

## Supplementary material

One wonders whether for strong shear Lumley-type scaling (Lumley, 1967; Lohse, 1994; Biferale & Procaccia, 2005)  $E_u(k) \sim k^{-7/3}$  and  $E_\theta(k) \sim k^{-4/3}$  for the velocity and temperature spectrum may show up in our spectra. However, those theoretical predictions were made for *homogeneous* turbulent shear flow. Given the plates, the detachment of plumes from them, and the relatively low Reynolds numbers we can numerically treat and the correspondingly short inertial range, we do not expect to see pronounced Lumley-type shear in our data. This is confirmed from figures 1 and 2.

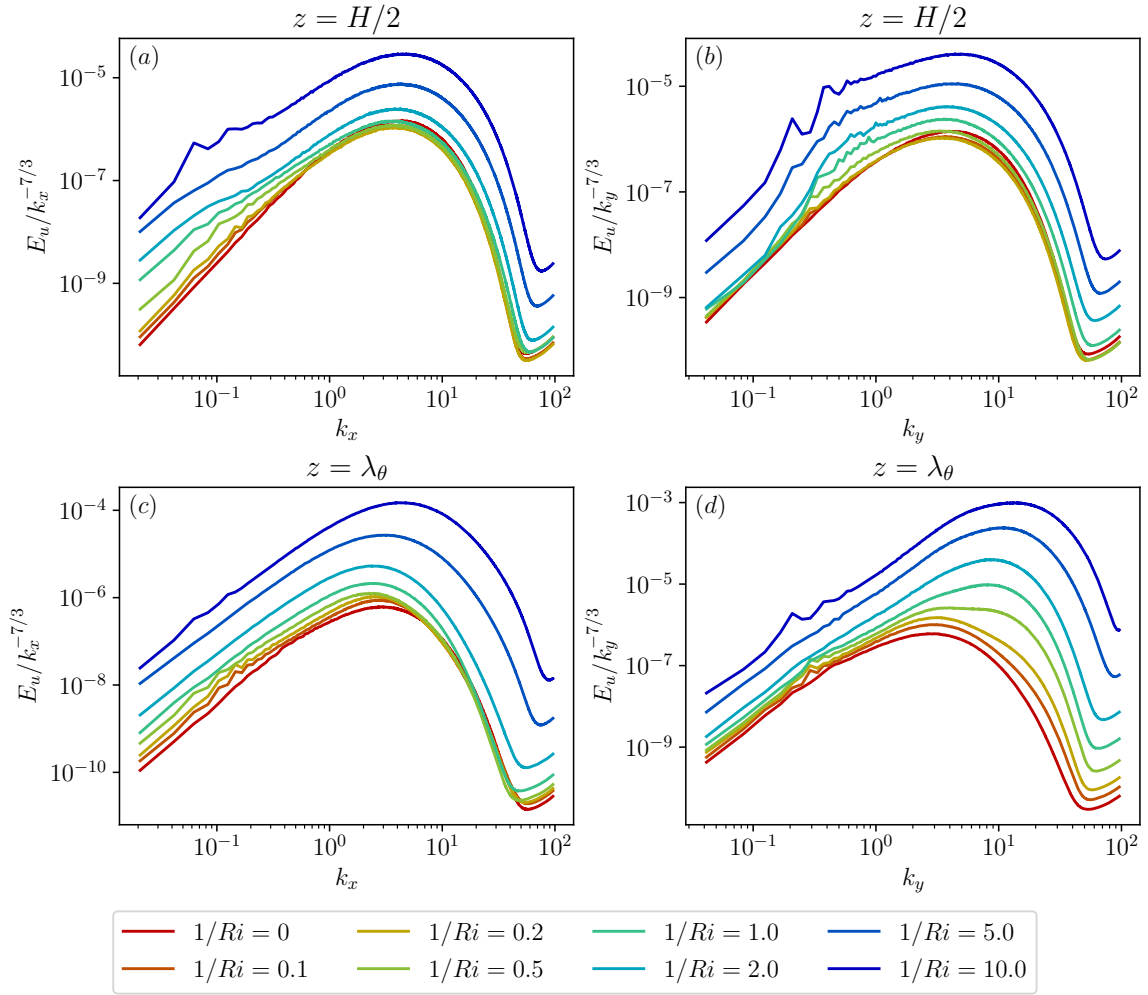


Figure 1: 1D velocity spectra in (a) streamwise direction, and (b) spanwise direction at mid-height, and (c) streamwise direction, and (d) spanwise direction at the thermal boundary layer height.

