The Origin of Enhanced Skin Friction in Transitional Boundary Layer

Supplementary Material

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This document includes a table and movie about the 48 analyzed events that lend additional support to the discussions in the main text.

1. Coordinates of events and contribution from different terms

Based on the criteria described in the main text, we extracted 48 events in the transitional region that represent enhanced instantaneous wall stress. The time, location and vorticity vector of all the events are summarized in table 1. Detailed analysis of event 43 was presented in §3 of the main text. For each event, 10^5 Lagrangian particles were integrated in backward time until $\delta s = -20$. The relative contribution of different terms in the stochastic Cauchy invariant at $\delta s = -20$ are also summarized in table 1.

event	t	x	z	ω_x	ω_z	$\frac{\widetilde{L}_z}{\omega_z(t)}$	$\frac{\widetilde{D}_{zx}\omega_x}{\omega_z(t)}$	$\frac{\widetilde{D}_{zy}\omega_y}{\omega_z(t)}$	$\frac{(\widetilde{D}_{zz}\omega_z)_{\rm nw}}{\omega_z(t)}$	$\frac{(\widetilde{D}_{zz}\omega_z)_{\rm out}}{\omega_z(t)}$
01	106.50	165.6	109.0	0.14	-2.82	0.40	0.00	-0.01	0.46	0.15
02	124.75	213.5	231.5	-0.88	-2.80	-0.03	0.01	0.06	1.00	-0.05
03	129.50	210.3	124.7	0.19	-2.81	0.02	0.01	-0.10	1.00	0.07
04	186.25	199.2	199.1	-0.14	-2.81	0.02	0.00	0.01	0.98	0.00
05	198.50	310.7	17.1	-0.01	-2.82	-0.02	0.00	-0.08	0.80	0.29
06	207.25	343.8	142.4	2.65	-2.83	-0.23	0.04	0.00	1.18	0.00
07	215.25	265.2	34.4	0.36	-2.82	-0.04	0.01	0.03	0.78	0.21
08	225.00	282.1	37.9	-0.46	-2.81	0.08	0.00	-0.16	1.00	0.07
09	265.00	252.6	54.4	-0.44	-2.81	-0.04	0.00	0.01	0.94	0.09
10	266.00	293.2	160.6	0.27	-2.83	-0.05	0.01	0.01	0.90	0.14
11	275.50	280.1	48.3	-1.05	-2.80	0.04	0.01	-0.04	0.96	0.03
12	377.50	267.8	18.7	0.73	-2.82	-0.14	0.01	-0.01	1.12	0.02
13	393.00	266.6	145.6	-0.64	-2.80	0.02	0.00	-0.03	0.80	0.20
14	412.75	254.1	130.7	-0.54	-2.84	0.27	0.00	-0.01	0.57	0.17
15	431.00	205.6	178.4	-0.62	-2.81	-0.16	0.01	-0.03	0.77	0.41
16	455.50	288.0	79.1	-0.57	-2.80	0.15	0.00	-0.10	0.56	0.38
17	470.75	169.4	25.9	0.41	-2.84	-0.15	0.01	0.18	0.97	0.00
18	478.25	116.5	68.0	-0.20	-2.80	0.02	0.00	-0.01	0.84	0.15
19	495.25	287.4	38.0	-0.02	-2.80	0.10	0.00	-0.12	0.85	0.16
20	519.75	295.0	61.7	-0.07	-2.82	0.00	0.00	-0.14	0.71	0.42

