Supplemental Materials for Asymmetry of motion: vortex rings crossing a density gradient

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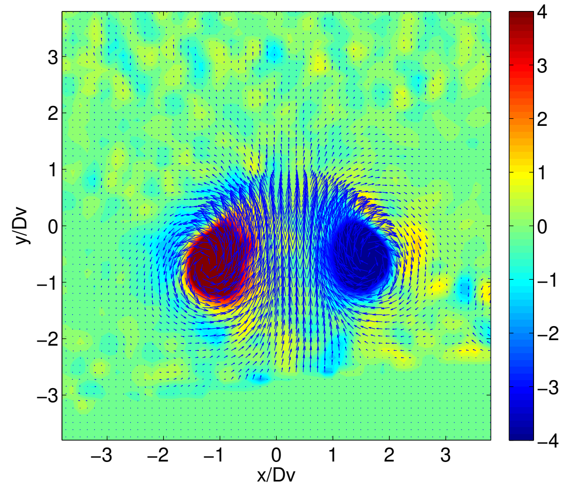
DIRECT MEASUREMENT OF THE BAROCLINIC VORTICITY

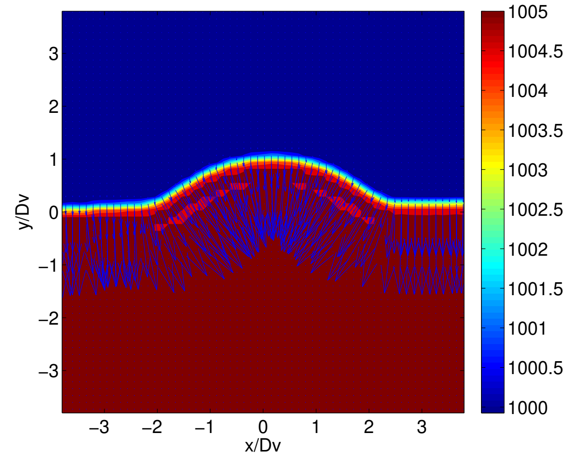
In recent years, there have been several studies attempting to conduct simultaneous measurements of both velocity and density fields using Particle Image Velocimetry (PIV) and Planar Laser-Induced Fluorescence (PLIF) (e.g., [1]). Such measurements imply the use of two cameras (one for PIV, the other for PLIF) and a series of optical filters and splitters to separate the signals for each measurement. Here, in contrast, simultaneous measurements of both velocity and density fields are obtained from the same imagery of fluids containing small tracer particles as well as fluorescent dye. A representative image is presented in Fig. 1(a).

The density gradient can be readily obtained by tracking the interface using the technique of planar laser-induced fluorescence (PLIF) (as in [2]). To track the evolution of the interface between the two fluids, a small amount (20 ppm) of fluorescent dye (Rhodamine 6G, Sigma-Aldrich) was added to the dense fluid. In this manner, it was possible to track the position of the interface in a plane. Note that in this case, the degree of mixing during the initial instants of the interaction of the vortex ring with the interface is very small. Therefore, the location of the interface was used to calculate the density gradient, $\nabla\rho$, which had a zero value everywhere except at the interface. At the interface, the direction and size of the density gradient vector are given by the shape of the interface and the value of the density contrast. Fig. 1(b) shows a typical measurement of the density around the interface with the arrows showing the density gradients. To calculate the pressure field, ∇P, the scheme proposed by [3] was adopted in which PIV velocity fields are used to solve for the pressure gradient fields from the Navier-Stokes equations. The corresponding pressure field of the velocity field in Fig. 1(a) is shown in Fig. 1(c).

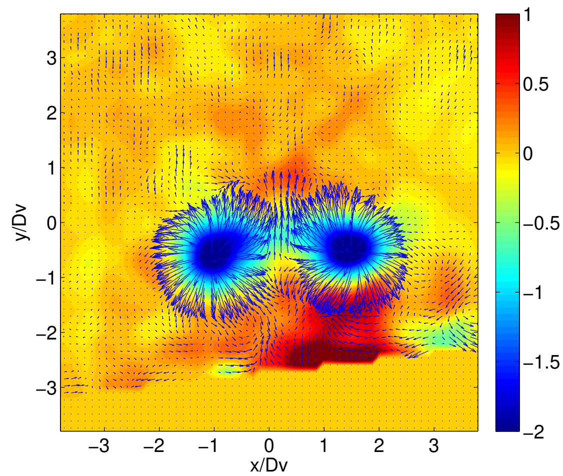
Using the pressure gradient field and density gradient field, according to Marshall &



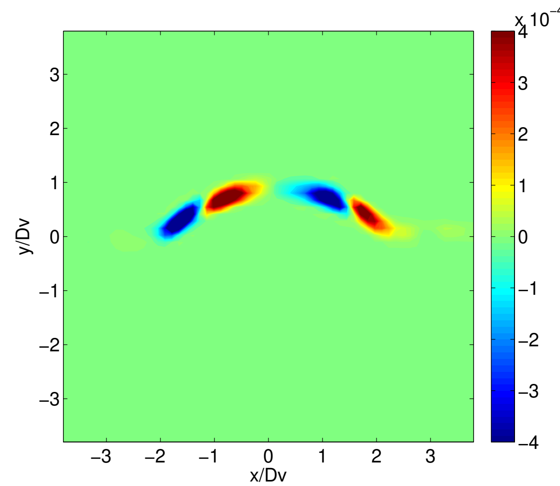
(a) Velocity field by PIV



(b) Density field by PLIF



(c) Pressure field



(d) Baroclinic vorticity

FIG. 1. Typical velocity field (a) and density field (b) of a vortex ring crossing the density interface obtained via PIV and PLIF measurements, respectively. The blue arrows in (b) show the density gradient, and the blue arrows in (c) show the corresponding pressure gradient of the velocity field (a). The associated baroclinic vorticity around the vortex ring is shown in (d), which is calculated from Eqn. 1 using the velocity and density gradients in (b) and (c).

Plumb [4], the baroclinic vorticity is calculated as

$$\frac{\partial \omega_B}{\partial t} = \frac{1}{\rho^2} \nabla \rho \times \nabla P \tag{1}$$

where ω_B is baroclinic vorticity, P is the pressure, and ρ is the density. The baroclinic vorticity field of the example covered in this section is shown in Fig. 1(d). Note how it is generated at the density interface between the two layers of fluids.

[1] Charogiannis, A., An, J. S. & Markides, C. N. 2015 A simultaneous planar laser-induced fluorescence, particle image velocimetry and particle tracking velocimetry technique for the investigation of thin liquid-film flows. *Exp. Them. Fluid Sci.* **68**, 516–536.

[2] Crimaldi, J.P. 2008 Planar laser induced fluorescence in aqueous flows. *Exp. Fluids* **44**, 851–863.

[3] Dabiri, John O, Bose, Sanjeeb, Gemmell, Brad J, Colin, Sean P & Costello, John H 2014 An algorithm to estimate unsteady and quasi-steady pressure fields from velocity field measurements. *J. Exp. Biol.* **217**, 331–336.

[4] Marshall, John & Plumb, R Alan 2016 *Atmosphere, ocean and climate dynamics: an introductory text.* Academic Press.