1 Resolution Checks

Here, we confirm the accuracy of the results by comparing solutions for different truncations (N, M, P). We perform detailed resolution check at various values of the parameters Re, α and A.

The error comparison is done by computing absolute and relative errors using infinity norm for each velocity component u, v, w in Fourier space separately. We denote u=$\{u_{kl}(r),\ 0 \leq k \leq M,\ 0 \leq l \leq P\}$ and define v and w in the same manner.

Let $\mathbf{v}_1$ be a solution at truncation (N_1, M_1, P_1) and $\mathbf{v}_2$ at (N_2, M_2, P_2) where $N_1 \leq N_2$, $M_1 \leq M_2$ and $P_1 \leq P_2$. The comparison is made by taking the difference of these two solutions by injecting the lower resolution solution $\mathbf{v}_1$ into the higher dimension by adding zeros.

Results shown in the tables below confirm that truncation (N_1, M_1, P_1) provides good accuracy for the given set of parameters (A, Re, α). In these tables, we compare the given solution (N_1, M_1, P_1) with a solution at new truncation levels obtained by significantly increasing N , M and P one at a time within computational limitations.

1.1 Two-fold rotational symmetric travelling waves

In this section, we present resolution checks for two-fold rotational symmetric solutions with $k_0 = 2$ at various values of aspect ratio A when Re^*=7500 and α^*=1.55. Tables (1), (2) and (3) show at least 5 digits accuracy of the phase speed and 3 digits accuracy of each velocity component.

(N_2, M_2, P_2)	$(95, 24, 12)$	$(95, 16, 8)$	$(145, 24, 8)$
c	1.1695e-07	1.1435e-07	4.7445e-12
$\|\|u\|\|$	3.4999e-05	3.5281e-05	1.8075e-04
$\|\|v\|\|$	7.4493e-05	7.5101e-05	2.1472e-04
$\|\|w\|\|$	4.9040e-06	4.9237e-06	5.5970e-06

Table 1: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (95, 24, 8)$ for two-fold rotationally symmetric TW at A=1.76 where $c = 0.81762$.

(N_2, M_2, P_2)	(165, 24, 8)	(125, 24, 12)	(125, 32, 8)
c	2.4133e-07	3.7031e-08	7.1715e-06
$\|\|u\|\|$	0.0002301	0.00023255	0.00057591
$\|\|v\|\|$	0.00047082	0.00023357	0.00026923
$\|\|w\|\|$	1.2311e-05	5.0758e-06	5.856e-05

Table 2: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (125, 24, 8)$ for two-fold symmetric TW at A=3.36 where $c = 0.8583$.

(N_2, M_2, P_2)	(175, 36, 5)	(145, 36, 8)	(145, 24, 5)
c	7.8941e-05	0.00065222	3.3535e-08
$\|\|u\|\|$	5.3926e-04	0.0017163	5.1168e-06
$\|\|v\|\|$	9.9952e-04	0.0060749	5.3974e-07
$\|\|w\|\|$	5.2937e-04	0.00582472	7.9781e-06

Table 3: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (145, 36, 5)$ for two-fold symmetric TW at A=5.51 where $c = 0.863$.

1.2 Rotationally asymmetric travelling waves

In this section, we present resolution checks for rotational asymmetric solutions with $k_0 = 1$ when α^*=1.437 at various values of aspect ratio A and Re^*. Tables (4)- (10) show at least 4 digits accuracy of the phase speed and 3 digits accuracy of each velocity component.

(N_2, M_2, P_2)	(195, 52, 5)	(135, 52, 8)	(135, 92, 5)
c	4.4396e-10	1.9677e-08	7.1768e-05
$\|\|u\|\|$	4.2123e-05	8.7381e-06	0.0032
$\|\|v\|\|$	4.2135e-05	3.5203e-06	4.8262e-04
$\|\|w\|\|$	6.7129e-07	4.2709e-07	9.6421e-04

Table 4: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (135, 52, 5)$ for A=5.382 with Re^*=5000 where $c = 0.926$.

