Supplementary material for Asymptotics of stream-wise Reynolds stress in wall turbulence

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1 Alternative asymptotic sequences for the inner expansion of $\langle uu\rangle^+$

1.1 First alternative sequence $[1, 1/\ln(\mathrm{Re}_\tau), ...]$

The first alternative asymptotic expansion of the near-wall stream-wise normal stress,

$$\langle uu\rangle^+_{\mathrm{DNS}} = \tilde{f}(y^+) + \tilde{g}(y^+)/\ln(\mathrm{Re}_\tau) \tag{1}$$

corresponds essentially to the one proposed by Monkewitz & Nagib (2015). Using the DNS profiles in table 1 of the main paper to determine the two terms of the asymptotic expansion (1) results in figure 1 of this supplement. The figure shows, that the different DNS pairs yield consistent results for $\tilde{f}(y^+)$ and $\tilde{g}(y^+)$ between the wall and $y^+ \approx 200$, similar to figure 1 of the main paper.

The expansion (1) also yields an inner peak height of

$$\left[\tilde{f} + \tilde{g}/\ln(\mathrm{Re}_\tau)\right]_{\mathrm{IP}} = 14.0 - 40.9/\ln(\mathrm{Re}_\tau) \tag{2}$$

which is in excellent agreement with figure 2 of the main paper. However, the resulting coefficient of $(y^+)^2$ in the Taylor expansion of $\langle uu\rangle^+$ about $y^+ = 0$ is $(0.30 - 0.86/\ln \mathrm{Re}_\tau)$, i.e. significantly overshoots the maximum of $1/4$ posited by Chen & Sreenivasan (2021) for $\mathrm{Re}_\tau \to \infty$. Therefore, the inner expansion (1) is not further pursued here.

1.2 Second alternative sequence $[\ln(\mathrm{Re}_\tau), 1, ...]$

It is instructive to test the asymptotic sequence $[\ln(\mathrm{Re}_\tau), 1, ...]$ of the attached eddy model (Marusic & Monty, 2019) with the same methodology, i.e. decompose the channel DNS profiles of table 1 in the main paper according to

$$\langle uu\rangle^+_{\mathrm{DNS}} = \tilde{\tilde{f}}(y^+) + \tilde{\tilde{g}}(y^+)\ \ln(\mathrm{Re}_\tau) \tag{3}$$

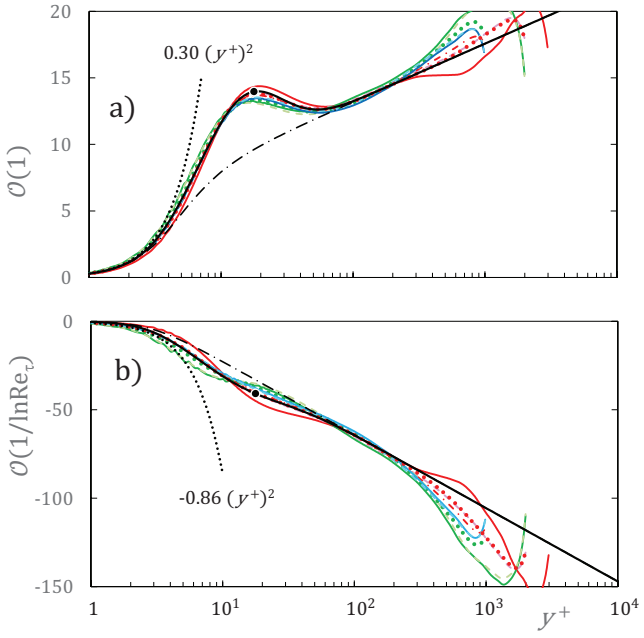

Figure 1: The $\mathcal{O}(1)$ (panel a) and $\mathcal{O}(1/\ln(\mathrm{Re}_\tau)$ (panel b) components of $\langle uu \rangle^+_{\mathrm{wall}}$, extracted from the same channel DNS pairs of table 1 in the main paper, using the same color scheme as in its figure 1: (red) profiles 1 and 2 (—), 1 and 3 (- - -), 1 and 4 ($\cdots$), 1 and 5 ($-\cdot-$); (green) 2 and 3 (—), 2 and 4 (- - -), 2 and 5 ($\cdots$); (blue) 3 and 5 (—), 4 and 5 (- - -). •, inner peak (equation 2 of this supplement); — and $-\cdot-$, fits similar to equation (2.2) of main paper.

The result is shown in figure 2. While the resulting inner peak at $y^+ \approxeq 15$ is quite well fitted by

$$\left[\tilde{\tilde{f}} + \tilde{\tilde{g}} \, \ln(\mathrm{Re}_\tau)\right]_{\mathrm{IP}} = 3.54 + 0.646 \, \ln(\mathrm{Re}_\tau) \quad , \tag{4}$$

given as equation (3.2) in Samie *et al.* (2018), the $\tilde{\tilde{f}}$'s and $\tilde{\tilde{g}}$'s from different profile pairs are seen to spread out, indicating that the asymptotic sequence $[\ln(\mathrm{Re}_\tau), 1, ...]$ is not appropriate (see also the formal argument against this scaling in the main text).

