Impact of polydispersity and hydrodynamics on diffusion in spherically confined colloidal suspensions — Supplementary material

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Hydrodynamic mobility functions

We report the mobility functions that capture the hydrodynamic interactions between spherical particles and the enclosing cavity relating velocity derivatives $(\mathbf{U}, \boldsymbol{\Omega}, \mathbf{E})$ to traction moments $(\mathbf{F}, \mathbf{L}, \mathbf{S})$ for particles of arbitrary colloidal particle sizes a_i and a_j and with particle-to-cavity size ratio $\lambda_{c_i} = a_i/R$ and $\lambda_{c_j} = a_j/R$, respectively. Each mobility function has three superscripts. The first, in capital letters, refers to the coupling between a velocity moment $(\mathbf{U}, \boldsymbol{\Omega}, \mathbf{E})$ and a traction moment $(\mathbf{F}, \mathbf{L}, \mathbf{S})$. The second, the letter c, references that the mobility function is the cavity contribution to the total mobility. The third, superscripts ii or ij, refers to an interacting pair of particles of sizes a_i and a_j. The particle-to-cavity size ratios λ_{c_i} and λ_{c_j} carry subscripts that refer to the driven and the entrained particles of size a_i and a_j. Each mobility function has indices that refer to position in the matrix. The scalar y is the distance between the center of the cavity and the center of the forced particle of size a_j, while the scalar x is the distance between the center of the cavity and the center of the entrained particle of size a_i. The symbol b is the cosine of the angle between the unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$, such that $b = \hat{\mathbf{x}} \cdot \hat{\mathbf{y}}$. The unit vectors carry subscript indices that couple to those of the mobility matrix. These mobility functions provide the far-field couplings between particles owing to truncation of the multipole expansion beyond the stresslet. Subsequent inversion produces the many-body reflected interactions between all particles, and superposition with the near-field functions completes the couplings for all separations and all interactions [2].

Self-mobility functions

The cavity contribution to the self-mobility for six couplings was reported previously by Aponte-Rivera & Zia (2016)[1]. However, an error in the calculation for two of those couplings for the self-mobility (which did not affect the results of that paper) is corrected here. The corrected versions are show below. Here, "y" is the magnitude of the distance between the center of the cavity and the center of a particle of size a_i.

$$
\begin{aligned}
6\pi\eta a_i^2 M_{kln}^{US,c,ii} = & \left[\lambda_{c_i}^2(-27\text{y})/(8(-1+\text{y}^2)^2) + \lambda_{c_i}^4(9\text{y}(23+7\text{y}^2))/(20(-1+\text{y}^2)^4)\right. \\
& \left. -\lambda_{c_i}^6(9\text{y}(35+42\text{y}^2+3\text{y}^4))/(40(-1+\text{y}^2)^6)\right]\hat{\text{y}}_k(\hat{\text{y}}_l\hat{\text{y}}_n - \tfrac{1}{3}\delta_{l,n}) \\
& + \left[\lambda_{c_i}^2(9\text{y}(-3+\text{y}^2))/(16(-1+\text{y}^2)^2)\right. \\
& -\lambda_{c_i}^4(3\text{y}(-138+59\text{y}^2-20\text{y}^4+3\text{y}^6))/(80(-1+\text{y}^2)^4) \\
& \left. -\lambda_{c_i}^6(3\text{y}(105+42\text{y}^2+18\text{y}^4-6\text{y}^6+\text{y}^8))/(80(-1+\text{y}^2)^6)\right] \\
& \times(\delta_{kl}\hat{\text{y}}_n + \delta_{kn}\hat{\text{y}}_l - 2\hat{\text{y}}_k\hat{\text{y}}_l\hat{\text{y}}_n) \quad (S1)
\end{aligned}
$$

$$
\begin{aligned}
6\pi\eta a_i^3 M_{nklr}^{ES,c,ii} = & \left[\lambda_{c_i}^3(27(5+3\text{y}^2))/(16(-1+\text{y}^2)^3)\right. \\
& -\lambda_{c_i}^5(81(7+22\text{y}^2+3\text{y}^4))/(40(-1+\text{y}^2)^5) \\
& \left. +\lambda_{c_i}^7(81(35+259\text{y}^2+177\text{y}^4+9\text{y}^6))/(400(-1+\text{y}^2)^7)\right]
\end{aligned}
$$

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$$
\times(\hat{y}_n\hat{y}_k - \tfrac{1}{3}\delta_{nk})(\hat{y}_l\hat{y}_r - \tfrac{1}{3}\delta_{lr})
$$

$$
+\Big[\lambda_{c_i}^3(9(5+\mathrm{y}^2))/(16(-1+\mathrm{y}^2)^3)
$$

$$
-\lambda_{c_i}^5(9(21+47\mathrm{y}^2-5\mathrm{y}^4+\mathrm{y}^6))/(40(-1+\mathrm{y}^2)^5)
$$

$$
+\lambda_{c_i}^7(9(105+609\mathrm{y}^2+222\mathrm{y}^4+30\mathrm{y}^6-7\mathrm{y}^8+\mathrm{y}^{10}))/(400(-1+\mathrm{y}^2)^7)\Big]
$$

$$
\times(\delta_{nl}\hat{y}_k\hat{y}_r+\delta_{nr}\hat{y}_k\hat{y}_l+\delta_{kl}\hat{y}_n\hat{y}_r+\delta_{kr}\hat{y}_n\hat{y}_l-4\hat{y}_n\hat{y}_l\hat{y}_k\hat{y}_r)
$$

$$
+\Big[\lambda_{c_i}^3(-9(-10+10\mathrm{y}^2-5\mathrm{y}^4+\mathrm{y}^6))/(32(-1+\mathrm{y}^2)^3)
$$

$$
-\lambda_{c_i}^5(9(42-20\mathrm{y}^2+15\mathrm{y}^4-6\mathrm{y}^6+\mathrm{y}^8))/(80(-1+\mathrm{y}^2)^5)
$$

$$
-\lambda_{c_i}^7(9(-210-210\mathrm{y}^2-75\mathrm{y}^4+21\mathrm{y}^6-7\mathrm{y}^8+\mathrm{y}^{10}))/(800(-1+\mathrm{y}^2)^7)\Big]
$$

$$
\times(\delta_{nl}\delta_{kr}+\delta_{nr}\delta_{kl}-\delta_{nl}\hat{y}_k\hat{y}_r-\delta_{nr}\hat{y}_k\hat{y}_l-\delta_{kl}\hat{y}_n\hat{y}_r-\delta_{kr}\hat{y}_n\hat{y}_l
$$

$$
+\hat{y}_n\hat{y}_k\hat{y}_r\hat{y}_l-\delta_{lr}\delta_{nk}+\delta_{kn}\hat{y}_l\hat{y}_r+\delta_{lr}\hat{y}_n\hat{y}_k) \tag{S2}
$$

Entrainment mobility functions

The cavity contribution to the entrainment mobility for six couplings for uniformly sized particles was reported previously by Aponte-Rivera & Zia (2016)[1]. We derived new expressions for these couplings to account for differences in the sizes of interacting particles. In this section, we report the cavity contribution to these entrainment mobility functions. Each mobility function is scaled on the hydrodynamic mobility of the cavity itself. All symbols carry subscripts, superscripts, and indices as defined above. For all the mobility matrix components, the symbol $\vartheta=\sqrt{1-2b\mathrm{xy}+\mathrm{x}^2\mathrm{y}^2}$ indicates the distance of the entrained particle to the image point of the forced particle, correcting a typographical error in the argument of the square root given in Aponte-Rivera & Zia (2016) [1].

$$
6\pi\eta R M_{kl}^{UF,c,ij} = M_1^{UF,c,ij}\hat{y}_k\hat{y}_l + M_2^{UF,c,ij}\hat{y}_k\hat{y}_l^\perp + M_3^{UF,c,ij}\hat{y}_k^\perp\hat{y}_l + M_4^{UF,c,ij}\hat{y}_k^\perp\hat{y}_l^\perp
$$
$$
+M_5^{UF,c,ij}\delta_{kl}, \tag{S3a}
$$

$$
\begin{aligned}
M_1^{UF,c,ij} = &\Big[3(4b^5\mathrm{x}^4\mathrm{y}^2+b^4\mathrm{x}^3\mathrm{y}(-2+(-11+\mathrm{x}^2)\mathrm{y}^2-3(-1+\mathrm{x}^2)\mathrm{y}^4)+\mathrm{xy}^3(2\\
&+\mathrm{x}^2(-6+8\mathrm{y}^2-3\mathrm{x}^2(-1+\mathrm{y}^2)(2+(-1+\mathrm{x}^2)\mathrm{y}^2)))-2b^3\mathrm{x}^2\mathrm{y}^2(-11+\mathrm{y}^2(7\\
&-6\vartheta)+6\vartheta+\mathrm{x}^2(-1+\mathrm{y}^2)(-11+6\vartheta))+b(\mathrm{x}^4\mathrm{y}^2(\mathrm{y}^2(23-9\vartheta)+3\mathrm{y}^4(-5+\vartheta)\\
&+6(-2+\vartheta))-3\mathrm{x}^6\mathrm{y}^4(-1+\mathrm{y}^2)(-5+\vartheta)+3(-1+\mathrm{y}^2)(-1+\vartheta)+\mathrm{x}^2(\mathrm{y}^2(11\\
&-9\vartheta)+3(-1+\vartheta)+2\mathrm{y}^4(-8+3\vartheta)))+b^2\mathrm{xy}(\mathrm{y}^2(13-12\vartheta)+3(-5+4\vartheta)+\mathrm{x}^2\\
&(17-28\mathrm{y}^2+19\mathrm{y}^4-12(-1+\mathrm{y}^2)^2\vartheta)+\mathrm{x}^4\mathrm{y}^2(23-12\vartheta+3\mathrm{y}^2(-7+4\vartheta))))\\
&/(8(-1+b^2)\mathrm{xy}\vartheta^5)\Big]+\lambda_{c_i}^2\Big[(3(4b^5\mathrm{x}^4\mathrm{y}^4(-2+\mathrm{y}^2)+\mathrm{xy}^3(-2-3\mathrm{x}^4\mathrm{y}^2(-1+\mathrm{y}^2)\\
&-2\mathrm{x}^2(-6+7\mathrm{y}^2)+\mathrm{x}^6(\mathrm{y}^4-\mathrm{y}^6))+b^4\mathrm{x}^3\mathrm{y}^3(17+3\mathrm{x}^2\mathrm{y}^4-\mathrm{y}^2(7\\
&+\mathrm{x}^2-8\vartheta)-8\vartheta)-2b^3\mathrm{x}^2\mathrm{y}^2(7-6\vartheta+\mathrm{x}^2\mathrm{y}^4(-1+6\vartheta)+\mathrm{y}^2(-3+\mathrm{x}^2\\
&+6\vartheta-6\mathrm{x}^2\vartheta))+b(-1+\mathrm{x}^4\mathrm{y}^6(29+\mathrm{x}^2(-7+\vartheta)-3\vartheta)-\mathrm{x}^6\mathrm{y}^8(-7+\vartheta)+\vartheta\\
&+\mathrm{x}^2\mathrm{y}^4(-3(-5+\vartheta)+\mathrm{x}^2(-25+3\vartheta))+\mathrm{y}^2(1-\vartheta+\mathrm{x}^2(-7+3\vartheta)))+b^2\mathrm{xy}(7\\
&-6\vartheta+3\mathrm{x}^4\mathrm{y}^6(-7+2\vartheta)+\mathrm{y}^2(-5+\mathrm{x}^2(6-12\vartheta)+6\vartheta)+\mathrm{x}^2\mathrm{y}^4(\mathrm{x}^2(19\\
&-6\vartheta)+2(-7+6\vartheta)))))/(8(-1+b^2)\mathrm{xy}\vartheta^7)\Big]+\lambda_{c_j}^2\Big[(3(4b^5\mathrm{x}^4\mathrm{y}^2(-1+2(-1+\\
&\mathrm{x}^2)\mathrm{y}^2)+\mathrm{x}^3\mathrm{y}^3(12-\mathrm{x}^6\mathrm{y}^4+\mathrm{x}^4\mathrm{y}^2(-5+\mathrm{y}^2)+\mathrm{x}^2(-14+3\mathrm{y}^2))-2b^3\mathrm{x}^2\mathrm{y}^2\\
&(7-6\vartheta+3\mathrm{x}^4\mathrm{y}^2(1+2\vartheta)+\mathrm{x}^2(-7+\mathrm{y}^2+6\vartheta-6\mathrm{y}^2\vartheta))+b(-1+\mathrm{x}^6\mathrm{y}^4(33+\\
&\mathrm{y}^2(-7+\vartheta)-3\vartheta)-\mathrm{x}^8\mathrm{y}^6(-7+\vartheta)+\vartheta+\mathrm{x}^4\mathrm{y}^2(11-3\vartheta+\mathrm{y}^2(-25+3\vartheta))+\mathrm{x}^2(1\\
&-\vartheta+\mathrm{y}^2(-7+3\vartheta)))+b^2\mathrm{xy}(7-6\vartheta+\mathrm{x}^6\mathrm{y}^4(-17+6\vartheta)+\mathrm{x}^2(-9+\mathrm{y}^2(6\\
&-12\vartheta)+6\vartheta)+\mathrm{x}^4\mathrm{y}^2(\mathrm{y}^2(19-6\vartheta)+2(-7+6\vartheta)))+b^4\mathrm{x}^3\mathrm{y}(2+\mathrm{x}^2(-1+\mathrm{x}^2)\mathrm{y}^4+
\end{aligned}
$$

$$y^2(17 - 8\vartheta + x^2(-7 + 8\vartheta)))))/(8(-1 + b^2)xy\vartheta^7)\Big] - \lambda_{c_j}^2\lambda_{c_i}^2\Big[(32b^6x^5y^5 +$$
$$x^3y^3(84 - 51x^2y^2 + 6x^4y^4 + x^6y^6) + 8b^5x^4y^4(-9 + 6x^2y^2$$
$$+2\vartheta) + b(-1 + 6x^4y^4(-27 + \vartheta) + x^8y^8(-9 + \vartheta) + \vartheta + 2x^2y^2(-11 + 2\vartheta) + 2x^6y^6$$
$$(5 + 2\vartheta)) - b^2xy(-9 + 8\vartheta + 3x^4y^4(-89 + 8\vartheta) + x^6y^6(-27 + 8\vartheta) + 3x^2y^2(21$$
$$+8\vartheta)) + 2b^3x^2y^2(-7 + 12\vartheta + 6x^2y^2(9 + 4\vartheta) + x^4y^4(-71 + 12\vartheta)) + b^4x^3y^3(63$$
$$+3x^4y^4 - 32\vartheta - 2x^2y^2(61 + 16\vartheta)))/(8(-1 + b^2)xy\vartheta^9)\Big] \tag{S3b}$$

