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Supplemental Material

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In this supplementary material, we discuss additional DNS that have been carried out to strengthen the validation of the numerical setup.

To add to the discussion on the influence of the size of the reservoirs, figure 1(a,b,c) shows the spatio-temporal diagrams of the TKE for the Bench., BR and SR at $Re = 650$, $\theta = 6$ (Wave regime).

These cases show good consistency, although we should not expect exact agreement in such a chaotic flow at Reynolds number 650. The turbulent structures are qualitatively similar, especially in the centre of the duct (in spatial extent, amplitude, temporal duration), showing that the reservoir geometry has little influence on the flow in the middle of the duct. Near the exit, the BR and benchmark cases can produce stationary waves induced by the sharp change of geometry. In SR, these waves are largely eliminated due to a lack of geometry change, which causes minor differences in the overall flow dynamics between SR and the other cases.

Our DNS were initialised with a fairly large uniform noise of amplitude $\varsigma = 0.5$ in the entire computational domain, to break the symmetry of simulations so that it can faithfully mimic experiments. This random noise decays exponentially as the simulation starts, and it thus barely interacts with the exchange flow that develops shortly after. To validate the choice of random noise, we perform an additional DNS in the BR using a significantly smaller noise $\varsigma = 0.005$ (panel d). The TKE of this case shows negligible difference when compared with the case with large noise (panel b), except perhaps for a slightly greater symmetry (around $x = 0$), as expected. Comparing panels (b) and (d) also illustrate that a change in initial noise generates subsequent variability in the flow that is roughly comparable with a change in reservoir conditions (comparing panels a,b,c).

To illustrate the influence of the spanwise length of the reservoirs, we also show in panel (e) the DNS in BR with a wider reservoir $L_y = 4$, i.e. twice as wide as the duct (mimicking the experiments more closely). Comparing panels (b) and (e) we find again good qualitative agreement, with differences being of the same order as those caused by a change in initial noise.

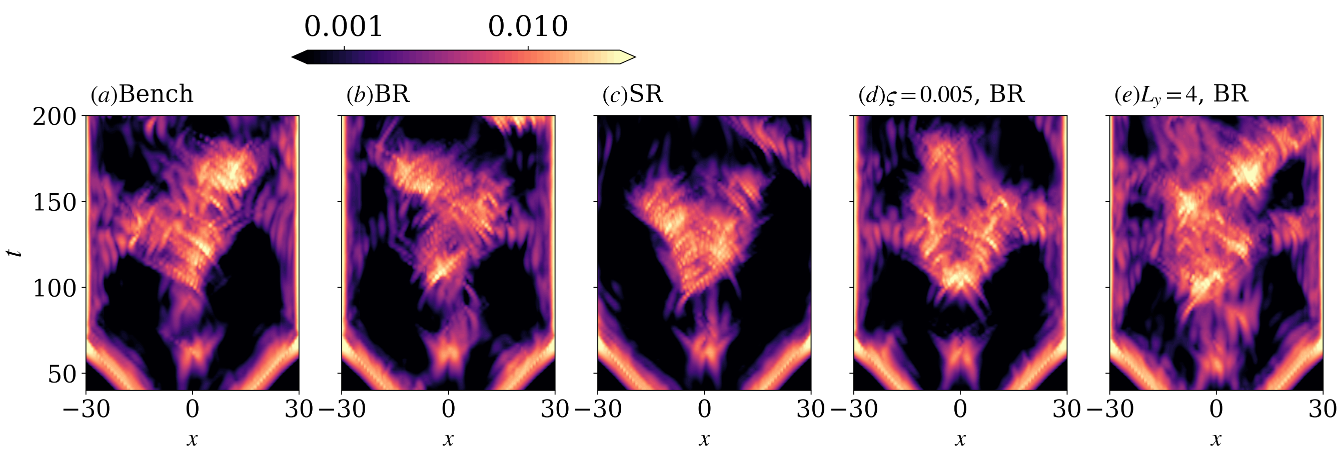

FIG. 1: Comparison of TKE $x - t$ diagrams in the W regime between (a) benchmark, (b) BR, (c) SR, (d) BR, $\varsigma = 0.005$, and (e) BR, $L_y = 4$.