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event	t	x	z	ω_x	ω_z	$\frac{\widetilde{L}_z}{\omega_z(t)}$	$\frac{\widetilde{D}_{zx}\omega_x}{\omega_z(t)}$	$\frac{\widetilde{D}_{zy}\omega_y}{\omega_z(t)}$	$\frac{(\widetilde{D}_{zz}\omega_z)_{\rm nw}}{\omega_z(t)}$	$\frac{(\widetilde{D}_{zz}\omega_z)_{\rm out}}{\omega_z(t)}$
21	531.25	332.7	217.2	-0.44	-2.86	-0.09	0.00	0.02	0.69	0.37
22	550.25	270.1	8.4	0.99	-2.86	-0.09	0.01	-0.02	0.87	0.22
23	553.00	247.9	215.7	-0.50	-2.80	0.04	0.01	-0.03	0.96	0.02
24	559.50	233.0	234.9	-0.50	-2.80	-0.04	0.01	-0.02	1.04	0.01
25	560.75	309.6	99.1	-0.54	-2.81	-0.06	0.03	-0.13	1.03	0.13
26	595.75	166.4	16.2	1.94	-2.81	-0.33	0.01	0.15	0.96	0.21
27	607.50	276.6	79.4	0.32	-2.81	-0.06	0.01	0.00	1.02	0.04
28	642.00	321.0	109.0	0.50	-2.80	-0.04	0.02	-0.09	1.02	0.09
29	654.25	253.5	187.9	-0.51	-2.80	0.01	0.00	0.01	0.76	0.22
30	655.25	297.9	119.6	0.78	-2.83	-0.13	0.01	0.01	0.91	0.20
31	662.75	345.2	110.2	0.77	-2.87	0.03	0.00	-0.13	0.75	0.34
32	664.75	307.2	119.5	0.38	-2.80	0.01	0.01	-0.01	0.93	0.06
33	678.50	305.2	148.5	0.28	-2.80	-0.04	0.00	0.05	0.94	0.04
34	678.25	226.3	119.8	0.43	-2.83	0.04	0.02	-0.12	1.02	0.05
35	694.00	199.2	81.4	-0.63	-2.80	0.03	0.00	-0.02	0.83	0.15
36	733.25	253.5	174.9	1.24	-2.81	0.04	0.05	-0.09	0.96	0.04
37	768.00	299.9	205.1	0.43	-2.81	-0.07	0.04	-0.07	0.97	0.13
38	783.75	234.5	165.2	-1.83	-2.80	-0.06	0.00	0.01	0.82	0.22
39	794.75	147.2	14.9	0.68	-2.83	-0.08	0.04	-0.03	1.06	0.01
40	864.00	228.9	234.1	2.04	-2.85	-0.22	0.01	0.03	0.96	0.22
41	880.25	227.2	208.5	-0.43	-2.82	0.06	0.01	0.02	0.77	0.14
42	909.00	197.7	58.9	0.38	-2.81	0.06	0.01	-0.07	0.97	0.03
43	916.25	203.5	219.6	0.71	-2.81	-0.20	0.02	-0.03	1.13	0.08
44	927.00	298.2	194.8	1.48	-2.81	-0.26	0.05	0.20	0.77	0.25
45	950.75	331.2	109.1	0.06	-2.80	-0.07	0.00	0.09	0.67	0.30
46	961.00	253.8	135	0.75	-2.81	-0.06	0.00	0.11	0.8	0.15
47	960.50	351.3	219.6	-0.30	-2.80	-0.03	0.00	-0.04	0.91	0.16
48	965.00	200.6	230.1	0.78	-2.80	0.07	0.020	0.11	0.85	-0.05

Table 1: (Left of the thick line) Time, location and vorticity of the analyzed event; (right of the thick line) contribution of different terms in the stochastic Cauchy invariant at $\delta s = -20$, and the full expressions are: $\mathbb{E}[\widetilde{L}_z(s)]/\omega_z(t)$, $\mathbb{E}[\widetilde{D}_{zx}\omega_x(s)]/\omega_z(t)$, $\mathbb{E}[\widetilde{D}_{zy}\omega_y(s)]/\omega_z(t)$ $f_{\rm nw}\mathbb{E}_{\rm nw}[\widetilde{D}_{zz}\omega_z(s)]/\omega_z(t)$, $f_{\rm out}\mathbb{E}_{\rm out}[\widetilde{D}_{zz}\omega_z(s)]/\omega_z(t)$.

2. Classification of near-wall and outer particles

The section includes a supplemental movie that depicts particle locations and classification results for each event at $\delta s = -20$. A sample snapshot for the event analyzed in §3 of the main text is shown in figure 1. Since the flow is statistically homogeneous in spanwise direction, we project all the 10^5 particles onto one x-y plane. The particle density peaks in the near-wall and outer layers, and the k-means clustering algorithm clearly captures two clouds of particles. These results hold in most of the events, as visualized in the movie. For a few events (e.g. event 08 and 45), the particles have not separated into two clusters, probably due to a relatively weaker upward velocity in reverse time. These results support and complement our conclusions in the main paper that the particles can be classified into near-wall and outer clusters for most events.

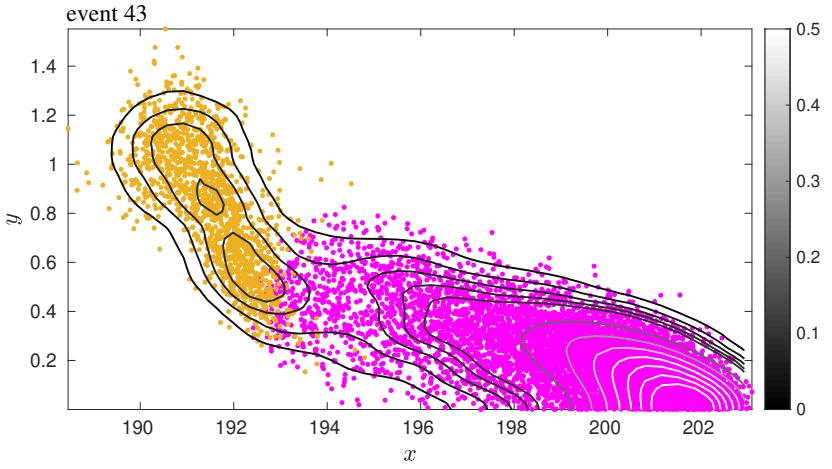

Figure 1: Particle locations projected onto a x-y plane at $\delta s = -20$ for event 43. Magenta and yellow colors correspond to near-wall and outer clusters. Gray-scale contours show the probability density function (PDF) of particle locations.