

## Description of files

The supplementary material consists of three MATLAB mat files: Type\_1\_Limit.mat, Type\_2\_Limit.mat, and Type\_3\_Limit.mat containing the original data of three profiles shown in Figure 5 in the article. Each mat file includes the following 18 quantities:

| quantity | meaning                             |
|----------|-------------------------------------|
| $a$      | Fourier coefficients of $q_1(\tau)$ |
| $\alpha$ | total arclength in half period      |
| $b$      | Fourier coefficients of $q_2(\tau)$ |
| $B$      | Bernoulli constant                  |
| $c$      | Fourier coefficients of $x'(\tau)$  |
| $d$      | Fourier coefficients of $y'(\tau)$  |
| $dx$     | Function values of $x'(\tau)$       |
| $dy$     | Function values of $y'(\tau)$       |
| $F$      | Froude number                       |
| $h$      | depth ratio                         |
| $H$      | wave height                         |
| $k$      | wave number                         |
| $N$      | number of Fourier modes             |
| $q1$     | Function values of $q_1(\tau)$      |
| $q2$     | Function values of $q_2(\tau)$      |
| $R$      | density ratio                       |
| $x$      | horizontal coordinate of interface  |
| $y$      | vertical coordinate of interface    |