Added mass and damping of structures with periodic angular shape

by J.N. Newman, Š. Malenica and C. Ouled Housseine

Supplementary material

This file includes more extensive tables of the added-mass coefficients for the floating offshore wind turbine and the configuration of three hemispheroids, to supplement the coefficients in Section 6 of the paper. Tables S1 and S4 include the 18×18 matrices $\mathbf{C}$ defined in Section 4, for the force and moment on each sector or body when they move independently. Tables S2 and S5 include the matrices $\mathbf{a}^{(n)}$ defined in Section 3, for the force and moment acting on each sector or body when they move together as a single rigid body, and the matrices $\boldsymbol{\alpha}^{(n)}$ defined in Section 4, for the force and moment on the entire structure when each sector or body moves independently. $\mathbf{a}^{(n)}$ and $\boldsymbol{\alpha}^{(n)}$ are evaluated from $\mathbf{C}$ using equations (4.1) and (4.2). Tables S3 and S6 show the matrices $\mathbf{A}$ for the entire structure, moving as a rigid body, evaluated from the sum (3.1) of $\mathbf{a}^{(n)}$ and similarly from the sum of $\boldsymbol{\alpha}^{(n)}$. These may be compared with tables 1 and 2 in the paper, that are computed directly.

The forces, moments, and modes of motion are defined with respect to the coordinate system $\mathbf{x}$ with the origin at the center of the structure, in the plane of the free surface, and the z-axis positive upwards. The centre of the first float or spheroid is on the $+x$-axis.

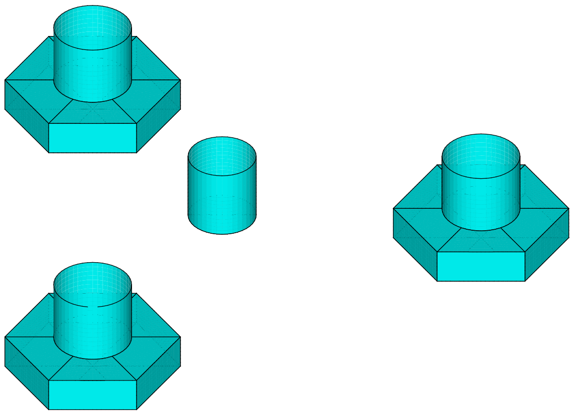

FIGURE S1. Perspective view showing the submerged surfaces of the floating offshore wind-turbine floats.

Floating offshore wind turbine

The outer floats are centered on a circle of radius $R_c = 23.9$m. These floats have a total draft of 9.185m, including upper cylinders of radius 3.6m and depth 5.885m and lower skirts with six rectangular sides 8.1m wide by 3.3m high. The radius of the inner cylinder is 3.15m and the draft is 6.6m. The added mass is nondimensionalized by the displaced mass ρV where ρ is the fluid density and V=2612.12m^3 is the volume; the additional normalizing factors R_c and R_c^2 are used for the cross-coupling coefficients and the moments due to rotation. The calculations are performed with the fluid depth equal to 65m, the frequency ω=1 radian-per-second, and gravity g=9.80665 m^2/sec.