$$M_2^{UF,c,ij} = \Big[3(-3bx^7y^5(-1 + y^2) + 3x^6y^4(-1 + y^2)(5b^2 - \vartheta) + 3(-1 + y^2)(-1 + \vartheta)$$
$$-3bxy(-1 + y^2)(-5 + 4\vartheta) + bx^3y(13 - 28y^2 + 9y^4 + b^2(2 - 17y^2 + 21y^4)$$
$$-12(-1 + y^2)^2\vartheta) + bx^5y^3(7 + b^2(23 - 21y^2) - 12\vartheta + 3y^2(-4 + y^2 + 4\vartheta))$$
$$+x^4y^2(-4b^4 + 6\vartheta + b^2(-26 + y^2(43 - 15y^2 - 12\vartheta) + 12\vartheta) + y^2(2 + 3(-3 +$$
$$y^2)\vartheta)) + x^2(y^2(5 + 4b^2(7 - 3\vartheta) - 9\vartheta) + 3(-1 + \vartheta) + 6y^4(\vartheta + b^2(-5$$
$$+2\vartheta))))/(8\sqrt{1 - b^2}xy\vartheta^5)\Big] + \lambda_{c_i}^2\Big[(3(-1 + 4b^4x^4y^4(-6 + 7y^2) + \vartheta + x^4(-3$$
$$+x^2)y^6\vartheta - x^6y^8\vartheta + x^2y^4(-3(3 + \vartheta) + x^2(-2 + 3\vartheta)) + y^2(1 - \vartheta$$
$$+x^2(7 + 3\vartheta)) + b^3x^3y^3(43 - 19x^2y^4 - 8\vartheta + y^2(-49 + 17x^2 + 8\vartheta))$$
$$+bxy(7 - x^6y^8 - 6\vartheta + x^4y^6(-2 + x^2 + 6\vartheta) + 2x^2y^4(7 + x^2(2 - 3\vartheta) +$$
$$6\vartheta) - y^2(7 - 6\vartheta + 4x^2(2 + 3\vartheta))) + b^2x^2y^2(7x^4y^6 + x^2y^4(7 - 7x^2 - 12\vartheta)$$
$$+4(-7 + 3\vartheta) + 3y^2(10 - 4\vartheta + x^2(-3 + 4\vartheta))))))/(8\sqrt{1 - b^2}xy\vartheta^7)\Big] + \lambda_{c_j}^2\Big[(3(-1$$
$$-bx^9y^7 + bxy(7 - 6\vartheta) + x^8y^6(7b^2 - \vartheta) + \vartheta + bx^7y^5(-4 - 17b^2$$
$$+y^2 + 6\vartheta) + x^6y^4(24b^4 + b^2(11 - 7y^2 - 12\vartheta) + (-3 + y^2)\vartheta) + bx^5y^3(y^2(4$$
$$-6\vartheta) + 2(7 + 6\vartheta) + b^2(-49 + 17y^2 + 8\vartheta)) - x^4y^2(4b^4(-1 + 6y^2) + y^2(2 - 3\vartheta)$$
$$+3(3 + \vartheta) + b^2(-26 + y^2(9 - 12\vartheta) + 12\vartheta)) + x^2(1 - \vartheta + y^2(7 + 3\vartheta + 4b^2(-7 + 3\vartheta)))$$
$$-bx^3y(5 - 6\vartheta + 4y^2(2 + 3\vartheta) + b^2(2 + y^2(-43 + 8\vartheta)))))/(8\sqrt{1 - b^2}xy\vartheta^7)\Big]$$
$$+\lambda_{c_j}^2\lambda_{c_i}^2\Big[(1 - 96b^5x^5y^5 + 3x^4y^4(27 - 2\vartheta) + 8b^4x^4y^4(27$$
$$+2x^2y^2 - 2\vartheta) - \vartheta - x^8y^8\vartheta - 4x^2y^2(12 + \vartheta) - 2x^6y^6(5$$
$$+2\vartheta) + bxy(-9 - x^8y^8 + 8\vartheta + 12x^4y^4(-9 + 2\vartheta) + x^6y^6(-3 + 8\vartheta)$$
$$+3x^2y^2(35 + 8\vartheta)) + b^3x^3y^3(-189 - 33x^4y^4 + 32\vartheta + 2x^2y^2(39$$
$$+16\vartheta)) + 3b^2x^2y^2(28 + 3x^6y^6 + 2x^4y^4(13 - 4\vartheta) - 8\vartheta - x^2y^2(57$$
$$+16\vartheta))))/(8\sqrt{1 - b^2}xy\vartheta^9)\Big] \tag{S3c}$$

$$M_3^{UF,c,ij} = \Big[3\sqrt{1 - b^2}x(4b^2x^2y + bx(-2 + (-7 + x^2)y^2 - 3(-1 + x^2)y^4)$$
$$+y(5 - 3y^2 + x^2(-3 + 5y^2)))/(8\vartheta^5)\Big] + \lambda_{c_i}^2\Big[(3\sqrt{1 - b^2}xy(-7 + (5 + 3x^2)y^2$$
$$-5x^2y^4 + 4b^2x^2y^2(-2 + y^2) + bxy(13 - (7 + x^2)y^2$$
$$+3x^2y^4)))/(8\vartheta^7)\Big] + \lambda_{c_j}^2\Big[(3\sqrt{1 - b^2}x(4b^2x^2y(-1 + 2(-1$$
$$+x^2)y^2) + bx(2 + (13 - 7x^2)y^2 + x^2(-1 + x^2)y^4) + y(-7 - 5x^4y^2$$
$$+x^2(5 + 3y^2))))/(8\vartheta^7)\Big] - \lambda_{c_j}^2\lambda_{c_i}^2\Big[\sqrt{1 - b^2}xy$$
$$(32b^3x^3y^3 + 24b^2x^2y^2(-3 + 2x^2y^2) + 3bxy(21 - 38x^2y^2$$
$$+x^4y^4) - 5(7 - 18x^2y^2 + 3x^4y^4))/(8\vartheta^9)\Big] \tag{S3d}$$

$$M_4^{UF,c,ij} = \Big[3(-4b^5x^4y^2 + b^4x^3y(2 + (-21 + 23x^2)y^2 - 21(-1 + x^2)y^4) +$$

3

$$\begin{aligned}
&x^3y(2 + y^2(6(-1 + y^2) + x^2(8 + 3(-3 + x^2)y^2 - 3(-1 + x^2)y^4))) + b(6(-1 \\
&+y^2)(-1 + \vartheta) - 3x^6y^4(-1 + y^2)(-5 + 2\vartheta) + x^4y^2(-19 + 6y^2(5 - 3\vartheta) + 12\vartheta \\
&+3y^4(-5 + 2\vartheta)) + 3x^2(-1 + y^2)(2 - 2\vartheta + y^2(-5 + 4\vartheta))) - b^2xy(1 \\
&+x^2y^2)(3x^4y^2(-1 + y^2) + 6(-1 + y^2)(-5 + 4\vartheta) + x^2(-26 + 24\vartheta - 3y^2(-11 \\
&+y^2 + 8\vartheta))) + b^3x^2y^2(15x^4y^2(-1 + y^2) + 3(-1 + y^2)(-15 + 8\vartheta) + x^2(-37 \\
&+24\vartheta - 3y^2(-20 + 5y^2 + 8\vartheta)))))/(8(-1 + b^2)xy\vartheta^5)\Big] + \lambda_{c_i}^2\Big[(3(4b^5x^4y^4(-6 \\
&+7y^2) + x^3y^3(12 + (-14 + 3x^2)y^2 + x^2(-5 + x^2)y^4 - x^4y^6) \\
&+b^4x^3y^3(47 - 19x^2y^4 - 16\vartheta + y^2(-49 + 17x^2 + 16\vartheta)) + b^2xy(14 - x^6y^8 \\
&-12\vartheta + x^4y^6(-18 + x^2 + 12\vartheta) + x^2y^4(-7 + x^2(22 - 12\vartheta) + 24\vartheta) + y^2(x^2(11 \\
&-24\vartheta) + 2(-7 + 6\vartheta))) + b(x^2y^4(7 + 6x^2(-4 + \vartheta) - 6\vartheta) + x^6y^8(7 - 2\vartheta) + 2(-1 \\
&+\vartheta) + x^4y^6(28 - 6\vartheta + x^2(-7 + 2\vartheta)) + y^2(2 - 2\vartheta + x^2(-7 + 6\vartheta))) + b^3x^2y^2(-35 \\
&+7x^4y^6 + x^2y^4(14 - 7x^2 - 24\vartheta) + 24\vartheta + y^2(35 - 24\vartheta + x^2(-22 + 24\vartheta))))) \\
&/(8(-1 + b^2)xy\vartheta^7)\Big] + \lambda_{c_j}^2\Big[3(4b^5x^4y^2(1 + 6(-1 + x^2)y^2) + x^3y(-2 - 2(-6 \\
&+7x^2)y^2 - 3x^2(-1 + x^2)y^4 + (x^4 - x^6)y^6) + b(x^4y^2(11 + 6y^2(-4 + \vartheta) \\
&-6\vartheta) + x^8y^6(7 - 2\vartheta) + 2(-1 + \vartheta) + x^6y^4(-6(-4 + \vartheta) + y^2(-7 + 2\vartheta)) + x^2(2 \\
&-2\vartheta + y^2(-7 + 6\vartheta))) + b^2xy(14 - x^8y^6 - 12\vartheta + x^6y^4(-22 + y^2 \\
&+12\vartheta) + x^4y^2(-7 + y^2(22 - 12\vartheta) + 24\vartheta) + x^2(y^2(11 - 24\vartheta) + 2(-5 + 6\vartheta))) \\
&+b^4x^3y(-2 - 17x^2(-1 + x^2)y^4 + y^2(47 - 16\vartheta + x^2(-49 + 16\vartheta))) + b^3x^2y^2(-35 \\
&+7x^6y^4 + x^4y^2(22 - 7y^2 - 24\vartheta) + 24\vartheta + x^2(27 - 24\vartheta + y^2(-22 \\
&+24\vartheta))))/(8(-1 + b^2)xy\vartheta^7)\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[(-96b^6x^5y^5 - x^3y^3(84 - 51x^2y^2 + 6x^4y^4 + \\
&x^6y^6) + 8b^5x^4y^4(27 + 2x^2y^2 - 4\vartheta) - b(2(-1 + \vartheta) + x^8y^8(-9 + 2\vartheta) \\
&+3x^4y^4(-51 + 4\vartheta) + x^2y^2(-9 + 8\vartheta) + x^6y^6(5 + 8\vartheta)) + b^2xy(-(x^8y^8) \\
&+2(-9 + 8\vartheta) + 3x^4y^4(-87 + 16\vartheta) + x^6y^6(-33 + 16\vartheta) + 3x^2y^2(35 + 16\vartheta)) \\
&+b^4x^3y^3(-189 - 33x^4y^4 + 64\vartheta + 2x^2y^2(27 + 32\vartheta)) + b^3x^2y^2(63 + 9x^6y^6 \\
&+x^4y^4(157 - 48\vartheta) - 48\vartheta - 3x^2y^2(39 + 32\vartheta)))/(8(-1 + b^2)xy\vartheta^9)\Big]
\end{aligned}$$

(S3e)

$$\begin{aligned}
M_5^{UF,c,ij} &= \Big[3(4b^3x^2y^2 + xy(6 - 4y^2 + x^2(-4 - 3(-3 + x^2)y^2 + 3(-1 + x^2)y^4)) \\
&+b^2xy(3 - 5y^2 + x^2(-5 + 3y^2) - 6(-1 + x^2)(-1 + y^2)\vartheta) + b(3x^4y^2(-1 \\
&+y^2)(-3 + \vartheta) - 3(-1 + y^2)(-1 + \vartheta) + x^2(3 - 16y^2 + 9y^4 - 3(-1 \\
&+y^2)^2\vartheta)))/(8(-1 + b^2)xy\vartheta^3)\Big] + \lambda_{c_i}^2\Big[(xy(-10 + (12 - 7x^2)y^2 - 3x^2(-3 + x^2)y^4 \\
&+3x^4y^6) + b(3 + y^2(x^2(26 - 6\vartheta) + 3(-1 + \vartheta)) - 3x^2(-2 + x^2)y^4(-5 + \vartheta) \\
&+3x^4y^6(-5 + \vartheta) - 3\vartheta) + 4b^3x^2y^2(1 + 3(-1 + y^2)\vartheta) + b^2xy(-5 + 12\vartheta \\
&-3x^2y^4(-7 + 4\vartheta) + y^2(3 - 12\vartheta + x^2(-23 + 12\vartheta)))))/(8(-1 + b^2)xy\vartheta^5)\Big] \\
&+\lambda_{c_j}^2\Big[(xy(-10 + 3x^6y^4 + x^2(12 - 7y^2) - 3x^4y^2(-3 + y^2)) + b(3 + x^2(y^2(26 - 6\vartheta) \\
&+3(-1 + \vartheta)) + 3x^6y^4(-5 + \vartheta) - 3x^4y^2(-2 + y^2)(-5 + \vartheta) - 3\vartheta) + 4b^3x^2y^2(1 \\
&+3(-1 + x^2)\vartheta) + b^2xy(-5 + 12\vartheta - 3x^4y^2(-7 + 4\vartheta) + x^2(3 - 12\vartheta + y^2(-23 \\
&+12\vartheta)))))/(8(-1 + b^2)xy\vartheta^5)\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[(xy(10 - 7x^2y^2 \\
&+4x^4y^4 + x^6y^6) - 8b^4x^3y^3\vartheta + 12b^3x^2y^2(x^2y^2(-2 + \vartheta) \\
&+\vartheta) + b(-1 + 3x^2y^2(-7 + \vartheta) + x^6y^6(-7 + \vartheta) + \vartheta + x^4y^4(-11 + 3\vartheta)) \\
&-b^2xy(3 + 6\vartheta + 6x^2y^2(-7 + 2\vartheta) + x^4y^4(-17 + 6\vartheta)))/(8(-1 + b^2)xy\vartheta^7)\Big]
\end{aligned}$$

(S3f)

$$6\pi\eta R^2 M_{kl}^{UL,c,ij} = M_1^{UL,c,ij}\hat{y}_k(\hat{y}_n\epsilon_{nlr}\hat{y}_r^\perp) + M_2^{UL,c,ij}\hat{y}_k^\perp(\hat{y}_n\epsilon_{nlr}\hat{y}_r^\perp)$$
$$+ M_3^{UL,c,ij}\epsilon_{klr}\hat{y}_r + M_4^{UL,c,ij}\epsilon_{klr}\hat{y}_r^\perp \tag{S4a}$$

$$M_1^{UL,c,ij} = \Big[(9(-1+\mathrm{x}^2))/(8\sqrt{1-b^2}\mathrm{x}) + (9(-1+\mathrm{x}^2)(-1+\mathrm{x}^2\mathrm{y}^2 + 8b^3\mathrm{x}^3\mathrm{y}^3 - b^2\mathrm{x}^2\mathrm{y}^2$$
$$(11+5\mathrm{x}^2\mathrm{y}^2) + b\mathrm{xy}(5+2\mathrm{x}^2\mathrm{y}^2 + \mathrm{x}^4\mathrm{y}^4)))/(8\sqrt{1-b^2}\mathrm{x}\vartheta^5)\Big]$$
$$+\lambda_{c_i}^2\Big[(3/(8\sqrt{1-b^2}\mathrm{x}) - (3(1 - 14\mathrm{x}^2\mathrm{y}^2 + 5\mathrm{x}^4\mathrm{y}^4 + 32b^4\mathrm{x}^4\mathrm{y}^4$$
$$-8b^3\mathrm{x}^3\mathrm{y}^3(7+2\mathrm{x}^2\mathrm{y}^2) + b^2\mathrm{x}^2\mathrm{y}^2(35 - 2\mathrm{x}^2\mathrm{y}^2 + 7\mathrm{x}^4\mathrm{y}^4)$$
$$-b\mathrm{xy}(7 - 21\mathrm{x}^2\mathrm{y}^2 + 5\mathrm{x}^4\mathrm{y}^4 + \mathrm{x}^6\mathrm{y}^6)))/(8\sqrt{1-b^2}\mathrm{x}\vartheta^7))\Big] \tag{S4b}$$