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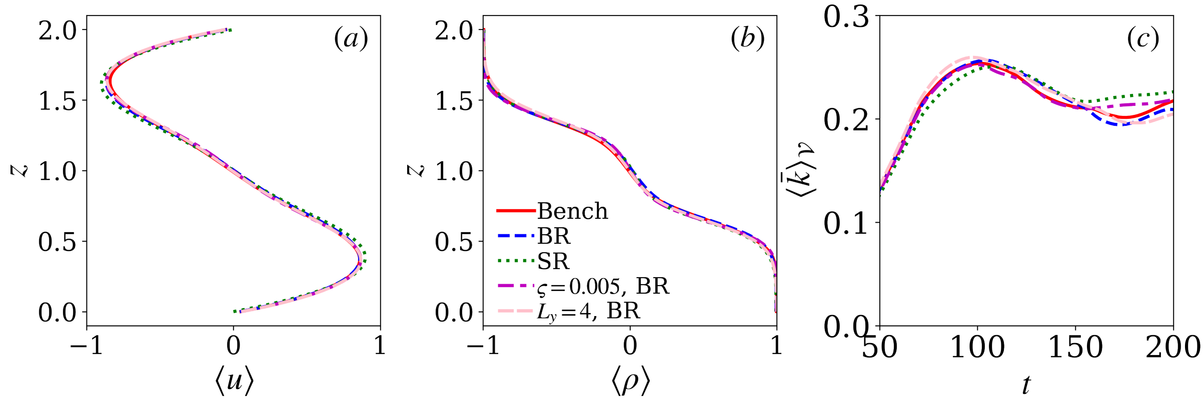

FIG. 2: Comparison of integrated flow statistics in the W regime.

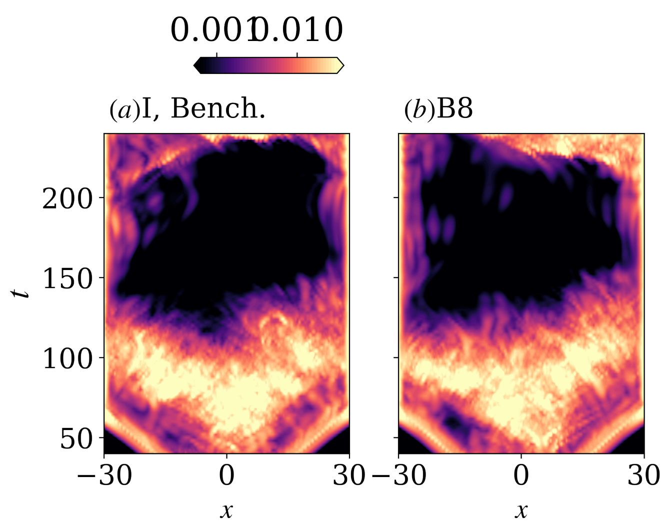

FIG. 3: Comparison of TKE $x - t$ diagrams in the I regime between (a) benchmark abd (b) BR.

In fig. 2 we plot the mean velocity profile (panel a), density profile (panel b) and volume-averaged TKE time series (panel c) in these five same DNS. The agreement between them is excellent, confirming that the mean statistics are even less dependent than the spatio-temporal TKE on variations in reservoir geometry or initial noise.

To evaluate the capability of the forcing method to simulate more turbulent regimes, we conducted an additional benchmark case at the I regime (B8). Figure 3 presents the $x - t$ diagrams of TKE for both the benchmark and the BR cases. The dynamic cycle between active and quiet phases, as well as the development and propagation of waves from the duct exit towards the center, are highly similar between the two cases. This indicates that the forcing terms have little effect on the instability near the exit and inside the duct. Furthermore, the presence of instabilities and waves near the exits in both cases confirms that these phenomena are not artifacts of the forcing method, but rather physical phenomena that are triggered by hydraulic effects[1].

[1] A. Atoufi, L. Zhu, A. Lefauve, J. R. Taylor, R. R. Kerswell, S. B. Dalziel, G. Lawrence, P. Linden, *et al.*, "Stratified inclined duct: two-layer hydraulics and instabilities," arXiv preprint arXiv:2301.13035 (2023).