0.2425	0.0000	-0.0135	0.0000	-0.0482	0.0000	0.0100	0.0029	0.0099	0.0092	0.0022	-0.0099	0.0100	-0.0029	0.0099	-0.0092	0.0022	0.0099
0.0000	0.2211	0.0000	0.0569	0.0000	0.2149	-0.0200	0.0002	-0.0140	-0.0122	-0.0015	0.0184	0.0200	0.0002	0.0140	-0.0122	0.0015	0.0184
-0.0135	0.0000	0.4326	0.0000	-0.4243	0.0000	-0.0171	0.0016	0.0123	0.0098	0.0099	0.0132	-0.0171	-0.0016	0.0123	-0.0098	0.0099	-0.0132
0.0000	0.0569	0.0000	0.0210	0.0000	0.0563	-0.0056	-0.0001	-0.0037	-0.0032	-0.0003	0.0049	0.0056	-0.0001	0.0037	-0.0032	0.0003	0.0049
-0.0482	0.0000	-0.4243	0.0000	0.4424	0.0000	0.0143	-0.0022	-0.0134	-0.0119	-0.0102	-0.0103	0.0143	0.0022	-0.0134	0.0119	-0.0102	0.0103
0.0000	0.2149	0.0000	0.0563	0.0000	0.2162	-0.0208	-0.0006	-0.0132	-0.0114	-0.0009	0.0179	0.0208	-0.0006	0.0132	-0.0114	0.0009	0.0179
0.0100	-0.0200	-0.0171	-0.0056	0.0143	-0.0208	0.2264	-0.0093	0.0068	0.0038	-0.0547	-0.1861	-0.0048	0.0114	0.0072	-0.0031	0.0051	-0.0110
0.0029	0.0002	0.0016	-0.0001	-0.0022	-0.0006	-0.0093	0.2371	-0.0117	0.0504	-0.0038	-0.1075	-0.0114	0.0149	0.0156	-0.0093	0.0109	-0.0177
0.0099	-0.0140	0.0123	-0.0037	-0.0134	-0.0132	0.0067	-0.0117	0.4326	0.3675	0.2122	0.0000	0.0072	-0.0156	0.0123	-0.0134	0.0035	0.0132
0.0092	-0.0122	0.0098	-0.0032	-0.0119	-0.0114	0.0038	0.0504	0.3675	0.3370	0.1825	-0.0281	0.0031	-0.0093	0.0134	-0.0137	0.0058	0.0064
0.0022	-0.0014	0.0099	-0.0003	-0.0102	-0.0009	-0.0547	-0.0038	0.2122	0.1825	0.1263	0.0487	0.0051	-0.0109	0.0035	-0.0058	0.0003	0.0094
-0.0099	0.0184	0.0132	0.0049	-0.0103	0.0179	-0.1861	-0.1075	0.0000	-0.0281	0.0487	0.2162	0.0110	-0.0177	-0.0132	0.0064	-0.0094	0.0179
0.0100	0.0200	-0.0171	0.0056	0.0143	0.0208	-0.0048	-0.0114	0.0072	0.0031	0.0051	0.0110	0.2264	0.0092	0.0068	-0.0038	-0.0547	0.1861
-0.0029	0.0002	-0.0016	-0.0001	0.0022	-0.0006	0.0114	0.0149	-0.0156	-0.0093	-0.0109	-0.0177	0.0092	0.2371	0.0117	0.0504	0.0038	-0.1075
0.0099	0.0140	0.0123	0.0037	-0.0134	0.0132	0.0072	0.0156	0.0123	0.0134	0.0035	-0.0132	0.0067	0.0117	0.4326	-0.3675	0.2122	0.0000
-0.0092	-0.0122	-0.0098	-0.0032	0.0119	-0.0114	-0.0031	-0.0093	-0.0134	-0.0137	-0.0058	0.0064	-0.0038	0.0504	-0.3675	0.3370	-0.1825	-0.0281
0.0022	0.0014	0.0099	0.0003	-0.0102	0.0009	0.0051	0.0109	0.0035	0.0058	0.0003	-0.0094	-0.0547	0.0038	0.2122	-0.1825	0.1263	-0.0487
0.0099	0.0184	-0.0132	0.0049	0.0103	0.0179	-0.0110	-0.0177	0.0132	0.0064	0.0094	0.0179	0.1861	-0.1075	0.0000	-0.0281	-0.0487	0.2162

Table S1. Added-mass matrix **C** for the floating offshore wind-turbine floats. For each row $i = (1-18)$ the coefficients C_{ij} are listed on the first line for $j = (1-12)$ and on the second line for $j = (13-18)$.