$$M_2^{UL,c,ij} = \Big[(9b(-1+\mathrm{x}^2))/(4(-1+b^2)\mathrm{x}) + (9(-1+\mathrm{x}^2)(8b^4\mathrm{x}^3\mathrm{y}^3 - 5b^3\mathrm{x}^2\mathrm{y}^2(3+\mathrm{x}^2\mathrm{y}^2)$$
$$+\mathrm{x}^3\mathrm{y}^3(3+\mathrm{x}^2\mathrm{y}^2) + b^2\mathrm{xy}(10+9\mathrm{x}^2\mathrm{y}^2 + \mathrm{x}^4\mathrm{y}^4) - b(2+5\mathrm{x}^2\mathrm{y}^2$$
$$+5\mathrm{x}^4\mathrm{y}^4)))/(8(-1+b^2)\mathrm{x}\vartheta^5)\Big] + \lambda_{c_i}^2\Big[((3b)/(4(-1+b^2)\mathrm{x}) - (3(32b^5\mathrm{x}^4\mathrm{y}^4$$
$$-8b^4\mathrm{x}^3\mathrm{y}^3(7+2\mathrm{x}^2\mathrm{y}^2) - \mathrm{x}^3\mathrm{y}^3(21+2\mathrm{x}^2\mathrm{y}^2 + \mathrm{x}^4\mathrm{y}^4) + b^3\mathrm{x}^2\mathrm{y}^2(35+6\mathrm{x}^2\mathrm{y}^2$$
$$+7\mathrm{x}^4\mathrm{y}^4) - b^2\mathrm{xy}(14 - 7\mathrm{x}^2\mathrm{y}^2 + 24\mathrm{x}^4\mathrm{y}^4 + \mathrm{x}^6\mathrm{y}^6) + b(2+7\mathrm{x}^2\mathrm{y}^2$$
$$+32\mathrm{x}^4\mathrm{y}^4 + 7\mathrm{x}^6\mathrm{y}^6)))/(8(-1+b^2)\mathrm{x}\vartheta^7))\Big] \tag{S4c}$$

$$M_3^{UL,c,ij} = \Big[(9b(-1+\mathrm{x}^2))/(8(-1+b^2)\mathrm{x}) - (3(2b^3\mathrm{x}^2 + b^2(4\mathrm{xy} - 6\mathrm{x}^3\mathrm{y}) + b(-3+9\mathrm{x}^4\mathrm{y}^2$$
$$+\mathrm{x}^2(1-9\mathrm{y}^2)) + \mathrm{xy}(5 - 3\mathrm{x}^4\mathrm{y}^2 + 3\mathrm{x}^2(-1+\mathrm{y}^2))))/(8(-1+b^2)\mathrm{x}\vartheta^3)\Big]$$
$$+\lambda_{c_i}^2\Big[((3b)/(8(-1+b^2)\mathrm{x}) + (3(8b^2\mathrm{x}^3\mathrm{y}^3 + \mathrm{xy}(5+2\mathrm{x}^2\mathrm{y}^2 + \mathrm{x}^4\mathrm{y}^4)$$
$$-b(1+10\mathrm{x}^2\mathrm{y}^2 + 5\mathrm{x}^4\mathrm{y}^4)))/(8(-1+b^2)\mathrm{x}\vartheta^5))\Big] \tag{S4d}$$

$$M_4^{UL,c,ij} = \Big[-3\sqrt{1-b^2}\mathrm{x}/(4\vartheta^3)\Big] \tag{S4e}$$

$$6\pi\eta R^2 M_{klm}^{US,c,ij} = M_1^{US,c,ij}\hat{y}_k(\hat{y}_l\hat{y}_m - \tfrac{1}{3}\delta_{lm}) + M_2^{US,c,ij}\hat{y}_k^\perp(\hat{y}_l\hat{y}_m - \tfrac{1}{3}\delta_{lm})$$
$$+ M_3^{US,c,ij}\hat{y}_k(\hat{y}_l^\perp\hat{y}_m^\perp - \tfrac{1}{3}\delta_{lm}) + M_4^{US,c,ij}\hat{y}_k^\perp(\hat{y}_l^\perp\hat{y}_m^\perp - \tfrac{1}{3}\delta_{lm})$$
$$+ M_5^{US,c,ij}(\hat{y}_k\hat{y}_l\hat{y}_m^\perp + \hat{y}_k\hat{y}_l^\perp\hat{y}_m) + M_6^{US,c,ij}(\hat{y}_k^\perp\hat{y}_l\hat{y}_m^\perp + \hat{y}_k\hat{y}_l^\perp\hat{y}_m)$$
$$+ M_7^{US,c,ij}(\delta_{kl}\hat{y}_m + \delta_{km}\hat{y}_l - \tfrac{2}{3}\delta_{ml}\hat{y}_k)$$
$$+ M_8^{US,c,ij}(\delta_{kl}\hat{y}_m^\perp + \delta_{km}\hat{y}_l^\perp - \tfrac{2}{3}\delta_{ml}\hat{y}_k^\perp) \tag{S5a}$$

$$M_1^{US,c,ij} = \Big[(9(4b(-1+\mathrm{x}^2) - 28b^2\mathrm{x}(-1+\mathrm{x}^2)\mathrm{y} - 2b(-1+(8+35b^2)\mathrm{x}^2 + (-7-36b^2$$
$$+b^4)\mathrm{x}^4)\mathrm{y}^2 + 2b^2\mathrm{x}(-7+7(6+5b^2)\mathrm{x}^2 + 2(-19-17b^2 + b^4)\mathrm{x}^4)\mathrm{y}^3 + b\mathrm{x}^2(6+36b^2$$
$$-(30+127b^2 + 25b^4)\mathrm{x}^2 + (30+11b^2(9+b^2))\mathrm{x}^4)\mathrm{y}^4 + \mathrm{x}^3(2 - 36(b^2+b^4) + (1$$
$$+b^2)(4+73b^2)\mathrm{x}^2 - (8+19b^2(3+b^2))\mathrm{x}^4)\mathrm{y}^5 + b\mathrm{x}^4(10+47b^2 + 13b^4 - (21+64b^2$$
$$+13b^4)\mathrm{x}^2 + 7(3+b^2)\mathrm{x}^4)\mathrm{y}^6 - \mathrm{x}^5(4 - 5\mathrm{x}^2 + 3\mathrm{x}^4 - 19b^4(-1+\mathrm{x}^2) + b^2(19 - 22\mathrm{x}^2$$
$$+\mathrm{x}^4))\mathrm{y}^7 - 7b(1+b^2)\mathrm{x}^6(-1+\mathrm{x}^2)\mathrm{y}^8 + (1+b^2)\mathrm{x}^7(-1+\mathrm{x}^2)\mathrm{y}^9))/(8(-1+b^2)\mathrm{xy}^2\vartheta^7)$$
$$-(9b(-1+\mathrm{x}^2)(-2+\mathrm{y}^2)(-1+\mathrm{xy}(2b-\mathrm{xy}))^3)/(4(-1+b^2)\mathrm{xy}^2\vartheta^6)\Big]$$

$$+\lambda_{c_i}^2 \Big[3(4b^6\text{x}^5\text{y}^5(-18+13\text{y}^2) + b^5\text{x}^4\text{y}^4(187 + (-117+97\text{x}^2)\text{y}^2 - 87\text{x}^2\text{y}^4)$$

$$+b^4\text{x}^3\text{y}^3(-198 - 3(-36+73\text{x}^2)\text{y}^2 + (145\text{x}^2 - 27\text{x}^4)\text{y}^4 + 27\text{x}^4\text{y}^6) + b^3\text{x}^2\text{y}^2(126$$

$$+(-68+203\text{x}^2)\text{y}^2 + 3\text{x}^2(-33+34\text{x}^2)\text{y}^4 + \text{x}^4(-8+9\text{x}^2)\text{y}^6 - 9\text{x}^6\text{y}^8)$$

$$+b^2\text{xy}(-36 - 6(-3+23\text{x}^2)\text{y}^2 + (74\text{x}^2-135\text{x}^4)\text{y}^4 - \text{x}^4(3+106\text{x}^2)\text{y}^6 - \text{x}^6(-42$$

$$+\text{x}^2)\text{y}^8 + \text{x}^8\text{y}^{10}) + \text{x}^3\text{y}^5(-14 + \text{x}^6\text{y}^4(-3+\text{y}^2) + \text{x}^4\text{y}^2(-11+3\text{y}^2)$$

$$+\text{x}^2(-78+58\text{y}^2)) + b(4 + 2\text{x}^2(-2+57\text{x}^2)\text{y}^4 + \text{x}^4(-36+137\text{x}^2)\text{y}^6 + \text{x}^6(-73$$

$$+27\text{x}^2)\text{y}^8 - 9\text{x}^8\text{y}^{10} - 4\vartheta^9 + 2\text{y}^2(-1+9\text{x}^2+\vartheta^9)))/(8(-1+b^2)\text{xy}^2\vartheta^9)\Big]$$

$$+\lambda_{c_j}^2 \Big[9(4b^6\text{x}^5\text{y}^3(-5+18(-1+\text{x}^2)\text{y}^2) + b^5\text{x}^4\text{y}^2(10 + (187-117\text{x}^2)\text{y}^2$$

$$-97\text{x}^2(-1+\text{x}^2)\text{y}^4) + \text{x}^5\text{y}^5(-78 + 3\text{x}^6\text{y}^4 - 11\text{x}^2(-8+\text{y}^2) - 3\text{x}^4\text{y}^2(-7+\text{y}^2))$$

$$+b^4\text{x}^3\text{y}^3(-198 + 27\text{x}^6\text{y}^4 + \text{x}^2(188-219\text{y}^2) - 3\text{x}^4\text{y}^2(-43+9\text{y}^2)) + b^3\text{x}^2\text{y}^2(126$$

$$-9\text{x}^8\text{y}^6 + \text{x}^6\text{y}^4(-52+9\text{y}^2) + 3\text{x}^4\text{y}^2(-81+34\text{y}^2) + \text{x}^2(-136+203\text{y}^2))$$

$$+b^2\text{xy}(-36 + \text{x}^{10}\text{y}^8 - \text{x}^8\text{y}^6(-96+\text{y}^2) - 6\text{x}^2(-6+23\text{y}^2) - 3\text{x}^4\text{y}^2(-56$$

$$+45\text{y}^2) + \text{x}^6(215\text{y}^4 - 106\text{y}^6)) + b(4 - 27\text{x}^{10}\text{y}^8 + 6\text{x}^4\text{y}^2(-3+19\text{y}^2) + \text{x}^8\text{y}^6(-187$$

$$+27\text{y}^2) + \text{x}^6\text{y}^4(-144+137\text{y}^2) - 4\vartheta^9 + 2\text{x}^2(-2+9\text{y}^2+2\vartheta^9)))$$

$$/(40(-1+b^2)\text{xy}^2\vartheta^9)\Big] + \lambda_{c_j}^2\lambda_{c_i}^2 \Big[((3b)/(10(-1+b^2)\text{xy}^2) - (3(288b^7\text{x}^6\text{y}^6$$

$$-8b^6\text{x}^5\text{y}^5(99+34\text{x}^2\text{y}^2) - 3\text{x}^5\text{y}^5(286 - 127\text{x}^2\text{y}^2 + 8\text{x}^4\text{y}^4 + \text{x}^6\text{y}^6)$$

$$-3b^4\text{x}^3\text{y}^3(126 + 77\text{x}^2\text{y}^2 - 104\text{x}^4\text{y}^4 + 21\text{x}^6\text{y}^6) + b^5(891\text{x}^4\text{y}^4$$

$$+454\text{x}^6\text{y}^6 - 161\text{x}^8\text{y}^8) + b^3\text{x}^2\text{y}^2(198 - 363\text{x}^2\text{y}^2 + 39\text{x}^4\text{y}^4 + 915\text{x}^6\text{y}^6$$

$$+11\text{x}^8\text{y}^8) - b^2\text{xy}(44 + 282\text{x}^2\text{y}^2 - 33\text{x}^4\text{y}^4 + 1741\text{x}^6\text{y}^6 + 133\text{x}^8\text{y}^8$$

$$+\text{x}^{10}\text{y}^{10}) + b(4 + 22\text{x}^2\text{y}^2 + 792\text{x}^4\text{y}^4 + 1067\text{x}^6\text{y}^6 - 94\text{x}^8\text{y}^8$$

$$+33\text{x}^{10}\text{y}^{10})))/(40(-1+b^2)\text{xy}^2\vartheta^{11}))\Big] \tag{S5b}$$

$$M_2^{US,c,ij} = \Big[(-9(-2b^2(-1+\text{x}^2) + 14b^3\text{x}(-1+\text{x}^2)\text{y} + b^2(-2 + (9+35b^2)\text{x}^2 + (-5-39b^2$$

$$+2b^4)\text{x}^4)\text{y}^2 + b\text{x}(14b^2 - (8+33b^2+43b^4)\text{x}^2 + (4+23b^2+47b^4-4b^6)\text{x}^4)\text{y}^3$$

$$-\text{x}^2(-2+11b^2+33b^4 - (5+18b^2+68b^4+21b^6)\text{x}^2 + (5+b^2+53b^4+11b^6)\text{x}^4)\text{y}^4$$

$$+b\text{x}^3(35(b^2+b^4) - (13+37b^2+62b^4)\text{x}^2 + (5+18b^2+19b^4)\text{x}^4)\text{y}^5 - \text{x}^4(3-5\text{x}^2$$

$$-13b^6(-1+\text{x}^2) + b^2(7-10\text{x}^2+7\text{x}^4) + b^4(47+7\text{x}^2(-8+\text{x}^2)))\text{y}^6 + b\text{x}^5(-1+\text{x}^2)$$

$$(-5-19b^4+\text{x}^2+b^2(-18+\text{x}^2))\text{y}^7 + 7b^2(1+b^2)\text{x}^6(-1+\text{x}^2)\text{y}^8 - b(1+b^2)\text{x}^7$$

$$(-1+\text{x}^2)\text{y}^9))/(8(1-b^2)^{3/2}\text{xy}^2\vartheta^7) - (9b^2(-1+\text{x}^2)(-1+\text{y}^2)(-1+\text{xy}(2b-\text{xy}))^3)$$

$$/(4(1-b^2)^{3/2}\text{xy}^2\vartheta^6)\Big] + \lambda_{c_i}^2 \Big[3(4b^7\text{x}^5\text{y}^5(-18+13\text{y}^2) + b^6\text{x}^4\text{y}^4(167 +$$

$$(-117+97\text{x}^2)\text{y}^2 - 87\text{x}^2\text{y}^4) + b^5\text{x}^3\text{y}^3(-153 + (113-132\text{x}^2)\text{y}^2 + (152\text{x}^2$$

$$-27\text{x}^4)\text{y}^4 + 27\text{x}^4\text{y}^6) + b^4\text{x}^2\text{y}^2(63 + (-53+36\text{x}^2)\text{y}^2 - 2\text{x}^2(63$$

$$+10\text{x}^2)\text{y}^4 + 9\text{x}^6\text{y}^6 - 9\text{x}^6\text{y}^8) + b\text{x}^3\text{y}^3(-48 + (8-87\text{x}^2)\text{y}^2$$

$$+67\text{x}^2\text{y}^4 - \text{x}^6\text{y}^6 + \text{x}^6\text{y}^8) + b^3\text{xy}(-18 + 3(6+11\text{x}^2)\text{y}^2 + \text{x}^2(47+39\text{x}^2)\text{y}^4$$

$$-\text{x}^4(19+45\text{x}^2)\text{y}^6 - \text{x}^6(-45+\text{x}^2)\text{y}^8 + \text{x}^8\text{y}^{10}) + 5\text{x}^2\text{y}^4(2 + \text{x}^2(11-9\text{y}^2)$$

$$+3\text{x}^4\text{y}^2(-1+\text{y}^2)) + b^2(2 - \text{x}^2(29+6\text{x}^2)\text{y}^4 + 2\text{x}^4(18+53\text{x}^2)\text{y}^6 + (-96\text{x}^6$$

$$+9\text{x}^8)\text{y}^8 - 9\text{x}^8\text{y}^{10} - 2\vartheta^9 + \text{y}^2(-2+9\text{x}^2+2\vartheta^9)))/(8(1-b^2)^{3/2}\text{xy}^2\vartheta^9)\Big]$$

$$+\lambda_{c_j}^2 \Big[9(4b^7\text{x}^5\text{y}^3(-5+18(-1+\text{x}^2)\text{y}^2) + b^6\text{x}^4\text{y}^2(10 + (167-117\text{x}^2)\text{y}^2 - 97\text{x}^2(-1$$

$$+\text{x}^2)\text{y}^4) + b\text{x}^3\text{y}^3(-48 + 47\text{x}^4\text{y}^2 - \text{x}^6\text{y}^6 + \text{x}^8\text{y}^6 + \text{x}^2(28-87\text{y}^2)) + 5\text{x}^4\text{y}^4(11$$

$$+5\text{x}^4\text{y}^2 - 3\text{x}^2(3+\text{y}^2)) + b^5\text{x}^3\text{y}^3(-153 + 27\text{x}^6\text{y}^4 + \text{x}^2(173-132\text{y}^2)$$

$$+\text{x}^4(92\text{y}^2 - 27\text{y}^4)) + b^3\text{xy}(-18 + \text{x}^{10}\text{y}^8 - \text{x}^8\text{y}^6(-45+\text{y}^2) + 13\text{x}^4\text{y}^2(-1+3\text{y}^2)$$