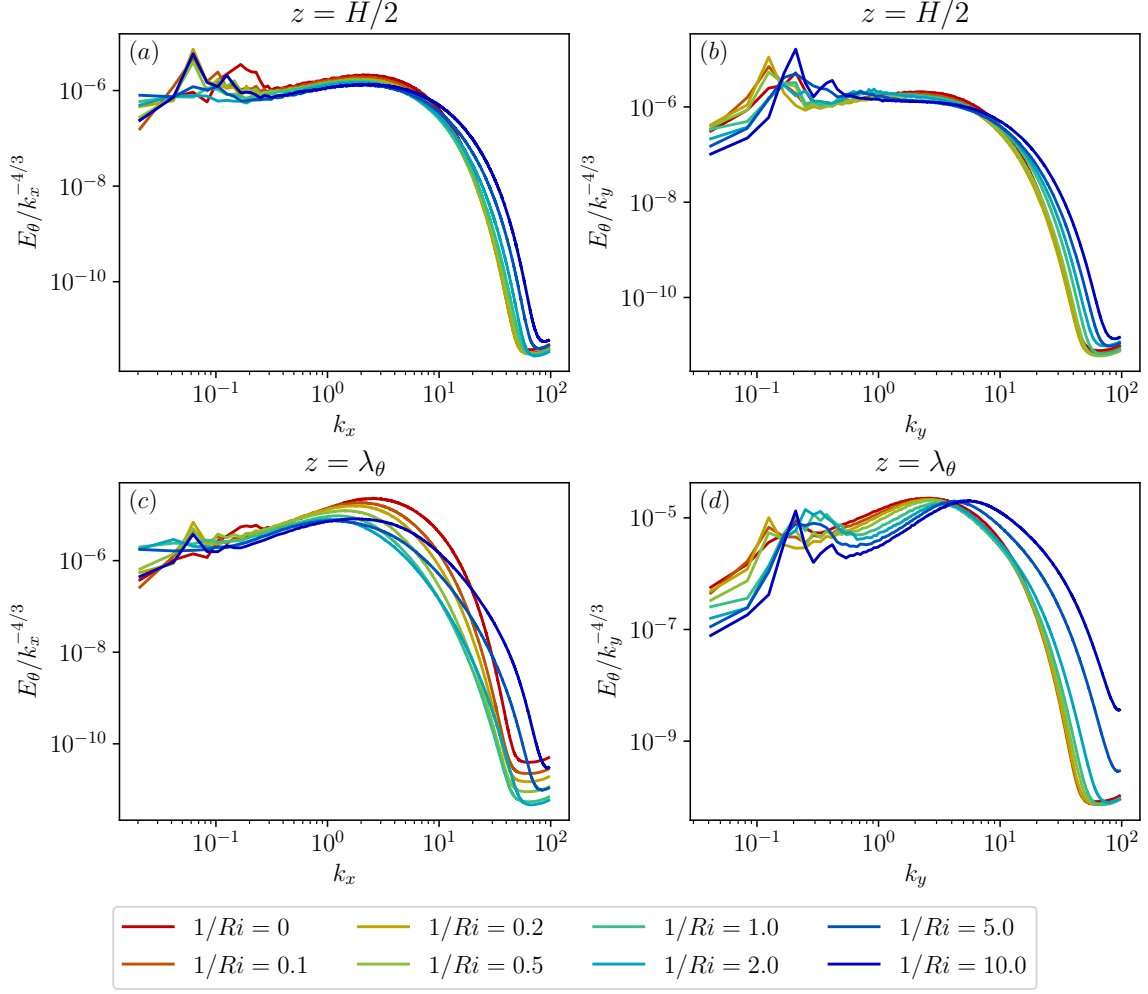


Figure 2: 1D temperature spectra in (a) streamwise direction, and (b) spanwise direction at mid-height, and (c) streamwise direction, and (d) spanwise direction at the thermal boundary layer height.