$\mathbf{a}^{(1)}$						$\boldsymbol{\alpha}^{(1)}$					
0.2625	0.0000	0.0064	0.0000	-0.0438	0.0000	0.2625	0.0000	-0.0477	0.0000	-0.0195	0.0000
0.0000	0.2214	0.0000	0.0325	0.0000	0.2516	0.0000	0.2214	0.0000	0.0568	0.0000	0.2137
-0.0477	0.0000	0.4571	0.0000	-0.4046	0.0000	0.0064	0.0000	0.4571	0.0000	-0.4511	0.0000
0.0000	0.0568	0.0000	0.0147	0.0000	0.0661	0.0000	0.0325	0.0000	0.0147	0.0000	0.0336
-0.0195	0.0000	-0.4511	0.0000	0.4220	0.0000	-0.0438	0.0000	-0.4046	0.0000	0.4220	0.0000
0.0000	0.2137	0.0000	0.0336	0.0000	0.2520	0.0000	0.2516	0.0000	0.0661	0.0000	0.2520

$\mathbf{a}^{(2)}$						$\boldsymbol{\alpha}^{(2)}$					
0.2317	-0.0178	-0.0032	-0.0049	-0.0353	-0.2179	0.2317	-0.0178	0.0239	0.0161	-0.0474	-0.1851
-0.0178	0.2522	0.0055	0.0410	0.0049	-0.1258	-0.0178	0.2522	-0.0413	0.0288	-0.0161	-0.1068
0.0238	-0.0413	0.4571	0.3504	0.2023	0.0000	-0.0032	0.0055	0.4571	0.3906	0.2255	0.0000
0.0161	0.0288	0.3906	0.3202	0.1764	-0.0330	-0.0049	0.0410	0.3504	0.3202	0.1764	-0.0168
-0.0474	-0.0161	0.2255	0.1764	0.1165	0.0573	-0.0353	0.0049	0.2023	0.1764	0.1165	0.0291
-0.1851	-0.1068	0.0000	-0.0168	0.0291	0.2520	-0.2179	-0.1258	0.0000	-0.0331	0.0573	0.2520

$\mathbf{a}^{(3)}$						$\boldsymbol{\alpha}^{(3)}$					
0.2317	0.0178	-0.0032	0.0049	-0.0353	0.2179	0.2317	0.0178	0.0239	-0.0161	-0.0474	0.1851
0.0178	0.2522	-0.0055	0.0410	-0.0049	-0.1258	0.0178	0.2522	0.0413	0.0288	0.0161	-0.1068
0.0238	0.0413	0.4571	-0.3504	0.2023	0.0000	-0.0032	-0.0055	0.4571	-0.3906	0.2255	0.0000
-0.0161	0.0288	-0.3906	0.3202	-0.1764	-0.0331	0.0049	0.0410	-0.3504	0.3202	-0.1764	-0.0168
-0.0474	0.0161	0.2255	-0.1764	0.1165	-0.0573	-0.0353	-0.0049	0.2023	-0.1764	0.1165	-0.0291
0.1851	-0.1068	0.0000	-0.0168	-0.0291	0.2520	0.2179	-0.1258	0.0000	-0.0331	-0.0573	0.2520

TABLE S2. Added-mass matrices $\mathbf{a}^{(n)}$ and $\boldsymbol{\alpha}^{(n)}$ for the floating offshore wind-turbine floats evaluated from the sums of coefficients in table S1.

$\mathbf{a}^{(1)}+\mathbf{a}^{(2)}+\mathbf{a}^{(3)}$						$\boldsymbol{\alpha}^{(1)}+\boldsymbol{\alpha}^{(2)}+\boldsymbol{\alpha}^{(3)}$					
0.7258	0.0000	0.0000	0.0000	-0.1144	0.0000	0.7258	0.0000	0.0000	0.0000	-0.1144	0.0000
0.0000	0.7258	0.0000	0.1144	0.0000	0.0000	0.0000	0.7258	0.0000	0.1144	0.0000	0.0000
0.0000	0.0000	1.3713	0.0000	0.0000	0.0000	0.0000	0.0000	1.3713	0.0000	0.0000	0.0000
0.0000	0.1144	0.0000	0.6550	0.0000	0.0000	0.0000	0.1144	0.0000	0.6550	0.0000	0.0000
-0.1144	0.0000	0.0000	0.0000	0.6550	0.0000	-0.1144	0.0000	0.0000	0.0000	0.6550	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.7559	0.0000	0.0000	0.0000	0.0000	0.0000	0.7559