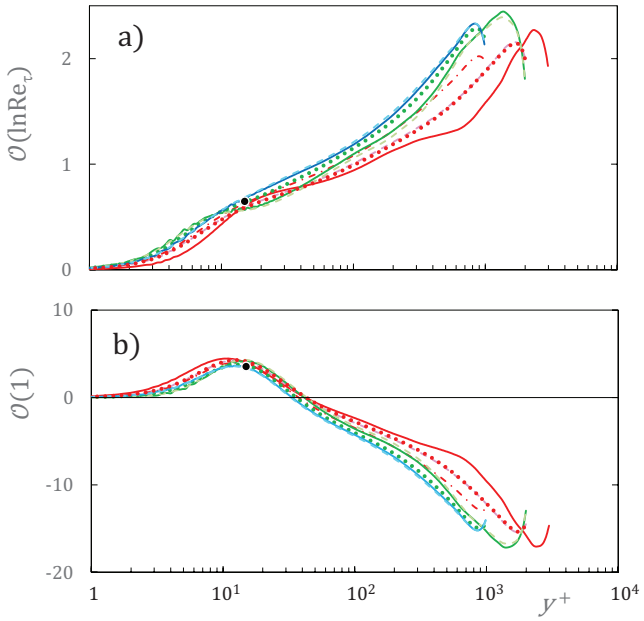

Figure 2: The $\mathcal{O}(\ln(\mathrm{Re}_\tau))$ (panel a) and $\mathcal{O}(1)$ (panel b) components of $\langle uu \rangle^+$, extracted from the same channel DNS pairs of table 1 in the main paper, using the same color scheme as in its figure 1. •, inner peak of Samie *et al.* (2018) (equation (4) of this supplement).

2 Comparison of the present MAE with the fit of Marusic and Kunkel for $\langle uu \rangle^+$

The composite MAE profiles of figure 3 of the main paper are compared to the fits of Marusic & Kunkel (2003), inspired by the attached eddy model (see also Monkewitz *et al.*, 2017). Note, that the inner and outer parts of these fits are **not matched across an overlap region**, but patched across the interval $30 \leq y^+ \leq 150$ with a cubic in $\ln(y^+)$.

All the parameters are as in Marusic & Kunkel (2003) and Marusic & Perry (1997), except for

$$(A1, B1) = (1.26, 2.1) \quad , \tag{5}$$

instead of the original (1.03, 2.39). This value of $A1$ has been proposed by Marusic *et al.* (2013) and the value of $B1$, which is well within the range of their table 2, has been chosen to provide a good fit to the outer part of the ZPG TBL profile of Samie *et al.* (2018) at $\mathrm{Re}_\tau = 20000$. Beyond this, no attempt has been made to modify the parameters of Marusic & Kunkel (2003), but simultaneously matching all the high quality profiles included in figure 3 appears difficult.

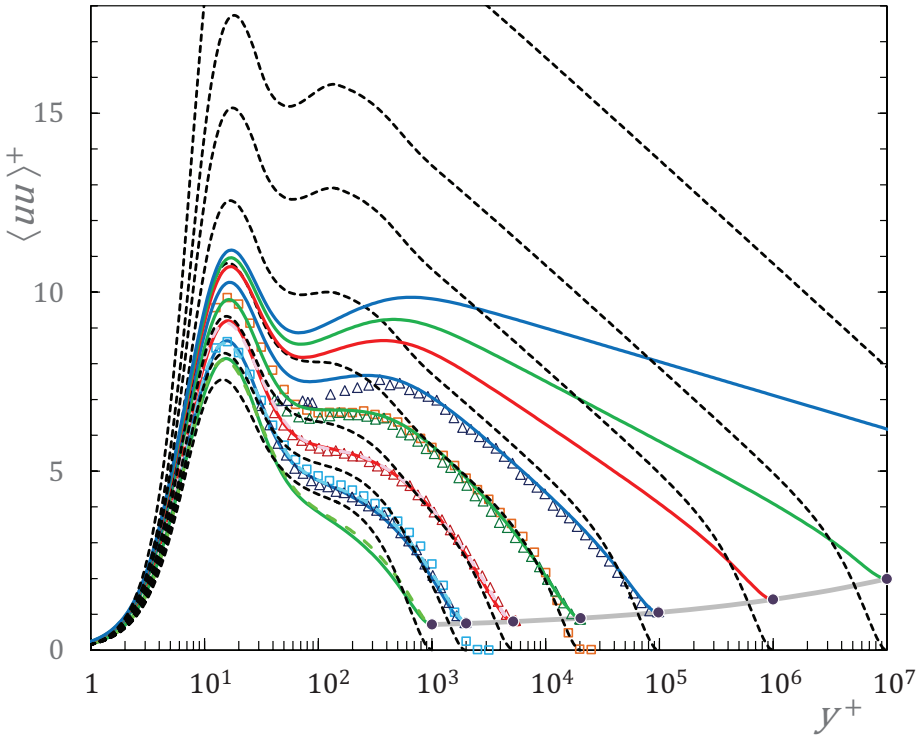

Figure 3: Solid lines: composite profiles of $\langle uu \rangle^+$ as in figure 3 of the main paper for $\mathrm{Re}_\tau = 1000$ (green), 1995 (blue), 5186 (red), 20'250 (green), 98'190 (blue), 10^6 (red), 10^7 (green), 10^9 (blue). - - - (black) corresponding fits of Marusic & Kunkel (2003). DNS and experimental profiles: - - -, DNS profiles 5 (green), 4 (blue) and 1 (red) of table 1 in the main paper. △△△ , NSTAP Superpipe profiles of Hultmark *et al.* (2012) at $\mathrm{Re}_\tau = 1985$, 5411, 20'250 and 98'187. □, ZPG TBL profile of Sillero *et al.* (2013) for $\mathrm{Re}_\tau = 1989$ (blue) and of Samie *et al.* (2018) for $\mathrm{Re}_\tau = 20'000$ (orange).

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