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$$+3x^2(6+11y^2)+x^6(41y^4-45y^6))+b^4x^2y^2(63-9x^8y^6-2x^4y^2(63+10y^2)$$
$$+x^2(-83+36y^2)+x^6(30y^4+9y^6))+b^2(2-9x^{10}y^8+9x^8y^6(-14$$
$$+y^2)+2x^6y^4(18+53y^2)+x^4(y^2-6y^4)-2\vartheta^9+x^2(-2+9y^2$$
$$+2\vartheta^9)))/(40(1-b^2)^{3/2}xy^2\vartheta^9)\Big]+\lambda_{c_j}^2\lambda_{c_i}^2\Big[((3b^2)/(20(1-b^2)^{3/2}xy^2)$$
$$-(3(288b^8x^6y^6-8b^7x^5y^5(99+34x^2y^2)+15x^4y^4(33-46x^2y^2$$
$$+5x^4y^4)-7b^5x^3y^3(81-33x^2y^2-97x^4y^4+9x^6y^6)$$
$$+b^6(891x^4y^4+286x^6y^6-161x^8y^8)-bx^3y^3(336+429x^2y^2$$
$$-729x^4y^4+19x^6y^6+x^8y^8)+b^4x^2y^2(99-363x^2y^2-740x^4y^4+793x^6y^6+11x^8y^8)$$
$$-b^3xy(22-573x^2y^2-66x^4y^4+1796x^6y^6+28x^8y^8+x^{10}y^{10})$$
$$+b^2(2+11x^2y^2-363x^4y^4+1780x^6y^6-377x^8y^8+11x^{10}y^{10})))$$
$$/(40(1-b^2)^{3/2}xy^2\vartheta^{11}))\Big]\tag{S5c}$$

$$M_3^{US,c,ij}=\Big[(9(4b(-1+x^2)-28b^2x(-1+x^2)y+2b(2-(9+35b^2)x^2+(8+33b^2+b^4)x^4)y^2$$
$$-2x(14b^2-(4+41b^2+39b^4)x^2+(5+27b^2+36b^4+2b^6)x^4)y^3+bx^2(14+70b^2$$
$$-(45+148b^2+31b^4)x^2+(37+66b^2+37b^4)x^4)y^4-x^3(10(1+5b^2+8b^4)-(15$$
$$+96b^2+113b^4)x^2+7(1+6b^2+5b^4)x^4)y^5-7b(1+b^2)x^4(5+5b^2-2x^2)(-1+x^2)y^6$$
$$+(1+b^2)x^5(7+35b^2-2x^2)(-1+x^2)y^7-14b(1+b^2)x^6(-1+x^2)y^8+2(1+b^2)x^7$$
$$(-1+x^2)y^9))/(8(-1+b^2)xy^2\vartheta^7)-(9b(-1+x^2)(-1+y^2)(-1+xy(2b$$
$$-xy))^3)/(2(-1+b^2)xy^2\vartheta^6)\Big]+\lambda_{c_i}^2\Big[3(20b^6x^5y^5(-6+7y^2)-5b^5x^4y^4(-19+21y^2)$$
$$(3+x^2y^2)+b^4x^3y^3(-270-5(-56+27x^2)y^2-21x^2(-5+3x^2)y^4$$
$$+63x^4y^6)+x^3y^3(-60+(70+3x^2)y^2+(7x^2-9x^4)y^4+(9x^4-2x^6)y^6+2x^6y^8)$$
$$-2b^3x^2y^2(-63+(63-30x^2)y^2-94x^4y^4+(84x^4-9x^6)y^6+9x^6y^8)$$
$$+2b^2xy(-18-3(-6+x^2)y^2-7(x^2+18x^4)y^4-18x^4(-7+2x^2)y^6-x^6(-36$$
$$+x^2)y^8+x^8y^{10})+b(4+3x^2(-6+53x^2)y^4+x^4(-189+53x^2)y^6+9x^6(-7$$
$$+2x^2)y^8-18x^8y^{10}-4\vartheta^9+2y^2(-2+9x^2+2\vartheta^9)))/(8(-1+b^2)xy^2\vartheta^9)\Big]$$
$$+\lambda_{c_j}^2\Big[9(20b^6x^5y^3(1+6(-1+x^2)y^2)-5b^5x^4y^2(2+(-57+63x^2)y^2$$
$$+19x^2(-1+x^2)y^4)+2b^2xy(-18+x^{10}y^8-x^8y^6(-36+y^2)-3x^2(-6+y^2)$$
$$-4x^6y^4(-29+9y^2)+x^4(3y^2-126y^4))+b^4x^3y^3(-270+63x^6y^4$$
$$-15x^2(-16+9y^2)+x^4(145y^2-63y^4))-2b^3x^2y^2(-63-94x^4y^4$$
$$+9x^8y^6+x^2(53-30y^2)+x^6(94y^4-9y^6))+x^3y^3(-60+2x^8y^6$$
$$+x^2(70+3y^2)+x^4(7y^2-9y^4)+x^6(9y^4-2y^6))+b(4-18x^{10}y^8$$
$$+x^8y^6(-53+18y^2)+x^6y^4(-189+53y^2)+x^4y^2(-28+159y^2)-4\vartheta^9+2x^2(-2$$
$$+9y^2+2\vartheta^9)))/(40(-1+b^2)xy^2\vartheta^9)\Big]+\lambda_{c_j}^2\lambda_{c_i}^2\Big[((3b)/(10(-1+b^2)xy^2)$$
$$-(3(480b^7x^6y^6+40b^6x^5y^5(-33+2x^2y^2)+b^5x^4y^4(1485-790x^2y^2+281x^4y^4)$$
$$-b^4x^3y^3(882-1485x^2y^2+1504x^4y^4+99x^6y^6)-x^3y^3(420-693x^2y^2$$
$$+156x^4y^4+11x^6y^6+2x^8y^8)+2b^3x^2y^2(99-561x^2y^2+1456x^4y^4$$
$$+115x^6y^6+11x^8y^8)-2b^2xy(22-321x^2y^2+1353x^4y^4-130x^6y^6$$
$$+55x^8y^8+x^{10}y^{10})+b(4+22x^2y^2+957x^4y^4-754x^6y^6$$
$$+149x^8y^8+22x^{10}y^{10})))/(40(-1+b^2)xy^2\vartheta^{11}))\Big]\tag{S5d}$$

$$M_4^{US,c,ij}=\Big[(-9(-2(1+3b^2)(-1+x^2)+14b(1+3b^2)x(-1+x^2)y-(2+6b^2$$

$$-(9+62b^2+105b^4)\mathrm{x}^2+(5+62b^2+99b^4+2b^6)\mathrm{x}^4)\mathrm{y}^2+b\mathrm{x}(14+42b^2-7(7$$
$$+26b^2+15b^4)\mathrm{x}^2+(31+152b^2+93b^4+4b^6)\mathrm{x}^4)\mathrm{y}^3+\mathrm{x}^2(-7(1+8b^2+15b^4)$$
$$+7(1+23b^2+35b^4+5b^6)\mathrm{x}^2-(-2+111b^2+134b^4+37b^6)\mathrm{x}^4)\mathrm{y}^4+7b\mathrm{x}^3(-1$$
$$+\mathrm{x}^2)(-5+3\mathrm{x}^2+5b^4(-3+\mathrm{x}^2)+4b^2(-5+4\mathrm{x}^2))\mathrm{y}^5+7b^2(3+b^2)\mathrm{x}^4(5+5b^2$$
$$-2\mathrm{x}^2)(-1+\mathrm{x}^2)\mathrm{y}^6-b(3+b^2)\mathrm{x}^5(7+35b^2-2\mathrm{x}^2)(-1+\mathrm{x}^2)\mathrm{y}^7+14b^2(3+b^2)\mathrm{x}^6$$
$$(-1+\mathrm{x}^2)\mathrm{y}^8-2b(3+b^2)\mathrm{x}^7(-1+\mathrm{x}^2)\mathrm{y}^9))/(8(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^7)$$
$$-(9(1+3b^2)(-1+\mathrm{x}^2)(-1+\mathrm{y}^2)(-1+\mathrm{xy}(2b-\mathrm{xy}))^3)/(4(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^6)\Big]$$
$$+\lambda_{c_i}^2\Big[3(2-\mathrm{x}^2(9+53\mathrm{x}^2)\mathrm{y}^4+\mathrm{x}^4(63+10\mathrm{x}^2)\mathrm{y}^6+20b^7\mathrm{x}^5\mathrm{y}^5(-6+7\mathrm{y}^2)$$
$$-5b^6\mathrm{x}^4\mathrm{y}^4(-61+(63-19\mathrm{x}^2)\mathrm{y}^2+21\mathrm{x}^2\mathrm{y}^4)+3b^5\mathrm{x}^3\mathrm{y}^3(-105$$
$$-15(-7+6\mathrm{x}^2)\mathrm{y}^2+(70\mathrm{x}^2-21\mathrm{x}^4)\mathrm{y}^4+21\mathrm{x}^4\mathrm{y}^6)-3b^4\mathrm{x}^2\mathrm{y}^2$$
$$(-63+(63-115\mathrm{x}^2)\mathrm{y}^2+(105\mathrm{x}^2-136\mathrm{x}^4)\mathrm{y}^4-6\mathrm{x}^4(-21+\mathrm{x}^2)\mathrm{y}^6$$
$$+6\mathrm{x}^6\mathrm{y}^8)+2b^3\mathrm{xy}(-27-3(-9+49\mathrm{x}^2)\mathrm{y}^2+(147\mathrm{x}^2-306\mathrm{x}^4)\mathrm{y}^4$$
$$+(336\mathrm{x}^4-99\mathrm{x}^6)\mathrm{y}^6-\mathrm{x}^6(-99+\mathrm{x}^2)\mathrm{y}^8+\mathrm{x}^8\mathrm{y}^{10})+b\mathrm{xy}$$
$$(-18-9(-2+7\mathrm{x}^2)\mathrm{y}^2+(63\mathrm{x}^2-6\mathrm{x}^4)\mathrm{y}^4-\mathrm{x}^4(14+27\mathrm{x}^2)\mathrm{y}^6+(27\mathrm{x}^6-6\mathrm{x}^8)\mathrm{y}^8$$
$$+6\mathrm{x}^8\mathrm{y}^{10})-2\vartheta^9+\mathrm{y}^2(-2+9\mathrm{x}^2+2\vartheta^9)-3b^2((30\mathrm{x}^2-137\mathrm{x}^4)\mathrm{y}^4$$
$$+(147\mathrm{x}^4-53\mathrm{x}^6)\mathrm{y}^6-9\mathrm{x}^6(-7+2\mathrm{x}^2)\mathrm{y}^8+18\mathrm{x}^8\mathrm{y}^{10}+2(-1+\vartheta^9)$$
$$-2\mathrm{y}^2(-1+15\mathrm{x}^2+\vartheta^9)))/(8(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^9)\Big]+\lambda_{c_j}^2\Big[9(2-10\mathrm{x}^8\mathrm{y}^6$$
$$+\mathrm{x}^6\mathrm{y}^4(63+10\mathrm{y}^2)+20b^7\mathrm{x}^5\mathrm{y}^3(1+6(-1+\mathrm{x}^2)\mathrm{y}^2)+\mathrm{x}^4(\mathrm{y}^2-53\mathrm{y}^4)$$
$$-5b^6\mathrm{x}^4\mathrm{y}^2(2+(-61+63\mathrm{x}^2)\mathrm{y}^2+19\mathrm{x}^2(-1+\mathrm{x}^2)\mathrm{y}^4)+3b^5\mathrm{x}^3\mathrm{y}^3(-105$$
$$+21\mathrm{x}^6\mathrm{y}^4+\mathrm{x}^2(85-90\mathrm{y}^2)+\mathrm{x}^4(90\mathrm{y}^2-21\mathrm{y}^4))+2b^3\mathrm{xy}(-27+$$
$$\mathrm{x}^{10}\mathrm{y}^8-\mathrm{x}^8\mathrm{y}^6(-99+\mathrm{y}^2)-3\mathrm{x}^2(-9+49\mathrm{y}^2)+\mathrm{x}^4(177\mathrm{y}^2-306\mathrm{y}^4)$$
$$+\mathrm{x}^6(306\mathrm{y}^4-99\mathrm{y}^6))-3b^4\mathrm{x}^2\mathrm{y}^2(-63+6\mathrm{x}^8\mathrm{y}^6+\mathrm{x}^2(53-115\mathrm{y}^2)$$
$$+\mathrm{x}^4(105\mathrm{y}^2-136\mathrm{y}^4)+\mathrm{x}^6(136\mathrm{y}^4-6\mathrm{y}^6))+b\mathrm{xy}(-18+6\mathrm{x}^{10}\mathrm{y}^8$$
$$-9\mathrm{x}^2(-2+7\mathrm{y}^2)+\mathrm{x}^4(43\mathrm{y}^2-6\mathrm{y}^4)+\mathrm{x}^6(6\mathrm{y}^4-27\mathrm{y}^6)+\mathrm{x}^8(27\mathrm{y}^6$$
$$-6\mathrm{y}^8))-2\vartheta^9+\mathrm{x}^2(-2+9\mathrm{y}^2+2\vartheta^9)-3b^2(18\mathrm{x}^{10}\mathrm{y}^8+\mathrm{x}^4(40\mathrm{y}^2-137\mathrm{y}^4)+$$
$$\mathrm{x}^6(147\mathrm{y}^4-53\mathrm{y}^6)+\mathrm{x}^8(53\mathrm{y}^6-18\mathrm{y}^8)+2(-1+\vartheta^9)-2\mathrm{x}^2(-1+15\mathrm{y}^2+\vartheta^9)))$$
$$/(40(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^9)\Big]+\lambda_{c_j}^2\lambda_{c_i}^2\Big[((3(1+3b^2))/(20(1-b^2)^{3/2}\mathrm{xy}^2)$$
$$-(3(2+11\mathrm{x}^2\mathrm{y}^2-594\mathrm{x}^4\mathrm{y}^4+607\mathrm{x}^6\mathrm{y}^6+480b^8\mathrm{x}^6\mathrm{y}^6-50\mathrm{x}^8\mathrm{y}^8$$
$$+40b^7\mathrm{x}^5\mathrm{y}^5(-33+2\mathrm{x}^2\mathrm{y}^2)+b^6\mathrm{x}^4\mathrm{y}^4(1485-430\mathrm{x}^2\mathrm{y}^2+281\mathrm{x}^4\mathrm{y}^4)$$
$$-3b^5\mathrm{x}^3\mathrm{y}^3(231-165\mathrm{x}^2\mathrm{y}^2+773\mathrm{x}^4\mathrm{y}^4+33\mathrm{x}^6\mathrm{y}^6)+b^4\mathrm{x}^2\mathrm{y}^2(297$$
$$-528\mathrm{x}^2\mathrm{y}^2+4647\mathrm{x}^4\mathrm{y}^4+642\mathrm{x}^6\mathrm{y}^6+22\mathrm{x}^8\mathrm{y}^8)-2b^3\mathrm{xy}(33+264\mathrm{x}^2\mathrm{y}^2$$
$$+1749\mathrm{x}^4\mathrm{y}^4-21\mathrm{x}^6\mathrm{y}^6+154\mathrm{x}^8\mathrm{y}^8+\mathrm{x}^{10}\mathrm{y}^{10})-b\mathrm{xy}(22+99\mathrm{x}^2\mathrm{y}^2-627\mathrm{x}^4\mathrm{y}^4$$
$$+443\mathrm{x}^6\mathrm{y}^6+33\mathrm{x}^8\mathrm{y}^8+6\mathrm{x}^{10}\mathrm{y}^{10})+3b^2(2+44\mathrm{x}^2\mathrm{y}^2+759\mathrm{x}^4\mathrm{y}^4-536\mathrm{x}^6\mathrm{y}^6$$
$$+149\mathrm{x}^8\mathrm{y}^8+22\mathrm{x}^{10}\mathrm{y}^{10})))/(40(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^{11}))\Big]$$
(S5e)