TABLE S3. Added-mass matrix $\mathbf{A}$ for the floating offshore wind-turbine floats evaluated from the sums of coefficients in table S2.

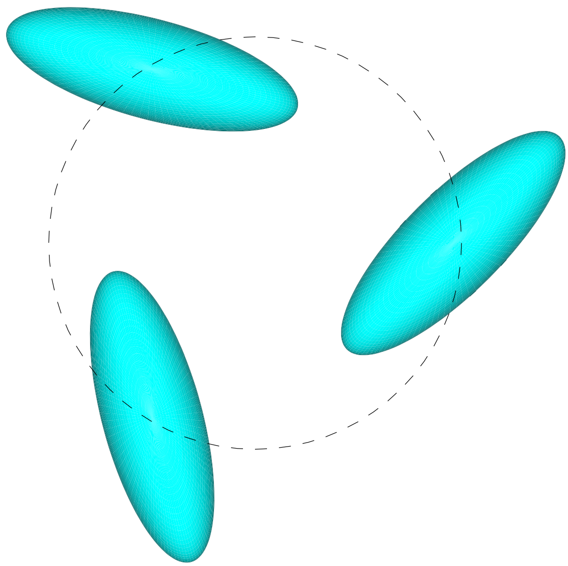

FIGURE S2. Aerial view of the configuration of three hemispheroids.

Configuration of three hemispheroids

The hemispheroids are prolate, with length 3m and maximum radius 0.5m. The centers are on a circle of radius $R_c = 2$m, shown by the dashed line in figure S2. The major axis of each hemispheroid is in the plane of the free surface, oriented at 45° from the tangent to the circle. The coefficients are normalized in the same manner as described for the floating offshore wind turbine, using the volume V =2.35619m^3. The fluid depth is infinite and the wavenumber $K = \omega^2/g = 1\text{m}^{-1}$.