(N_2, M_2, P_2)	$(125, 30, 8)$	$(95, 30, 12)$	$(95, 46, 8)$
c	8.3107e-13	3.8577e-09	1.7908e-06
$\|\|u\|\|$	3.215e-05	1.9333e-06	0.0049994
$\|\|v\|\|$	4.732e-05	1.9051e-06	0.0030439
$\|\|w\|\|$	2.194e-05	1.6063e-07	0.0014796

Table 5: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (95, 30, 8)$ for the S1 curve for A=0.36393 with Re^*=5000 where $c = 0.6862$.

(N_2, M_2, P_2)	$(165, 72, 5)$	$(135, 82, 5)$	$(135, 72, 8)$
c	1.0089e-07	0.00065222	3.3535e-08
$\|\|u\|\|$	7.6261e-05	0.00582472	7.9781e-06
$\|\|v\|\|$	9.6898e-05	0.0017163	5.1168e-06
$\|\|w\|\|$	3.287e-06	0.0060749	5.3974e-07

Table 6: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (135, 72, 5)$ for the M1 curve for A=9.382 with Re^*=5000 and α^*=1.437 where $c = 0.932$.

(N_2, M_2, P_2)	$(95, 102, 8)$	$(95, 82, 10)$	$(125, 82, 8)$
c	2.5205e-06	7.5713e-10	1.2593e-07
$\|\|u\|\|$	5.0315e-04	5.0610e-08	3.9533e-04
$\|\|v\|\|$	1.2453e-04	4.4992e-08	0.0011
$\|\|w\|\|$	7.1177e-05	8.2996e-08	1.0427e-04

Table 7: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (95, 82, 8)$ for the S1 curve for A=0.189 with Re^*=26428.4 and α^*=1.437 where $c = 0.71048$.

(N_2, M_2, P_2)	$(95, 50, 8)$	$(95, 30, 16)$	$(145, 30, 8)$
c	7.4559e-07	8.2716e-11	8.2695e-11
$\|\|u\|\|$	1.2213e-04	7.5981e-10	1.5316e-05
$\|\|v\|\|$	2.4944e-05	2.1198e-09	3.3458e-05
$\|\|w\|\|$	2.8526e-05	6.5586e-10	7.7556e-06

Table 8: The relative errors found by comparing the numerical solution at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (95, 30, 8)$ for the S1 curve for A=1/1.76 and Re^*=8934, α^*=1.437 where c = 0.71234.

(N_2, M_2, P_2)	$(95, 80, 8)$	$(125, 80, 5)$	$(95, 120, 5)$
c	4.1176e-05	5.8269e-14	5.8128e-06
$\|\|u\|\|$	1.5912e-04	2.0791e-05	5.2552e-05
$\|\|v\|\|$	1.2921e-04	4.6805e-05	3.3817e-05
$\|\|w\|\|$	1.5283e-04	2.4733e-06	1.4227e-05

Table 9: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (95, 80, 5)$ for the M1 curve for A=1/1.76 with Re^*=16312 and α^*= 1.437 where c = 0.52208.

(N_2, M_2, P_2)	$(85, 24, 8)$	$(85, 80, 8)$	$(105, 50, 8)$
c	0.0013	1.8652e-08	1.6455e-10
$\|\|u\|\|$	0.0050	2.7570e-07	5.6575e-05
$\|\|v\|\|$	0.0060	2.5382e-07	6.1339e-05
$\|\|w\|\|$	0.0033	4.5942e-08	2.2338e-06

Table 10: The relative errors found by comparing the numerical solutions at various levels of truncation (N_2, M_2, P_2) with the baseline calculations with $(N_1, M_1, P_1) = (85, 50, 8)$ for A=1/1.76 and Re^*=5292.5 and α^*=1.437 where c = 0.50692. This is a lower branch resolution check.