$$M_5^{US,c,ij}=\Big[(-9(2-2\mathrm{x}^2+14b\mathrm{x}(-1+\mathrm{x}^2)\mathrm{y}+(-1-b^2+4(2+9b^2)\mathrm{x}^2+(-7-33b^2-4b^4$$
$$+2b^6)\mathrm{x}^4)\mathrm{y}^2+b\mathrm{x}(7(1+b^2)+2(-19-25b^2+2b^4)\mathrm{x}^2+(27+47b^2-4b^6)\mathrm{x}^4)\mathrm{y}^3$$
$$+\mathrm{x}^2(-5+7\mathrm{x}^2+b^2(-18-19b^2-3(-27-9b^2+b^4)\mathrm{x}^2+(-57+13b^2(-2+b^2))\mathrm{x}^4))\mathrm{y}^4$$
$$+2b\mathrm{x}^3(9+17b^2+9b^4-2(9+17b^2+2b^4)\mathrm{x}^2+(5+25b^2-9b^4)\mathrm{x}^4)\mathrm{y}^5+\mathrm{x}^4(1+\mathrm{x}^2$$
$$+b^2(-42-24b^2-5b^4+(59+19b^2+5b^4)\mathrm{x}^2+7(-3+b^2)\mathrm{x}^4))\mathrm{y}^6-b\mathrm{x}^5(-1+\mathrm{x}^2)$$
$$(7-3\mathrm{x}^2+b^2(35+\mathrm{x}^2))\mathrm{y}^7+14b^2\mathrm{x}^6(-1+\mathrm{x}^2)\mathrm{y}^8-2b\mathrm{x}^7(-1+\mathrm{x}^2)\mathrm{y}^9))$$
$$/(8(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^7)+(9(-1+\mathrm{x}^2)(-2+(1+b^2)\mathrm{y}^2)(1+\mathrm{xy}(-2b+\mathrm{xy}))^3)$$

$$/(8(1-b^2)^{3/2}xy^2\vartheta^6)\Big] + \lambda_{c_i}^2\Big[3(2-x^2(12+53x^2)y^4+2x^4(27$$
$$+5x^2)y^6-5x^6y^8+4b^7x^5y^5(6+5y^2)+b^6x^4y^4(-49-(45+79x^2)y^2$$
$$+25x^2y^4)+2b^5x^3y^3(18+(25+27x^2)y^2+15x^2(-1+x^2)y^4)+bxy(-18$$
$$+(9-27x^2)y^2+x^2(29+39x^2)y^4-x^4(53+15x^2)y^6-3x^6(-3+x^2)y^8$$
$$+2x^8y^{10})-b^4x^2y^4(39+9x^6y^4+9x^2(-12+5y^2)+x^4y^2(-189$$
$$+160y^2))+b^3xy^3(9+x^8y^6+9x^4y^2(-41+35y^2)+x^2(-177+89y^2)+x^6(-87y^4+63y^6))-2\vartheta^9$$
$$+y^2(-1+9x^2+\vartheta^9)+b^2y^2(-1-21x^2(-3+y^2)-9x^8y^6(-3+2y^2)$$
$$-6x^4y^2(-41+36y^2)+x^6(48y^4-28y^6)+\vartheta^9))/(8(1-b^2)^{3/2}xy^2\vartheta^9)\Big]$$

$$-\lambda_{c_j}^2\Big[9(18bxy-9b^2(-3+b^2)x^{10}y^8+b(-3+b^2)x^{11}y^9-3bx^3y(6$$
$$+(-9-59b^2+12b^4)y^2)+bx^9y^7(30b^4+3(-5+y^2)-b^2(87+y^2))+b^2x^8y^6(68-79b^4$$
$$-27y^2+b^2(179+9y^2))-bx^5y^3(7+39y^2+b^2(197-369y^2)+4b^6(-5+6y^2)+2b^4(-8$$
$$+27y^2))+x^4y^2(9+53y^2+b^2(53-246y^2)-4b^4(-5+27y^2)+b^6(-10+49y^2))$$
$$+x^6y^4(-63-10y^2-24b^2(-9+2y^2)-9b^4(-22+21y^2)+b^6(-99+79y^2))$$
$$+bx^7y^5(79+24b^6+15y^2+b^4(94-30y^2)+b^2(-449+87y^2))+2(-1+\vartheta^9)-x^2(9(1$$
$$+7b^2)y^2+2(-1+\vartheta^9)))/(40(1-b^2)^{3/2}xy^2\vartheta^9)\Big]$$

$$+\lambda_{c_j}^2\lambda_{c_i}^2\Big[3/(20(1-b^2)^{3/2}xy^2)+(3(-2-11x^2y^2+594x^4y^4$$
$$-607x^6y^6+96b^8x^6y^6+50x^8y^8-8b^7x^5y^5(33+58x^2y^2)$$
$$-18b^5x^3y^3(14+11x^2y^2-80x^4y^4+3x^6y^6)+b^4x^4y^4(-264$$
$$-2961x^2y^2+54x^4y^4+11x^6y^6)+b^6(297x^4y^4+802x^6y^6$$
$$-35x^8y^8)+b^3x^3y^3(735+2112x^2y^2-894x^4y^4+152x^6y^6$$
$$-x^8y^8)-3b^2x^2y^2(33+429x^2y^2-582x^4y^4+133x^6y^6$$
$$+11x^8y^8)+bxy(22-153x^2y^2-726x^4y^4+578x^6y^6+12x^8y^8$$
$$+3x^{10}y^{10})))/(40(1-b^2)^{3/2}xy^2\vartheta^{11}))\Big] \tag{S5f}$$

$$
\begin{aligned}
M_6^{US,c,ij} &= \Big[(9(2b(-1+x^2)-14b^2x(-1+x^2)y+bx^2(-7(1+5b^2)+(9+31b^2+2b^4)x^2)y^2 \\
&\quad +x^3(-4+43b^2+31b^4-(-2+43b^2+25b^4+4b^6)x^2)y^3+bx^4(-7(1+8b^2+b^4) \\
&\quad +(13+44b^2+13b^4)x^2)y^4+x^3(5(-1+b^2)^2+(-3+34b^2+11b^4)x^2-2(2+10b^2 \\
&\quad +9b^4)x^4)y^5+bx^4(-1+x^2)(5(-1+b^2)^2+7(1+b^2)x^2)y^6-(1+b^2)x^7(-1 \\
&\quad +x^2)y^7))/(8(-1+b^2)xy^2\vartheta^7)+(9b(-1+x^2)(-1+xy(2b-xy))^3) \\
&\quad /(4(-1+b^2)xy^2\vartheta^6)\Big]-\lambda_{c_i}^2\Big[3(4b^6x^5y^5(6+5y^2)+b^5x^4y^4(-69-(45+79x^2)y^2 \\
&\quad +25x^2y^4)+b^4x^3y^3(81+7(5+27x^2)y^2+15x^2(-5+2x^2)y^4) \\
&\quad +x^3y^3(-24+(35+48x^2)y^2+x^2(-35+3x^2)y^4+x^6y^6) \\
&\quad -b^3x^2y^2(63+9x^6y^6+x^4y^4(31+50y^2)+x^2(177y^2-90y^4)) \\
&\quad +b^2xy(18+39x^6y^6+x^8y^8+x^2(111y^2-70y^4)+x^4(-9y^4 \\
&\quad +90y^6))-b(2+9x^2y^2+9x^8y^8+x^4(6y^4+45y^6)+x^6(58y^6-25y^8)-2\vartheta^9)) \\
&\quad /(8(-1+b^2)xy^2\vartheta^9)\Big]+\lambda_{c_j}^2\Big[9(4b^6x^5y^3(5+6(-1+x^2)y^2)+b^5x^4y^2(-10 \\
&\quad +(69-99x^2)y^2-79x^2(-1+x^2)y^4)+b^2xy(-18+x^{10}y^8-x^8y^6(-39 \\
&\quad +y^2)+3x^4y^2(37+3y^2)-3x^2(-6+37y^2)-x^6y^4(29+39y^2))+b^3x^2y^2(63 \\
&\quad -9x^8y^6+x^6y^4(-31+9y^2)+x^4y^2(-117+31y^2)+x^2(-43+177y^2)) \\
&\quad +b^4x^3y^3(-81+30x^6y^4+x^2(51-189y^2)+x^4(199y^2-30y^4)) \\
&\quad +x^3y^3(24+x^8y^6-x^6y^4(-3+y^2)-2x^2(7+24y^2)
\end{aligned}
$$

$$+x^4(58y^2 - 3y^4)) + b(2 - 9x^{10}y^8 + x^4y^2(-19 + 6y^2) + x^8y^6(-58$$
$$+9y^2) + x^6(-36y^4 + 58y^6) - 2\vartheta^9 + x^2(-2 + 9y^2 + 2\vartheta^9)))/(40(-1 + b^2)xy^2\vartheta^9)\Big]$$

$$+\lambda_{c_j}^2\lambda_{c_i}^2\Big[((3b)/(20(-1 + b^2)xy^2) - (3(96b^7x^6y^6 - 8b^6x^5y^5(33 + 58x^2y^2)$$
$$-b^4x^3y^3(63 + 1188x^2y^2 - 625x^4y^4 + 54x^6y^6) + b^5(297x^4y^4 + 1162x^6y^6 - 35x^8y^8)$$
$$-x^3y^3(-168 + 792x^2y^2 - 291x^4y^4 + 10x^6y^6 + x^8y^8) + b^3x^2y^2(99 + 330x^2y^2$$
$$-1226x^4y^4 + 466x^6y^6 + 11x^8y^8) - b^2xy(22 + 435x^2y^2$$
$$-1320x^4y^4 + 1112x^6y^6 + 46x^8y^8 + x^{10}y^{10}) + b(2 + 11x^2y^2$$
$$+33x^4y^4 + 892x^6y^6 - 101x^8y^8 + 11x^{10}y^{10})))/(40(-1 + b^2)xy^2\vartheta^{11}))\Big] \tag{S5g}$$

$$M_7^{US,c,ij} = \Big[(9(-1 + x^2)(-b + 5b^2xy - 5(b + b^3)x^2y^2 + x(1 + 2x^2 + b^2(-1 + 8x^2))y^3$$
$$+bx^2(-1 + b^2 - 5x^2)y^4 + x^5y^5))/(8(-1 + b^2)xy^2\vartheta^5) + (9b(-1$$
$$+x^2)(1 + xy(-2b + xy))^2)/(8(-1 + b^2)xy^2\vartheta^4)\Big] - \lambda_{c_i}^2\Big[3(-(xy^3(3 + 3x^4y^2$$
$$+x^6y^4 + x^2(12 - 7y^2))) + b^3x^2y^2(7 + 5x^2y^4 + y^2(-1$$
$$+x^2(13 - 12\vartheta)) - 12\vartheta) + 4b^4x^3y^3(-1 + 2\vartheta) - b(-1 - x^2y^2(14 + y^2 - 3\vartheta)$$
$$+x^6y^6(-7 + \vartheta) + \vartheta + x^4y^4(-22 + 5y^2 + 3\vartheta)) + b^2xy(-7 + x^2y^4(-7$$
$$+6x^2(-3 + \vartheta)) + 6\vartheta + y^2(3 + x^2(-19 + 12\vartheta)))))/(8(-1 + b^2)xy^2\vartheta^7)\Big]$$

$$+\lambda_{c_j}^2\Big[9(x^3y^3(12 - x^6y^4 + x^4y^2(-5 + y^2) + x^2(-14 + 3y^2))$$
$$+4b^4x^3y^3(1 + 2(-1 + x^2)\vartheta) + b^2xy(7 - 3x^4y^2(7 + 2y^2(-3 + \vartheta)$$
$$-4\vartheta) - 6\vartheta + 2x^6y^4(-8 + 3\vartheta) + x^2(-7 + y^2(19 - 12\vartheta) + 6\vartheta)) + b(-1 +$$
$$x^6y^4(28 + y^2(-7 + \vartheta) - 3\vartheta) - x^8y^6(-7 + \vartheta) + \vartheta + x^4y^2(16$$
$$-3\vartheta + y^2(-22 + 3\vartheta)) + x^2(1 - \vartheta + y^2(-14 + 3\vartheta))) + b^3x^2y^2(-7$$
$$+x^4y^2(7 - 12\vartheta) + 12\vartheta + x^2(5 - 12\vartheta + y^2(-13 + 12\vartheta)))))/(40(-1 + b^2)xy^2\vartheta^7)\Big]$$

$$+\lambda_{c_j}^2\lambda_{c_i}^2\Big[((-3b)/(40(-1 + b^2)xy^2) - (3(24b^4x^5y^5 + b^3(21x^2y^2 - 54x^4y^4$$
$$-79x^6y^6) + x^3y^3(84 - 51x^2y^2 + 6x^4y^4 + x^6y^6)$$
$$+3b^2xy(3 + 51x^4y^4 + 10x^6y^6) - b(1 + 57x^2y^2 + 72x^4y^4$$
$$+5x^6y^6 + 9x^8y^8)))/(40(-1 + b^2)xy^2\vartheta^9))\Big] \tag{S5h}$$

$$M_8^{US,c,ij} = \Big[(9(-1 + x^2)(-1 + y^2)(1 + b^2 - 5(b + b^3)xy + 5b^2(3 + b^2)x^2y^2 - 5b(1$$
$$+3b^2)x^3y^3 + 10b^2x^4y^4 - 2bx^5y^5))/(8(1 - b^2)^{3/2}xy^2\vartheta^5)$$
$$-(9(1 + b^2)(-1 + x^2)(-1 + y^2)(1 + xy(-2b + xy))^2)/(8(1 - b^2)^{3/2}xy^2\vartheta^4)\Big]$$

