B_{n-1}^m - B_{n+1}^m \right]$$
$$H_n^m = \mathcal{H}_n^m - k_n'' \left[\frac{1}{2n-1}\, B_{n-1}^m - \frac{1}{2n+3}\, B_{n+1}^m \right]$$

and
$$\left\{ (2n-1)\, k_{n-1}'' - (2n-3)\, k_n'' \right\} \left\{ \frac{n-|m|}{2n-1}\, B_{n-1}^m - \frac{n}{2n+1}\, B_n^m \right\} -$$
$$- \left\{ (2n+5)\, k_n'' - (2n+3)\, k_{n+1}'' \right\} \left\{ \frac{n+1}{2n+1}\, B_n^m - \frac{(n+|m|+1)}{2n+3}\, B_{n+1}^m \right\} = \mathcal{B}_n^m$$

where
$$k_n'' = (n+1/2) \coth(n+1/2)\alpha - \coth\alpha \,.$$

Here the non-zero integrals of (A5) for the velocity field $\mathbf{u}_0$ *given in section 4 give*

$$\mathcal{D}_n^{-1} = 4\alpha\, \nu_n\, \Omega_y$$
$$\mathcal{D}_n^{0} = 2\nu_n\, U_z \operatorname{csch}\alpha$$
$$\mathcal{D}_n^{+1} = -4\alpha\, \nu_n\, \Omega_x$$

$$\mathcal{F}_n^{-1} = 2\, U_x \lambda_n + 2\alpha\Omega_y \left[(n+1/2)\, \lambda_n \sinh\alpha - \tfrac{1}{2}\nu_n \right]$$
$$\mathcal{F}_n^{0} = -2n(n+1) \left[c\,\Omega_z \lambda_n - \tfrac{2}{3}(n+1/2)\, \frac{\omega c^2}{\ell} \lambda_n + U_z \nu_n \operatorname{csch}\alpha \right]$$

$$\mathcal{F}_n^{+1} = 2U_y\lambda_n - 2\alpha\Omega_x[(n+1/2)\lambda_n \sinh\alpha - \tfrac{1}{2}\nu_n] - \frac{4\omega\delta c}{\ell}(n+1/2)\lambda_n$$

$$\mathcal{H}_n^{-1} = -\mathcal{D}_n^{-1}$$

$$\mathcal{H}_n^{0} = \frac{\mathcal{F}_n^{0}}{n(n+1)}$$

$$\mathcal{H}_n^{+1} = -\mathcal{D}_n^{+1}$$

$$\mathcal{B}_n^{-1} = K_n\Big\{\mu_{n-1}\Big[U_x e^{\alpha} - \alpha\Omega_y\Big(\frac{2n-1}{2n+1}\Big)\Big] - 2\mu_n\Big[U_x - \alpha\Omega_y\frac{2n+1}{2}\cdot\Big(\frac{e^{\alpha}}{2n-1} + \frac{e^{-\alpha}}{2n+3}\Big)\Big] + \mu_{n+1}\Big[U_x e^{-\alpha} - \alpha\Omega_y\frac{2n+3}{2n+1}\Big]\Big\}$$

$$\mathcal{B}_n^{0} = U_z \operatorname{csch}\alpha\{-n(2n-3)\nu_{n-1} + (4n^2+4n-5)\nu_n - (n+1)(2n+5)\nu_{n+1}\}$$

$$\mathcal{B}_n^{+1} = K_n\Big\{\mu_{n-1}\Big[U_y e^{\alpha} + \alpha\Omega_x\Big(\frac{2n-1}{2n+1}\Big) - \frac{\omega\delta c}{\ell}(2n+1)e^{\alpha}\Big] - 2\mu_n\Big[U_y + \alpha\Omega_x\frac{(2n+1)}{2}\Big(\frac{e^{\alpha}}{2n-1} + \frac{e^{-\alpha}}{2n+3}\Big) - \frac{\omega\delta c}{\ell}(2n+1)\Big] + \mu_{n+1}\Big[U_y e^{-\alpha} + \alpha\Omega_x\Big(\frac{2n+3}{2n+1}\Big) - \frac{\omega\delta c}{\ell}(2n+3)e^{-\alpha}\Big]\Big\}$$

where

$$K_n = 2^{1/2}\exp[-(n+1/2)\alpha]$$

$$\mu_n = \operatorname{csch}(n+1/2)\alpha$$

$$\lambda_n = K_n\mu_n$$

and

$$\nu_n = -\frac{1}{2}\Big(\frac{K_{n-1}}{2n-1} - \frac{K_{n+1}}{2n+3}\Big)\mu_n \quad .$$