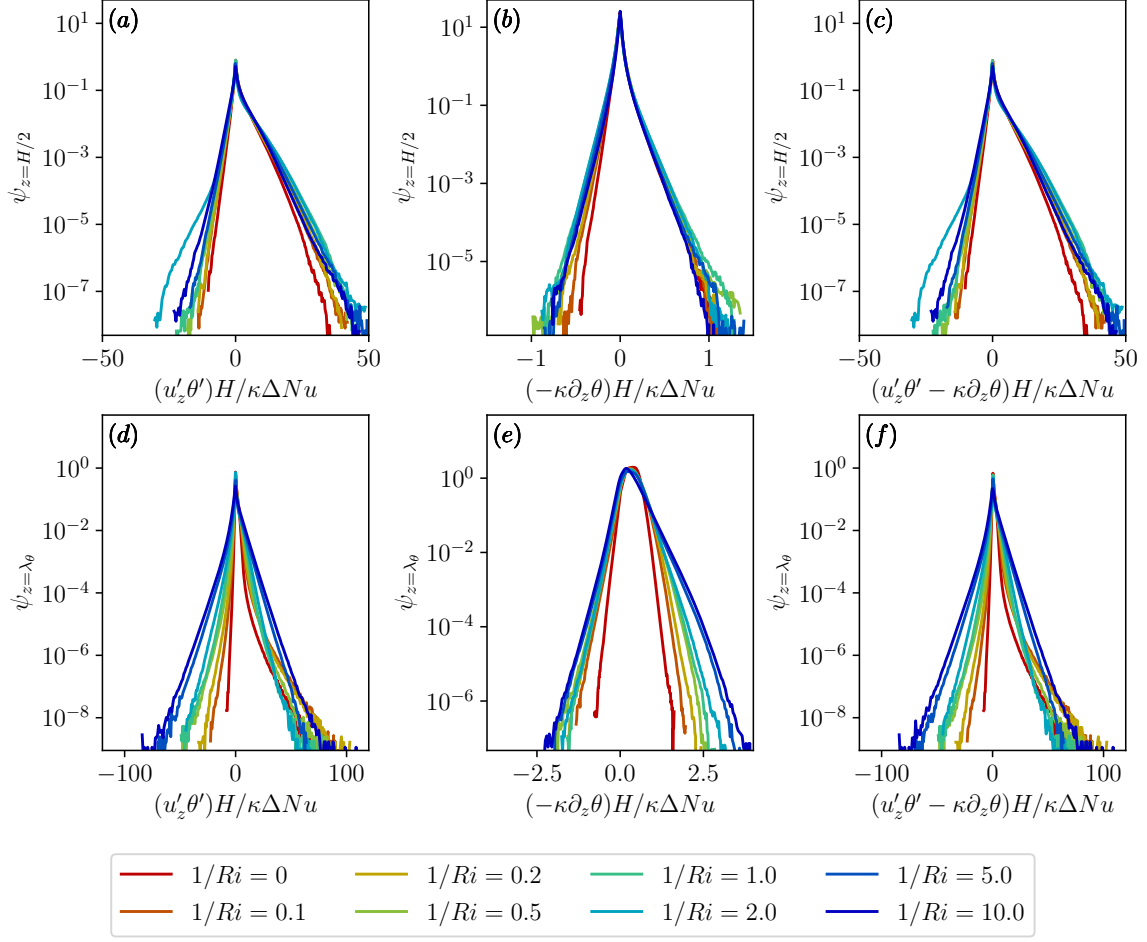


Figure 3: Probability distribution of (a) convective heat flux, (b) diffusive heat flux, and (c) total heat flux normalized with  $Nu$  at mid-height along with the probability distribution of (d) convective heat flux, (e) diffusive heat flux, and (f) total heat flux normalized with  $Nu$  at the thermal boundary layer height.