0.1931	-0.1561	-0.0362	-0.0135	0.0497	-0.1556	-0.0029	-0.0183	-0.0823	-0.0709	-0.0398	0.0157
						0.0000	0.0166	-0.0616	0.0430	-0.0336	-0.0064
-0.1561	0.2082	0.0119	-0.0187	0.0068	0.2081	-0.0035	-0.0115	0.0522	0.0451	0.0256	0.0048
						0.0018	-0.0145	0.0123	-0.0012	0.0087	0.0091
-0.0362	0.0119	0.2568	0.0083	-0.2651	0.0099	0.0202	-0.0595	-0.1100	-0.0890	-0.0316	0.0133
						0.0864	0.0452	-0.1100	0.0814	-0.0587	0.0521
-0.0135	-0.0187	0.0083	0.0139	-0.0222	-0.0188	0.0022	0.0100	0.0101	0.0087	0.0048	-0.0070
						-0.0008	-0.0007	0.0171	-0.0145	0.0086	-0.0012
0.0497	0.0068	-0.2651	-0.0222	0.2872	0.0089	-0.0224	0.0494	0.0999	0.0803	0.0268	-0.0063
						-0.0855	-0.0445	0.0929	-0.0669	0.0501	-0.0509
-0.1556	0.2081	0.0099	-0.0188	0.0089	0.2336	-0.0047	-0.0101	0.0521	0.0447	0.0244	0.0063
						-0.0037	-0.0160	0.0133	-0.0019	0.0092	0.0063
-0.0029	-0.0035	0.0202	0.0022	-0.0224	-0.0047	0.0693	0.0846	0.0078	0.0151	0.0353	-0.1025
						-0.0188	-0.0057	-0.0040	0.0037	-0.0019	-0.0120
-0.0183	-0.0115	-0.0595	0.0100	0.0494	-0.0101	0.0846	0.3320	-0.0373	-0.0332	-0.0219	-0.2388
						0.0091	0.0043	-0.0974	0.0829	-0.0491	0.0112
-0.0823	0.0522	-0.1100	0.0101	0.0999	0.0521	0.0078	-0.0373	0.2568	0.2254	0.1397	0.0099
						0.0414	0.0472	-0.1100	0.0719	-0.0613	0.0133
-0.0709	0.0451	-0.0890	0.0087	0.0803	0.0447	0.0151	-0.0332	0.2254	0.1997	0.1295	0.0017
						0.0323	0.0397	-0.0915	0.0592	-0.0512	0.0089
-0.0398	0.0256	-0.0316	0.0048	0.0268	0.0244	0.0353	-0.0219	0.1397	0.1295	0.1014	-0.0207
						0.0073	0.0193	-0.0412	0.0243	-0.0236	-0.0029
0.0157	0.0048	0.0133	-0.0070	-0.0063	0.0063	-0.1025	-0.2388	0.0099	0.0017	-0.0207	0.2336
						0.0111	0.0010	0.0521	-0.0435	0.0265	0.0063
0.0000	0.0018	0.0864	-0.0008	-0.0855	-0.0037	-0.0188	0.0091	0.0414	0.0323	0.0073	0.0111
						0.3396	0.0715	0.0284	-0.0117	0.0177	0.2581
0.0166	-0.0145	0.0452	-0.0007	-0.0445	-0.0160	-0.0057	0.0043	0.0472	0.0397	0.0193	0.0010
						0.0715	0.0618	0.0254	-0.0508	0.0050	0.0307
-0.0616	0.0123	-0.1100	0.0171	0.0929	0.0133	-0.0040	-0.0974	-0.1100	-0.0915	-0.0412	0.0521
						0.0284	0.0254	0.2568	-0.2337	0.1253	0.0099
0.0430	-0.0012	0.0814	-0.0145	-0.0669	-0.0019	0.0037	0.0829	0.0719	0.0592	0.0243	-0.0435
						-0.0117	-0.0508	-0.2337	0.2381	-0.1073	0.0171
-0.0336	0.0087	-0.0587	0.0086	0.0501	0.0092	-0.0019	-0.0491	-0.0613	-0.0512	-0.0236	0.0265
						0.0177	0.0050	0.1253	-0.1073	0.0630	0.0119
-0.0064	0.0091	0.0521	-0.0012	-0.0509	0.0063	-0.0120	0.0112	0.0133	0.0089	-0.0029	0.0063
						0.2581	0.0307	0.0099	0.0171	0.0119	0.2336

TABLE S4. Added-mass matrix **C** for the three hemispheroids. For each row $i = (1-18)$ the coefficients C_{ij} are listed on the first line for $j = (1-12)$ and on the second line for $j = (13-18)$.

$\mathbf{a}^{(1)}$

0.1902	-0.1578	-0.1801	-0.0415	-0.0236	-0.1462
-0.1578	0.1822	0.0764	0.0252	0.0410	0.2221
0.0703	-0.0023	0.0368	0.0007	-0.3554	0.0753
-0.0122	-0.0094	0.0355	0.0080	-0.0088	-0.0270
-0.0581	0.0117	-0.0723	-0.0088	0.3642	-0.0483
-0.1640	0.1820	0.0753	0.0239	0.0425	0.2461

$\boldsymbol{\alpha}^{(1)}$

0.1902	-0.1578	0.0703	-0.0122	-0.0581	-0.1640
-0.1578	0.1822	-0.0023	-0.0094	0.0117	0.1820
-0.1801	0.0764	0.0368	0.0355	-0.0723	0.0753
-0.0415	0.0252	0.0007	0.0080	-0.0087	0.0239
-0.0236	0.0410	-0.3554	-0.0088	0.3642	0.0425
-0.1462	0.2221	0.0753	-0.0270	-0.0483	0.2461