$$+\lambda_{c_i}^2\Big[3(-1 + \vartheta + x^4(-3 + x^2)y^6\vartheta - x^6y^8\vartheta$$
$$+4b^5x^3y^3(1 + 2(-1 + y^2)\vartheta) + x^2y^4(-3(3 + \vartheta) + x^2(-2 + 3\vartheta))$$
$$+y^2(1 - \vartheta + x^2(7 + 3\vartheta)) + b^3xy(7 - 6\vartheta + x^4y^6(-35 + 6\vartheta) + y^2$$
$$(-7 + x^2(62 - 20\vartheta) + 6\vartheta) + x^2y^4(-70 + x^2(35 - 6\vartheta) + 20\vartheta)) + bxy$$
$$(7 - 2x^6y^8 - 6\vartheta + x^4y^6(-7 + 2x^2 + 6\vartheta) + y^2(-7 + x^2(4 - 12\vartheta) + 6\vartheta)$$
$$+y^4(x^4(7 - 6\vartheta) + 12x^2\vartheta)) + b^4x^2y^2(-7 + x^2y^4(35 - 12\vartheta) + 12\vartheta + y^2(5$$
$$-12\vartheta + x^2(-37 + 12\vartheta))) + b^2(-1 + x^4y^6(35 + x^2(-14 + \vartheta) - 15\vartheta) - x^6y^8$$
$$(-14 + \vartheta) + \vartheta + y^2(1 - \vartheta + 3x^2(-14 + 5\vartheta)) + x^2y^4(46 - 15\vartheta + x^2(-31$$
$$+15\vartheta))))/(8(1 - b^2)^{3/2}xy^2\vartheta^7)\Big] + \lambda_{c_j}^2\Big[9(-1 + \vartheta - x^8y^6\vartheta + x^6y^4(-3 + y^2)\vartheta$$

$$+4b^5\mathrm{x}^3\mathrm{y}^3(1+2(-1+\mathrm{x}^2)\vartheta)+\mathrm{x}^4\mathrm{y}^2(-3(3+\vartheta)+\mathrm{y}^2(-2+3\vartheta))+\mathrm{x}^2(1$$
$$-\vartheta+\mathrm{y}^2(7+3\vartheta))+b^3\mathrm{xy}(7-6\vartheta+\mathrm{x}^6\mathrm{y}^4(-35+6\vartheta)+\mathrm{x}^2(-7+\mathrm{y}^2(62-20\vartheta)$$
$$+6\vartheta)+\mathrm{x}^4\mathrm{y}^2(-70+\mathrm{y}^2(35-6\vartheta)+20\vartheta))+b\mathrm{xy}(7-2\mathrm{x}^8\mathrm{y}^6-6\vartheta+\mathrm{x}^6\mathrm{y}^4$$
$$(-7+2\mathrm{y}^2+6\vartheta)+\mathrm{x}^2(-7+\mathrm{y}^2(4-12\vartheta)+6\vartheta)+\mathrm{x}^4(\mathrm{y}^4(7-6\vartheta)+12\mathrm{y}^2\vartheta))$$
$$+b^4\mathrm{x}^2\mathrm{y}^2(-7+\mathrm{x}^4\mathrm{y}^2(35-12\vartheta)+12\vartheta+\mathrm{x}^2(5-12\vartheta+\mathrm{y}^2(-37+12\vartheta)))$$
$$+b^2(-1+\mathrm{x}^6\mathrm{y}^4(35+\mathrm{y}^2(-14+\vartheta)-15\vartheta)-\mathrm{x}^8\mathrm{y}^6(-14+\vartheta)+\vartheta+\mathrm{x}^2(1-\vartheta$$
$$+3\mathrm{y}^2(-14+5\vartheta))+\mathrm{x}^4\mathrm{y}^2(46-15\vartheta+\mathrm{y}^2(-31+15\vartheta))))/(40(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^7)\Big]$$

$$+\lambda_{c_j}^2\lambda_{c_i}^2\Big[((-3(1+b^2))/(40(1-b^2)^{3/2}\mathrm{xy}^2)-(3(-1+48\mathrm{x}^2\mathrm{y}^2-81\mathrm{x}^4\mathrm{y}^4$$
$$+120b^5\mathrm{x}^5\mathrm{y}^5+10\mathrm{x}^6\mathrm{y}^6+b^4(21\mathrm{x}^2\mathrm{y}^2-270\mathrm{x}^4\mathrm{y}^4-95\mathrm{x}^6\mathrm{y}^6)$$
$$+3b^3\mathrm{xy}(3+63\mathrm{x}^2\mathrm{y}^2+25\mathrm{x}^4\mathrm{y}^4+21\mathrm{x}^6\mathrm{y}^6)+b\mathrm{xy}(9-21\mathrm{x}^2\mathrm{y}^2$$
$$+57\mathrm{x}^4\mathrm{y}^4+9\mathrm{x}^6\mathrm{y}^6+2\mathrm{x}^8\mathrm{y}^8)-b^2(1+141\mathrm{x}^2\mathrm{y}^2-99\mathrm{x}^4\mathrm{y}^4+83\mathrm{x}^6\mathrm{y}^6+18\mathrm{x}^8\mathrm{y}^8)))$$
$$/(40(1-b^2)^{3/2}\mathrm{xy}^2\vartheta^9))\Big]$$

$$\text{(S5i)}$$

$$6\pi\eta R^3 M_{kl}^{\Omega L,c,ij} = M_1^{\Omega L,c,ij}\hat{\mathrm{y}}_k\hat{\mathrm{y}}_l+M_2^{\Omega L,c,ij}\hat{\mathrm{y}}_k\hat{\mathrm{y}}_l^\perp+M_3^{\Omega L,c,ij}\hat{\mathrm{y}}_k^\perp\hat{\mathrm{y}}_l+M_4^{\Omega L,c,ij}\hat{\mathrm{y}}_k^\perp\hat{\mathrm{y}}_l^\perp$$
$$+M_5^{\Omega L,c,ij}\delta_{kl}$$
$$\text{(S6a)}$$

$$M_1^{\Omega L,c,ij} = \Big[9/(8(-1+b^2))-(9(1-2\mathrm{x}^2\mathrm{y}^2+b^4\mathrm{x}^2\mathrm{y}^2+b^2\mathrm{x}^2\mathrm{y}^2(11+5\mathrm{x}^2\mathrm{y}^2)-2b^3(\mathrm{xy}$$
$$+4\mathrm{x}^3\mathrm{y}^3)-b\mathrm{xy}(3+2\mathrm{x}^2\mathrm{y}^2+\mathrm{x}^4\mathrm{y}^4)))/(8(-1+b^2)\vartheta^5)\Big]$$
$$\text{(S6b)}$$

$$M_2^{\Omega L,c,ij} = \Big[(9b)/(8\sqrt{1-b^2})+(9(\mathrm{xy}-3b^3\mathrm{x}^2\mathrm{y}^2+3\mathrm{x}^3\mathrm{y}^3+\mathrm{x}^5\mathrm{y}^5+b^2\mathrm{xy}(4+7\mathrm{x}^2\mathrm{y}^2)$$
$$-b(1+7\mathrm{x}^2\mathrm{y}^2+5\mathrm{x}^4\mathrm{y}^4)))/(8\sqrt{1-b^2}\vartheta^5)\Big]$$
$$\text{(S6c)}$$

$$M_3^{\Omega L,c,ij} = \Big[(-9\sqrt{1-b^2}\mathrm{xy}(-1+b\mathrm{xy}))/(8\vartheta^5)\Big]$$
$$\text{(S6d)}$$

$$M_4^{\Omega L,c,ij} = \Big[(9(1+b^2))/(8(-1+b^2))-(9(1-2\mathrm{x}^2\mathrm{y}^2+3b^4\mathrm{x}^2\mathrm{y}^2-5b^3(\mathrm{xy}+3\mathrm{x}^3\mathrm{y}^3)$$
$$-b\mathrm{xy}(5+5\mathrm{x}^2\mathrm{y}^2+2\mathrm{x}^4\mathrm{y}^4)+b^2(1+19\mathrm{x}^2\mathrm{y}^2+10\mathrm{x}^4\mathrm{y}^4)))$$
$$/(8(-1+b^2)\vartheta^5)\Big]$$
$$\text{(S6e)}$$

$$M_5^{\Omega L,c,ij} = \Big[-9/(8(-1+b^2))-(3(-5+7\mathrm{x}^2\mathrm{y}^2+2b^3(\mathrm{xy}+12\mathrm{x}^3\mathrm{y}^3)+b^2(2-37\mathrm{x}^2\mathrm{y}^2$$
$$-15\mathrm{x}^4\mathrm{y}^4)+b\mathrm{xy}(13+6\mathrm{x}^2\mathrm{y}^2+3\mathrm{x}^4\mathrm{y}^4)))/(8(-1+b^2)\vartheta^5)\Big]$$
$$\text{(S6f)}$$

$$6\pi\eta R^3 M_{kln}^{\Omega S,c,ij} = \{M_1^{\Omega S,c,ij}\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_r(\hat{\mathrm{y}}_n\hat{\mathrm{y}}_l-\tfrac{1}{3}\delta_{nl})+M_2^{\Omega S,c,ij}(\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_r\hat{\mathrm{y}}_n^\perp\hat{\mathrm{y}}_l+\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_r\hat{\mathrm{y}}_n\hat{\mathrm{y}}_l^\perp)+$$
$$M_3^{\Omega S,c,ij}\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_r(\hat{\mathrm{y}}_n^\perp\hat{\mathrm{y}}_l^\perp-\tfrac{1}{3}\delta_{nl})+M_4^{\Omega S,c,ij}(\delta_{rn}\hat{\mathrm{y}}_m\hat{\mathrm{y}}_l+\delta_{rl}\hat{\mathrm{y}}_m\hat{\mathrm{y}}_n)$$
$$+M_5^{\Omega S,c,ij}\big[\delta_{rn}\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_l+\delta_{rl}\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_n-\tfrac{2}{3}\delta_{ln}(\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_r)\big]$$
$$+M_6^{\Omega S,c,ij}\big[\delta_{rn}\hat{\mathrm{y}}_m\hat{\mathrm{y}}_l^\perp+\delta_{rl}\hat{\mathrm{y}}_m\hat{\mathrm{y}}_n^\perp-\tfrac{2}{3}\delta_{ln}(\hat{\mathrm{y}}_m\hat{\mathrm{y}}_r^\perp)\big]$$
$$+M_7^{\Omega S,c,ij}(\delta_{rn}\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_l^\perp+\delta_{rl}\hat{\mathrm{y}}_m^\perp\hat{\mathrm{y}}_n^\perp)\}\epsilon_{kmr}$$
$$\text{(S7a)}$$

$$M_1^{\Omega S,c,ij} = \Big[(-9b(-2+b^2+\mathrm{y}^2))/(4(1-b^2)^{3/2}\mathrm{y}^2)+(9(4b^6\mathrm{x}^3\mathrm{y}^3(-5+\mathrm{y}^2)+b^5\mathrm{x}^2\mathrm{y}^2$$

$$(35 + (-7 + 25x^2)y^2 + 11x^2y^4) - b^2xy(-28 + (14 - 85x^2)y^2 + (43x^2$$
$$-44x^4)y^4 + x^4(18 + x^2)y^6 + x^6y^8) + b(-4 + (2 - 14x^2)y^2 - 45x^4y^4$$
$$+(11x^4 - 21x^6)y^6 + 7x^6y^8) + x^3y^5(6 + x^2(13 - 5y^2) - x^4y^2(-3$$
$$+y^2)) - b^4xy(14 + x^4y^4(15 + 19y^2) + x^2y^2(-5 + 37y^2)) + b^3(2$$
$$+7x^2y^2(-9 + 7y^2) + x^4(-50y^4 + 48y^6) + 7x^6(y^6$$
$$+y^8))))/(8(1 - b^2)^{3/2}y^2\vartheta^7)\Big] + \lambda_{c_j}^2\Big[((9b(-2 + b^2))/(20(1 - b^2)^{3/2}y^2)$$
$$-(9(40b^6x^5y^5 + x^5y^5(143 + 6x^2y^2 + 3x^4y^4) + b^5(63x^2y^2$$
$$-90x^4y^4 - 5x^6y^6) - 3b^4xy(6 + 21x^2y^2 + 13x^6y^6) + b^3(2 - 117x^2y^2$$
$$+117x^4y^4 - 11x^6y^6 + 9x^8y^8) - b(4 + 18x^2y^2 + 279x^4y^4 + 152x^6y^6$$
$$+27x^8y^8) + b^2(36xy + 231x^3y^3 + 69x^5y^5$$
$$+105x^7y^7 - x^9y^9)))/(40(1 - b^2)^{3/2}y^2\vartheta^9))\Big] \tag{S7b}$$

$$M_2^{\Omega S,c,ij} = \Big[(-9(1 + b^2)(-2 + y^2))/(8(-1 + b^2)y^2) - (9(2 + (-1 + 7x^2)y^2 - x^2(7$$
$$+5x^2)y^4 + 4x^4y^6 + 4b^5x^3y^3(-5 + 3y^2) + b^4x^2y^2(35 + (-21 + 65x^2)y^2$$
$$-31x^2y^4) + b^3xy(-14 + (7 - 100x^2)y^2 + (46x^2 - 70x^4)y^4 + 35x^4y^6)$$
$$+bxy(-14 + (7 - 20x^2)y^2 - 2x^2(-6$$
$$+7x^2)y^4 + (7x^4 - 4x^6)y^6 + 2x^6y^8) - b^2(-2 + (1 - 42x^2)y^2 + (14x^2$$
$$-80x^4)y^4 + (43x^4 - 28x^6)y^6 + 14x^6y^8)))/(8(-1 + b^2)y^2\vartheta^7)\Big]$$
$$+\lambda_{c_j}^2\Big[((-9(1 + b^2))/(20(-1 + b^2)y^2) - (9(-2 - 9x^2y^2 + 108x^4y^4$$
$$+120b^5x^5y^5 - 25x^6y^6 - b^4x^2y^2(63 + 270x^2y^2 + 235x^4y^4)$$
$$+6b^3xy(3 + 63x^2y^2 + 65x^4y^4 + 21x^6y^6) + 2bxy(9$$
$$-21x^2y^2 - 3x^4y^4 + 9x^6y^6 + 2x^8y^8) - 2b^2(1 + 36x^2y^2$$
$$+171x^4y^4 + 38x^6y^6 + 18x^8y^8)))/(40(-1 + b^2)y^2\vartheta^9))\Big] \tag{S7c}$$

$$M_3^{\Omega S,c,ij} = \Big[(-9b(3 + b^2)(-1 + y^2))/(4(1 - b^2)^{3/2}y^2) - (9(-1 + y^2)(20b^6x^3y^3$$
$$+x^3y^3(15 + 7x^2y^2 + 2x^4y^4) - 35b^5(x^2y^2 + 3x^4y^4) + b^4xy(14 + 185x^2y^2$$
$$+105x^4y^4) - b(6 + 21x^2y^2 + 35x^4y^4 + 14x^6y^6) - 2b^3(1 + 56x^2y^2 + 70x^4y^4$$
$$+21x^6y^6) + b^2(42xy + 60x^3y^3 + 56x^5y^5 + 6x^7y^7)))/(8(1 - b^2)^{3/2}y^2\vartheta^7)\Big]$$
$$+\lambda_{c_j}^2\Big[((-9b(3 + b^2))/(20(1 - b^2)^{3/2}y^2) - (9(280b^6x^5y^5 + x^3y^3(105$$
$$-28x^2y^2 + 9x^4y^4 + 2x^6y^6) - 63b^5(x^2y^2 + 10x^4y^4$$
$$+5x^6y^6) + 3b^4xy(6 + 231x^2y^2 + 140x^4y^4 + 63x^6y^6)$$
$$+6b^2xy(9 - 21x^2y^2 + 56x^4y^4 + 15x^6y^6 + x^8y^8) - 3b(2$$
$$+9x^2y^2 + 42x^4y^4 + 21x^6y^6 + 6x^8y^8) - 2b^3(1 + 99x^2y^2$$
$$+126x^4y^4 + 147x^6y^6 + 27x^8y^8)))/(40(1 - b^2)^{3/2}y^2\vartheta^9))\Big] \tag{S7d}$$

$$M_4^{\Omega S,c,ij} = \Big[(9b^2)/(8(-1 + b^2)y^2) + (9(-(x^2y^4) + b^4x^2y^2(-4 + y^2) + bxy^3(2 + 3x^2 +$$
$$x^4y^2) + b^3xy(5 + (-2 + 7x^2)y^2) - b^2(1 + 6x^2y^2 + 5x^4y^4)))$$
$$/(8(-1 + b^2)y^2\vartheta^5)\Big] + \lambda_{c_j}^2\Big[((-9b^2)/(40(-1 + b^2)y^2) + (9b(8b^3x^4y^4 - x^3y^3(21$$
$$+2x^2y^2 + x^4y^4) - b^2xy(7 + 14x^2y^2 + 19x^4y^4) + b(1 + 21x^2y^2$$