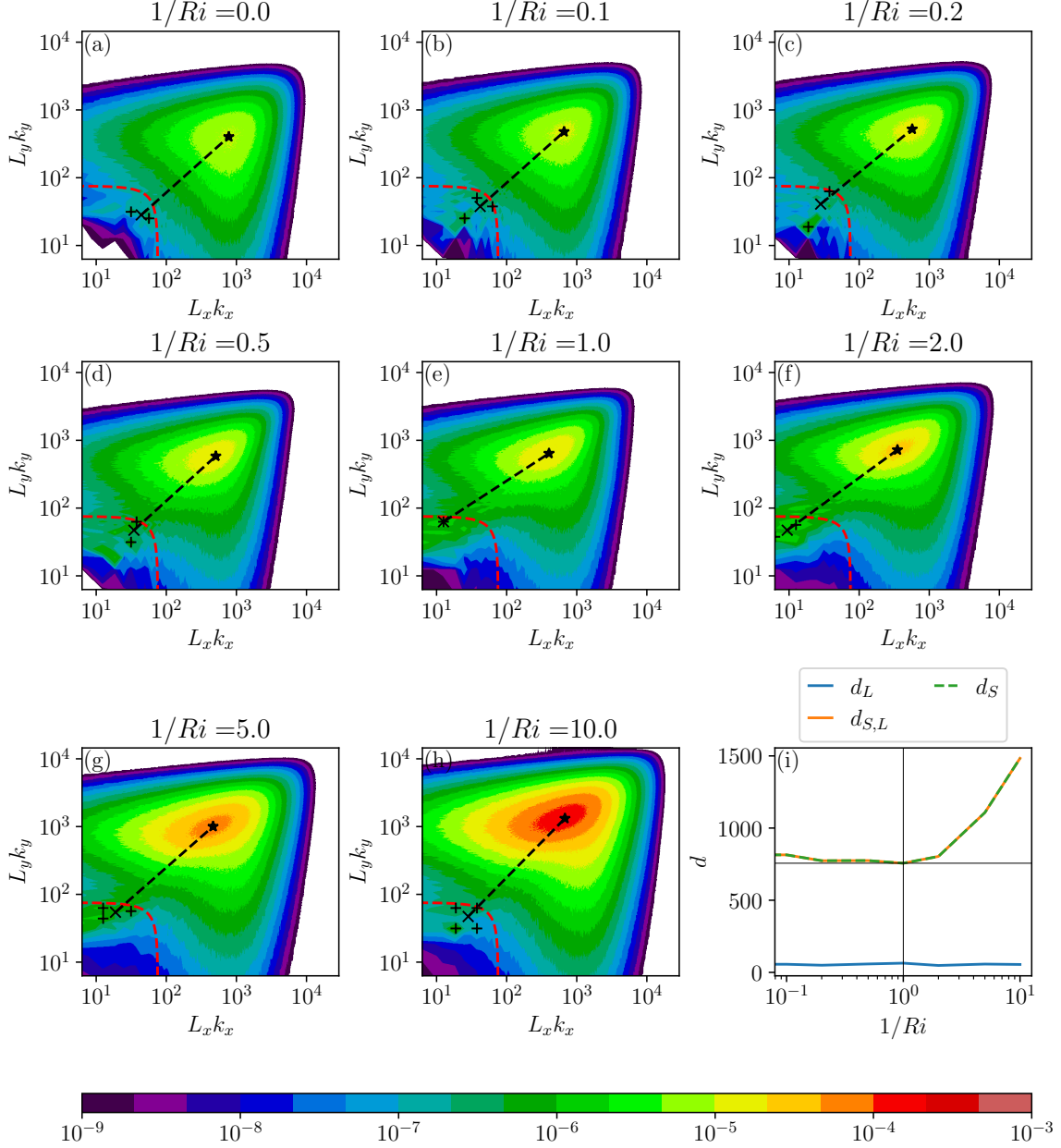


Figure 4: (a)-(h) 2D spectra of convective flux at the thermal boundary layer height. The plus markers indicate primary large-scale peaks detected within  $d = \sqrt{(L_x k_x)^2 + (L_y k_y)^2} < \pi L_y$  (shown with the red dashed line), the centroid of these peaks, indicated with cross marker, is considered to be the location of the primary peak associated with the large scale structures. The star marker indicates the secondary peak associated with small scale structures. The distance between the small and the large scale peaks are shown with black dashed line. (i) The radial distance of the small scale peaks  $d_S = \sqrt{(L_x k_{x,S})^2 + (L_y k_{y,S})^2}$ , the radial distance of the large scale peaks  $d_L = \sqrt{(L_x k_{x,L})^2 + (L_y k_{y,L})^2}$  and the distance between the small scale and large scale peaks  $d_{S,L} = \sqrt{(L_x k_{x,S} - L_x k_{x,L})^2 + (L_y k_{y,S} - L_y k_{y,L})^2}$  is plotted against  $1/Ri$ .

## References

- BIFERALE, L. & PROCACCIA, I. 2005 Anisotropy in turbulent flows and in turbulent transport. *Phys. Reports* **414**, 43–164.
- LOHSE, D. 1994 Temperature spectra in shear flow and thermal convection. *Phys. Lett. A* **196**, 70.
- LUMLEY, J. L. 1967 Similarity and turbulent energy spectrum. *Phys. Fluids* **10**, 855.