$\mathbf{a}^{(2)}$

0.0476	0.0754	0.0239	0.0211	0.0109	-0.1192
0.0754	0.3249	-0.1942	0.0597	-0.0216	-0.2377
-0.0331	0.0621	0.0368	0.3074	0.1783	0.0753
-0.0235	0.0516	0.0449	0.2676	0.1586	0.0553
0.0028	0.0230	0.0669	0.1586	0.1046	0.0008
-0.0757	-0.2330	0.0753	-0.0488	-0.0006	0.2461

$\boldsymbol{\alpha}^{(2)}$

0.0476	0.0754	-0.0331	-0.0235	0.0028	-0.0757
0.0754	0.3249	0.0621	0.0516	0.0230	-0.2330
0.0239	-0.1942	0.0368	0.0449	0.0669	0.0753
0.0211	0.0597	0.3074	0.2676	0.1586	-0.0488
0.0109	-0.0216	0.1783	0.1586	0.1046	-0.0006
-0.1192	-0.2377	0.0753	0.0553	0.0008	0.2461

$\mathbf{a}^{(3)}$

0.3209	0.0823	0.1562	0.0197	-0.0605	0.2654
0.0823	0.0516	0.1178	-0.0117	-0.0202	0.0156
-0.0372	-0.0597	0.0368	-0.3082	0.1771	0.0753
0.0349	0.0309	-0.0804	0.2827	-0.1498	-0.0283
-0.0179	-0.0354	0.0054	-0.1498	0.0895	0.0475
0.2396	0.0510	0.0753	0.0249	-0.0420	0.2461

$\boldsymbol{\alpha}^{(3)}$

0.3209	0.0823	-0.0372	0.0349	-0.0179	0.2396
0.0823	0.0516	-0.0597	0.0309	-0.0354	0.0510
0.1562	0.1178	0.0368	-0.0804	0.0054	0.0753
0.0197	-0.0117	-0.3082	0.2827	-0.1498	0.0249
-0.0605	-0.0202	0.1771	-0.1498	0.0895	-0.0420
0.2654	0.0156	0.0753	-0.0283	0.0475	0.2461

TABLE S5. Added-mass matrices $\mathbf{a}^{(n)}$ and $\boldsymbol{\alpha}^{(n)}$ for the three hemispheroids evaluated from the sums of coefficients in table S4.

$\mathbf{a}^{(1)} + \mathbf{a}^{(2)} + \mathbf{a}^{(3)}$

0.5587	0.0000	0.0000	-0.0008	-0.0732	0.0000
0.0000	0.5587	0.0000	0.0732	-0.0008	0.0000
0.0000	0.0000	0.1104	0.0000	0.0000	0.2259
-0.0008	0.0732	0.0000	0.5583	0.0000	0.0000
-0.0732	-0.0008	0.0000	0.0000	0.5583	0.0000
0.0000	0.0000	0.2259	0.0000	0.0000	0.7384

$\boldsymbol{\alpha}^{(1)} + \boldsymbol{\alpha}^{(2)} + \boldsymbol{\alpha}^{(3)}$

0.5587	0.0000	0.0000	-0.0008	-0.0732	0.0000
0.0000	0.5587	0.0000	0.0732	-0.0008	0.0000
0.0000	0.0000	0.1104	0.0000	0.0000	0.2259
-0.0008	0.0732	0.0000	0.5583	0.0000	0.0000
-0.0732	-0.0008	0.0000	0.0000	0.5583	0.0000
0.0000	0.0000	0.2259	0.0000	0.0000	0.7384

TABLE S6. Added-mass matrix $\mathbf{A}$ for the three hemispheroids evaluated from the sums of coefficients in table S5.