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$$+27\mathrm{x}^4\mathrm{y}^4 + 7\mathrm{x}^6\mathrm{y}^6)))/(40(-1+b^2)\mathrm{y}^2\vartheta^7))\Big] \tag{S7e}$$

$$
\begin{aligned}
M_5^{\Omega S,c,ij} = {} & \Big[(-9b)/(8\sqrt{1-b^2}\mathrm{y}^2) - (9(b^3\mathrm{x}^2\mathrm{y}^2(-4+\mathrm{y}^2) + \mathrm{xy}^3(1+3\mathrm{x}^2+\mathrm{x}^4\mathrm{y}^2) \\
& +b^2\mathrm{xy}(5+(-1+7\mathrm{x}^2)\mathrm{y}^2) - b(1+5\mathrm{x}^4\mathrm{y}^4+\mathrm{x}^2\mathrm{y}^2(6+\mathrm{y}^2)))) \\
& /(8\sqrt{1-b^2}\mathrm{y}^2\vartheta^5)\Big] + \lambda_{c_j}^2\Big[((9b)/(40\sqrt{1-b^2}\mathrm{y}^2) - (9(8b^3\mathrm{x}^4\mathrm{y}^4 \\
& -\mathrm{x}^3\mathrm{y}^3(21+2\mathrm{x}^2\mathrm{y}^2+\mathrm{x}^4\mathrm{y}^4) - b^2\mathrm{xy}(7+14\mathrm{x}^2\mathrm{y}^2+19\mathrm{x}^4\mathrm{y}^4) \\
& +b(1+21\mathrm{x}^2\mathrm{y}^2+27\mathrm{x}^4\mathrm{y}^4+7\mathrm{x}^6\mathrm{y}^6)))/(40\sqrt{1-b^2}\mathrm{y}^2\vartheta^7))\Big] \tag{S7f}
\end{aligned}
$$

$$
\begin{aligned}
M_6^{\Omega S,c,ij} = {} & \Big[(-9b(1+b^2)(-1+\mathrm{y}^2))/(8(1-b^2)^{3/2}\mathrm{y}^2) + (9(-(\mathrm{xy}^3)+b^5\mathrm{x}^2\mathrm{y}^2(-4+5\mathrm{y}^2) \\
& +b(-1+(1+\mathrm{x}^2)\mathrm{y}^2) + b^4\mathrm{xy}(5+3(-2+5\mathrm{x}^2)\mathrm{y}^2-15\mathrm{x}^2\mathrm{y}^4) + b^2\mathrm{xy}(5 \\
& +(-3+5\mathrm{x}^2)\mathrm{y}^2+\mathrm{x}^2(-5+2\mathrm{x}^2)\mathrm{y}^4-2\mathrm{x}^4\mathrm{y}^6) + b^3(-1+(1-17\mathrm{x}^2)\mathrm{y}^2 \\
& -5\mathrm{x}^2(-3+2\mathrm{x}^2)\mathrm{y}^4+10\mathrm{x}^4\mathrm{y}^6)))/(8(1-b^2)^{3/2}\mathrm{y}^2\vartheta^5)\Big] + \lambda_{c_j}^2\Big[((-9b(1+b^2)) \\
& /(40(1-b^2)^{3/2}\mathrm{y}^2) + (9b(1-14\mathrm{x}^2\mathrm{y}^2+5\mathrm{x}^4\mathrm{y}^4+40b^4\mathrm{x}^4\mathrm{y}^4 - 7b^3(\mathrm{xy} \\
& +10\mathrm{x}^3\mathrm{y}^3+5\mathrm{x}^5\mathrm{y}^5) - b\mathrm{xy}(7+7\mathrm{x}^4\mathrm{y}^4+2\mathrm{x}^6\mathrm{y}^6) + b^2(1+56\mathrm{x}^2\mathrm{y}^2 \\
& +25\mathrm{x}^4\mathrm{y}^4+14\mathrm{x}^6\mathrm{y}^6)))/(40(1-b^2)^{3/2}\mathrm{y}^2\vartheta^7))\Big] \tag{S7g}
\end{aligned}
$$

$$
\begin{aligned}
M_7^{\Omega S,c,ij} = {} & \Big[(9(1+b^2)(-1+\mathrm{y}^2))/(8(-1+b^2)\mathrm{y}^2) - (9(-1+(1+\mathrm{x}^2)\mathrm{y}^2 + b^4\mathrm{x}^2\mathrm{y}^2(-4 \\
& +5\mathrm{y}^2) - 5b^3\mathrm{xy}(-1+\mathrm{y}^2)(1+3\mathrm{x}^2\mathrm{y}^2) - b\mathrm{xy}(-1+\mathrm{y}^2)(5+5\mathrm{x}^2\mathrm{y}^2 \\
& +2\mathrm{x}^4\mathrm{y}^4) + b^2(-1+(1-17\mathrm{x}^2)\mathrm{y}^2-5\mathrm{x}^2(-3+2\mathrm{x}^2)\mathrm{y}^4+10\mathrm{x}^4\mathrm{y}^6)))/(8(-1 \\
& +b^2)\mathrm{y}^2\vartheta^5)\Big] + \lambda_{c_j}^2\Big[((9(1+b^2))/(40(-1+b^2)\mathrm{y}^2) - (9(1-14\mathrm{x}^2\mathrm{y}^2 \\
& +5\mathrm{x}^4\mathrm{y}^4+40b^4\mathrm{x}^4\mathrm{y}^4 - 7b^3(\mathrm{xy}+10\mathrm{x}^3\mathrm{y}^3+5\mathrm{x}^5\mathrm{y}^5) - b\mathrm{xy}(7+7\mathrm{x}^4\mathrm{y}^4 \\
& +2\mathrm{x}^6\mathrm{y}^6) + b^2(1+56\mathrm{x}^2\mathrm{y}^2+25\mathrm{x}^4\mathrm{y}^4+14\mathrm{x}^6\mathrm{y}^6)))/(40(-1+b^2)\mathrm{y}^2\vartheta^7))\Big] \tag{S7h}
\end{aligned}
$$

$$
\begin{aligned}
6\pi\eta R^3 M_{nmlk}^{ES,c,ij} = {} & M_1^{ES,c,ij}(\hat{y}_n\hat{y}_m - \tfrac{1}{3}\delta_{nm})(\hat{y}_l\hat{y}_k - \tfrac{1}{3}\delta_{lk}) \\
& +M_2^{ES,c,ij}(\hat{y}_l\hat{y}_k^\perp + \hat{y}_l^\perp\hat{y}_k)(\hat{y}_n\hat{y}_m - \tfrac{1}{3}\delta_{nm}) \\
& +M_3^{ES,c,ij}(\hat{y}_n\hat{y}_m^\perp + \hat{y}_n^\perp\hat{y}_m)(\hat{y}_l\hat{y}_k - \tfrac{1}{3}\delta_{lk}) \\
& +M_4^{ES,c,ij}(\hat{y}_n\hat{y}_m - \tfrac{1}{3}\delta_{nm})(\hat{y}_l^\perp\hat{y}_k^\perp - \tfrac{1}{3}\delta_{lk}) \\
& +M_5^{ES,c,ij}(\hat{y}_n\hat{y}_m^\perp\hat{y}_l\hat{y}_k^\perp + \hat{y}_n^\perp\hat{y}_m\hat{y}_l\hat{y}_k^\perp + \hat{y}_n\hat{y}_m^\perp\hat{y}_l^\perp\hat{y}_k + \hat{y}_n^\perp\hat{y}_m\hat{y}_l^\perp\hat{y}_k) \\
& +M_6^{ES,c,ij}(\hat{y}_n^\perp\hat{y}_m^\perp - \tfrac{1}{3}\delta_{nm})(\hat{y}_l\hat{y}_k - \tfrac{1}{3}\delta_{lk}) \\
& +M_7^{ES,c,ij}(\hat{y}_n\hat{y}_m^\perp + \hat{y}_n^\perp\hat{y}_m)(\hat{y}_l^\perp\hat{y}_k^\perp - \tfrac{1}{3}\delta_{lk}) \\
& +M_8^{ES,c,ij}(\hat{y}_n^\perp\hat{y}_m^\perp - \tfrac{1}{3}\delta_{nm})(\hat{y}_l\hat{y}_k^\perp + \hat{y}_l^\perp\hat{y}_k) \\
& +M_9^{ES,c,ij}(\hat{y}_n^\perp\hat{y}_m^\perp - \tfrac{1}{3}\delta_{nm})(\hat{y}_l^\perp\hat{y}_k^\perp - \tfrac{1}{3}\delta_{lk}) \\
& +M_{10}^{ES,c,ij}(\delta_{nl}\hat{y}_m\hat{y}_k + \delta_{ml}\hat{y}_n\hat{y}_k + \delta_{nk}\hat{y}_m\hat{y}_l + \delta_{mk}\hat{y}_n\hat{y}_l - \tfrac{4}{3}\delta_{nm}\hat{y}_l\hat{y}_k \\
& -\tfrac{4}{3}\delta_{lk}\hat{y}_n\hat{y}_m + \tfrac{4}{9}\delta_{nm}\delta_{lk}) + M_{11}^{ES,c,ij}(\delta_{nl}\hat{y}_m^\perp\hat{y}_k^\perp + \delta_{ml}\hat{y}_n^\perp\hat{y}_k^\perp + \delta_{nk}\hat{y}_m^\perp\hat{y}_l^\perp \\
& +\delta_{mk}\hat{y}_n^\perp\hat{y}_l^\perp - \tfrac{4}{3}\delta_{nm}\hat{y}_l^\perp\hat{y}_k^\perp - \tfrac{4}{3}\delta_{lk}\hat{y}_n^\perp\hat{y}_m^\perp + \tfrac{4}{9}\delta_{nm}\delta_{lk}) + M_{12}^{ES,c,ij}(\delta_{nl}\hat{y}_m\hat{y}_k^\perp \\
& +\delta_{ml}\hat{y}_n\hat{y}_k^\perp + \delta_{nk}\hat{y}_m\hat{y}_l^\perp + \delta_{mk}\hat{y}_n\hat{y}_l^\perp - \tfrac{2}{3}\delta_{nm}(\hat{y}_l\hat{y}_k^\perp + \hat{y}_l^\perp\hat{y}_k) \\
& -\tfrac{2}{3}\delta_{lk}(\hat{y}_n\hat{y}_m^\perp + \hat{y}_n^\perp\hat{y}_m)) + M_{13}^{ES,c,ij}(\delta_{nl}\hat{y}_m^\perp\hat{y}_k + \delta_{ml}\hat{y}_n^\perp\hat{y}_k + \delta_{nk}\hat{y}_m^\perp\hat{y}_l \\
& +\delta_{mk}\hat{y}_n^\perp\hat{y}_l - \tfrac{2}{3}\delta_{nm}(\hat{y}_l^\perp y_k + y_l\hat{y}_k^\perp) - \tfrac{2}{3}\delta_{lk}(\hat{y}_n^\perp\hat{y}_m + \hat{y}_n\hat{y}_m^\perp))
\end{aligned}
$$

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$$+M_{14}^{ES,c,ij}(\delta_{nl}\delta_{mk}+\delta_{nk}\delta_{ml}-\tfrac{2}{3}\delta_{nm}\delta_{lk}) \tag{S8a}$$

$$
\begin{aligned}
M_1^{ES,c,ij} = \Big[& (9(2b^4\mathrm{x}^2(-2+\mathrm{y}^2)-2(-1+\mathrm{x}^2)(-2+\mathrm{y}^2)+b^2(1+\mathrm{y}^2-3\mathrm{x}^2(-1+\mathrm{y}^2)))) \\
& /(4(-1+b^2)^2\mathrm{x}^2\mathrm{y}^2)+(9(8-4\mathrm{y}^2+4b^9\mathrm{x}^7\mathrm{y}^5(-18+13\mathrm{y}^2)-2\mathrm{x}^2(4-20\mathrm{y}^2 \\
& +9\mathrm{y}^4)+\mathrm{x}^6\mathrm{y}^4(-63+49\mathrm{y}^2+10\mathrm{y}^4)+b^8\mathrm{x}^6\mathrm{y}^4(312+(-143+97\mathrm{x}^2)\mathrm{y}^2 \\
& -3(5+29\mathrm{x}^2)\mathrm{y}^4)+\mathrm{x}^4(-36\mathrm{y}^2+81\mathrm{y}^4-39\mathrm{y}^6)+b^7\mathrm{x}^5\mathrm{y}^3(-418+(209 \\
& -507\mathrm{x}^2)\mathrm{y}^2+(33+164\mathrm{x}^2-27\mathrm{x}^4)\mathrm{y}^4+3\mathrm{x}^2(13+9\mathrm{x}^2)\mathrm{y}^6)-b^6\mathrm{x}^4\mathrm{y}^2(-252 \\
& -6(-35+66\mathrm{x}^2)\mathrm{y}^2+(78+163\mathrm{x}^2-455\mathrm{x}^4)\mathrm{y}^4+(17\mathrm{x}^2+50\mathrm{x}^4-9\mathrm{x}^6)\mathrm{y}^6 \\
& +9(\mathrm{x}^4+\mathrm{x}^6)\mathrm{y}^8)-b^4\mathrm{x}^2(-8+(67+153\mathrm{x}^2)\mathrm{y}^2+(63-339\mathrm{x}^2+720\mathrm{x}^4)\mathrm{y}^4 \\
& +4\mathrm{x}^2(81-159\mathrm{x}^2+106\mathrm{x}^4)\mathrm{y}^6+(142\mathrm{x}^4-211\mathrm{x}^6-99\mathrm{x}^8)\mathrm{y}^8+27\mathrm{x}^6(-1 \\
& +\mathrm{x}^2)\mathrm{y}^{10})+b^5\mathrm{x}^3\mathrm{y}(-72+(141+59\mathrm{x}^2)\mathrm{y}^2+(107-23\mathrm{x}^2+54\mathrm{x}^4)\mathrm{y}^4 \\
& -18\mathrm{x}^2(-8+5\mathrm{x}^2+20\mathrm{x}^4)\mathrm{y}^6-\mathrm{x}^4(108-107\mathrm{x}^2+\mathrm{x}^4)\mathrm{y}^8+(\mathrm{x}^6 \\
& +\mathrm{x}^8)\mathrm{y}^{10})+b\mathrm{x}\mathrm{y}(36(-2+\mathrm{y}^2)-2\mathrm{x}^{10}\mathrm{y}^8(-9+5\mathrm{y}^2)+\mathrm{x}^8\mathrm{y}^6(81-57\mathrm{y}^2 \\
& +8\mathrm{y}^4)+8\mathrm{x}^2(9-36\mathrm{y}^2+16\mathrm{y}^4)+\mathrm{x}^6\mathrm{y}^4(273-254\mathrm{y}^2+33\mathrm{y}^4) \\
& +\mathrm{x}^4\mathrm{y}^2(252-403\mathrm{y}^2+147\mathrm{y}^4))+b^3\mathrm{x}\mathrm{y}(18(1+\mathrm{y}^2)+\mathrm{x}^{10}\mathrm{y}^8(-11+3\mathrm{y}^2) \\
& +\mathrm{x}^8\mathrm{y}^6(522-299\mathrm{y}^2-3\mathrm{y}^4)+4\mathrm{x}^6\mathrm{y}^4(252-211\mathrm{y}^2+63\mathrm{y}^4)+\mathrm{x}^2(54 \\
& -411\mathrm{y}^2+269\mathrm{y}^4)+\mathrm{x}^4\mathrm{y}^2(611-1043\mathrm{y}^2+432\mathrm{y}^4))+b^2(-2(1+\mathrm{y}^2) \\
& +18\mathrm{x}^{10}\mathrm{y}^8(-9+5\mathrm{y}^2)-4\mathrm{x}^8\mathrm{y}^6(158-121\mathrm{y}^2+18\mathrm{y}^4)-3\mathrm{x}^2(2-83\mathrm{y}^2 \\
& +45\mathrm{y}^4)-3\mathrm{x}^4\mathrm{y}^2(93-254\mathrm{y}^2+105\mathrm{y}^4)+\mathrm{x}^6(-681\mathrm{y}^4+881\mathrm{y}^6-340\mathrm{y}^8)))) \\
& /(8(-1+b^2)^2\mathrm{x}^2\mathrm{y}^2\vartheta^9)\Big]+\lambda_{c_i}^2\Big[((-9(2(-2+\mathrm{y}^2)+b^2(1+\mathrm{y}^2)))/(20(-1 \\
& +b^2)^2\mathrm{x}^2\mathrm{y}^2)-(9(8+(-4+44\mathrm{x}^2)\mathrm{y}^2+11\mathrm{x}^2(-2+9\mathrm{x}^2)\mathrm{y}^4-\mathrm{x}^4(117 \\
& +497\mathrm{x}^2)\mathrm{y}^6+\mathrm{x}^6(481+70\mathrm{x}^2)\mathrm{y}^8-50\mathrm{x}^8\mathrm{y}^{10}+20b^9\mathrm{x}^7\mathrm{y}^7(-13+6\mathrm{y}^2) \\
& +b^8\mathrm{x}^6\mathrm{y}^6(445+(-327+463\mathrm{x}^2)\mathrm{y}^2-393\mathrm{x}^2\mathrm{y}^4)+b^7\mathrm{x}^5\mathrm{y}^5(-75+(363 \\
& +278\mathrm{x}^2)\mathrm{y}^2+(490\mathrm{x}^2-27\mathrm{x}^4)\mathrm{y}^4+27\mathrm{x}^4\mathrm{y}^6)-b^6\mathrm{x}^4\mathrm{y}^4(330+(274 \\
& +1609\mathrm{x}^2)\mathrm{y}^2+\mathrm{x}^2(95+1864\mathrm{x}^2)\mathrm{y}^4-\mathrm{x}^4(1228+11\mathrm{x}^2)\mathrm{y}^6+11\mathrm{x}^6\mathrm{y}^8) \\
& +b^5\mathrm{x}^3\mathrm{y}^3(231+(245+849\mathrm{x}^2)\mathrm{y}^2+\mathrm{x}^2(407+1656\mathrm{x}^2)\mathrm{y}^4+\mathrm{x}^4(-1492+281\mathrm{x}^2)\mathrm{y}^6 \\
& -\mathrm{x}^6(89+\mathrm{x}^2)\mathrm{y}^8+\mathrm{x}^8\mathrm{y}^{10})+b^4\mathrm{x}^2\mathrm{y}^2(-99+99(-1+9\mathrm{x}^2)\mathrm{y}^2+\mathrm{x}^2(-1021 \\
& +1848\mathrm{x}^2)\mathrm{y}^4+\mathrm{x}^4(-658+2277\mathrm{x}^2)\mathrm{y}^6-7\mathrm{x}^6(265+11\mathrm{x}^2)\mathrm{y}^8+33\mathrm{x}^8\mathrm{y}^{10}) \\
& +b^2(-2+(-2+385\mathrm{x}^2)\mathrm{y}^2+11\mathrm{x}^2(-19+120\mathrm{x}^2)\mathrm{y}^4+\mathrm{x}^4(-568+2585\mathrm{x}^2)\mathrm{y}^6+\mathrm{x}^6 \\
& (-2173+44\mathrm{x}^2)\mathrm{y}^8+4\mathrm{x}^8(20+33\mathrm{x}^2)\mathrm{y}^{10}-88\mathrm{x}^{10}\mathrm{y}^{12})-88\mathrm{x}^{10}\mathrm{y}^{12})+b^3\mathrm{x}\mathrm{y}(22+(22-825\mathrm{x}^2)\mathrm{y}^2 \\
& -41\mathrm{x}^2(-13+81\mathrm{x}^2)\mathrm{y}^4+(1793\mathrm{x}^4-4228\mathrm{x}^6)\mathrm{y}^6+(3476\mathrm{x}^6-503\mathrm{x}^8)\mathrm{y}^8+\mathrm{x}^8(339+ \\
& 7\mathrm{x}^2)\mathrm{y}^{10}-3\mathrm{x}^{10}\mathrm{y}^{12})+b\mathrm{x}\mathrm{y}(-88-44(-1+9\mathrm{x}^2)\mathrm{y}^2+(212\mathrm{x}^2-225\mathrm{x}^4)\mathrm{y}^4 \\
& +\mathrm{x}^4(209+574\mathrm{x}^2)\mathrm{y}^6-\mathrm{x}^6(614+81\mathrm{x}^2)\mathrm{y}^8+\mathrm{x}^8(53-12\mathrm{x}^2)\mathrm{y}^{10}+8\mathrm{x}^{10}\mathrm{y}^{12}))) \\
& /(40(-1+b^2)^2\mathrm{x}^2\mathrm{y}^2\vartheta^{11}))\Big]+\lambda_{c_j}^2\Big[((9(4-4\mathrm{x}^2+4b^4\mathrm{x}^2-b^2(1+3\mathrm{x}^2))) \\
& /(20(-1+b^2)^2\mathrm{x}^2\mathrm{y}^2)-(9(8+288b^{10}\mathrm{x}^8\mathrm{y}^6+11\mathrm{x}^4\mathrm{y}^2(-4+9\mathrm{y}^2)+7\mathrm{x}^8\mathrm{y}^6(81 \\
& +10\mathrm{y}^2)+\mathrm{x}^2(-8+44\mathrm{y}^2)-\mathrm{x}^6\mathrm{y}^4(99+497\mathrm{y}^2)-4b^9\mathrm{x}^7\mathrm{y}^5(198 \\
& +(65+68\mathrm{x}^2)\mathrm{y}^2)+b^8\mathrm{x}^6\mathrm{y}^4(1136+(445+481\mathrm{x}^2)\mathrm{y}^2+(463\mathrm{x}^2-161\mathrm{x}^4)\mathrm{y}^4) \\
& -b^7\mathrm{x}^5\mathrm{y}^3(910+(75+979\mathrm{x}^2)\mathrm{y}^2+(-278\mathrm{x}^2+496\mathrm{x}^4)\mathrm{y}^4+9\mathrm{x}^4(3+7\mathrm{x}^2)\mathrm{y}^6) \\
& +b^6\mathrm{x}^4\mathrm{y}^2(396+(-330+524\mathrm{x}^2)\mathrm{y}^2+\mathrm{x}^2(-1609+2135\mathrm{x}^2)\mathrm{y}^4+2\mathrm{x}^4(-932 \\
& +1099\mathrm{x}^2)\mathrm{y}^6+11(\mathrm{x}^6+\mathrm{x}^8)\mathrm{y}^8)+b^4\mathrm{x}^2(8-11(9+23\mathrm{x}^2)\mathrm{y}^2+(891\mathrm{x}^2 \\
& -2047\mathrm{x}^4)\mathrm{y}^4-8\mathrm{x}^4(-231+383\mathrm{x}^2)\mathrm{y}^6-23\mathrm{x}^6(-99+107\mathrm{x}^2)\mathrm{y}^8+11\mathrm{x}^8(-7 \\
& +11\mathrm{x}^2)\mathrm{y}^{10})-b^5\mathrm{x}^3\mathrm{y}(88-(231+269\mathrm{x}^2)\mathrm{y}^2+\mathrm{x}^2(-849+55\mathrm{x}^2)\mathrm{y}^4 \\
& +2\mathrm{x}^4(-828+1193\mathrm{x}^2)\mathrm{y}^6+\mathrm{x}^6(-281+523\mathrm{x}^2)\mathrm{y}^8+(\mathrm{x}^8+\mathrm{x}^{10})\mathrm{y}^{10}) \\
& +b\mathrm{x}\mathrm{y}(-88+18\mathrm{x}^{12}\mathrm{y}^{10}+3\mathrm{x}^{10}\mathrm{y}^8(33-4\mathrm{y}^2)-44\mathrm{x}^2(-2+9\mathrm{y}^2)
\end{aligned}
$$

$$-9x^4y^2(-44+25y^2)-3x^8y^6(232+27y^2)+7x^6y^4(33+82y^2))$$
$$+b^3xy(22-11x^{12}y^{10}+x^2(66-825y^2)+x^{10}y^8(817+7y^2)$$
$$+x^4(1235y^2-3321y^4)+x^6(4367y^4-4228y^6)+x^8(5830y^6-503y^8))$$
$$-b^2(2+198x^{12}y^{10}+x^2(6-385y^2)+2x^{10}y^8(283-66y^2)-33x^4y^2$$
$$(-13+40y^2)+x^6(1494y^4-2585y^6)+x^8(3179y^6-44y^8))))$$
$$/(40(-1+b^2)^2x^2y^2\vartheta^{11}))\Big]+\lambda_{c_j}^2\lambda_{c_i}^2\Big[(9(8+52x^2y^2+143x^4y^4$$
$$-7748x^6y^6+5663x^8y^8+2080b^{10}x^8y^8-350x^{10}y^{10}-8b^9x^7y^7(710$$
$$+467x^2y^2)+b^8x^6y^6(5785+1642x^2y^2-1883x^4y^4)+b^7x^5y^5(-2145$$
$$+11431x^2y^2+19909x^4y^4-139x^6y^6)+b^6x^4y^4(-1122-16627x^2y^2$$
$$-30455x^4y^4+5055x^6y^6+13x^8y^8)+b^4x^2y^2(-143+3102x^2y^2$$
$$+12441x^4y^4+55901x^6y^6-4954x^8y^8-91x^{10}y^{10})+b^5x^3y^3(429$$
$$+9438x^2y^2+8109x^4y^4-33577x^6y^6+674x^8y^8-x^{10}y^{10})$$
$$+b^3xy(26-1573x^2y^2-18876x^4y^4-37229x^6y^6+19511x^8y^8$$
$$-970x^{10}y^{10}+7x^{12}y^{12})-bxy(104+572x^2y^2-3861x^4y^4$$
$$-13073x^6y^6+6397x^8y^8+33x^{10}y^{10}+12x^{12}y^{12})-8\vartheta^{13}+b^2(-2$$
$$+559x^2y^2+2167x^4y^4+16445x^6y^6-27109x^8y^8+3848x^{10}y^{10}$$
$$+156x^{12}y^{12}+2\vartheta^{13})))/(200(-1+b^2)^2x^2y^2\vartheta^{13})\Big] \tag{S8b}$$

$$M_2^{ES,c,ij} = \Big[(9b(5-4y^2+x^2(-7+(5+b^2)y^2)))/(4(1-b^2)^{3/2}x^2y^2)+(9(4b^8x^7y^5(6$$
$$+5y^2)+b^7x^6y^4(-24-(63+79x^2)y^2+5(-7+5x^2)y^4)+b^6x^5y^3(6+(213$$
$$-151x^2)y^2+(-55+197x^2+30x^4)y^4)+b^4x^3y^3(543-415y^2+x^8y^6$$
$$+23x^6y^4(-12+11y^2)+x^2(-747+1358y^2-690y^4)-3x^4y^2(383-376y^2$$
$$+42y^4))+b(-10+8y^2+18x^{10}y^8(3-2y^2)+21x^4y^2(3-7y^2+3y^4)$$
$$+x^8y^6(199-182y^2+36y^4)+x^2(14-55y^2+36y^4)+x^6y^4(129$$
$$-228y^2+136y^4))+b^3x^2y^2(-317+252y^2-9x^8y^6(-9+8y^2)$$
$$+4x^6y^4(138-115y^2+9y^4)+3x^2(147-355y^2+210y^4)+x^4y^2(1041$$
$$-1224y^2+281y^4))+b^5x^4y^4(-480+315y^2-9x^6y^4+x^4(504y^2$$
$$-481y^4)+x^2(618-837y^2+290y^4))+b^2xy(90-72y^2+x^{10}y^8(-9$$
$$+8y^2)+x^2(-126+405y^2-262y^4)+x^8y^6(-231+165y^2-4y^4)$$
$$-2x^6y^4(216-233y^2+72y^4)+x^4(-435y^2+656y^4-219y^6))+x^3y^5(5$$
$$+x^2(41-44y^2)+x^4(-56+61y^2-18y^4)+x^6(-27y^2+24y^4-4y^6)$$
$$+x^8(-6y^4+4y^6))))/(8(1-b^2)^{3/2}x^2y^2\vartheta^9)\Big]$$
$$+\lambda_{c_j}^2\Big[9(96b^9x^6y^4-4b^8x^5y^3(66+(-25+116x^2)y^2)+b^7x^4y^2(192+(235$$
$$+207x^2)y^2+(313x^2-35x^4)y^4)-b^6x^3y(42+(1185-1727x^2)y^2$$
$$+(1857x^2-2525x^4)y^4+6x^4(7+9x^2)y^6)+b^5x^2y^2(1650+11x^8y^6$$
$$+x^6y^4(-639+11y^2)-3x^4y^2(1862+33y^2)+6x^2(-429+551y^2))$$
$$+(b(10-66x^{12}y^{10}+11x^4y^2(-7+30y^2)+x^2(-14+55y^2)+x^{10}y^8(-247$$
$$+66y^2)+x^8y^6(-1650+197y^2)+2x^6y^4(-171+733y^2)))/(x^2y^2)$$
$$-b^4xy(1155+x^{10}y^8+x^8y^6(-449+y^2)-3x^4y^2(1474+167y^2)$$
$$+x^6y^4(-513+169y^2)+3x^2(-567+932y^2))+x^3y^3(-546+6x^8y^6$$
$$+x^2(616+57y^2)+x^4(13y^2-33y^4)+x^6(33y^4-6y^6))+(b^2(-110$$
$$+9x^{12}y^{10}+x^2(154-495y^2)-3x^{10}y^8(-114+y^2)-93x^4y^2(-7$$
$$+y^2)-11x^6y^4(3+191y^2)+x^8(2033y^6-306y^8)))/(xy)-3b^3(-165$$

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$$+33x^{10}y^8 + x^2(231 - 440y^2) + x^4y^2(632 + 129y^2) - x^6y^4(155$$
$$+413y^2) + x^8(463y^6 - 11y^8)) + (2b(-5 + 7x^2)\vartheta^{11})/(x^2y^2))/(40(1$$
$$-b^2)^{3/2}\vartheta^{11})\Big] + \lambda_{c_i}^2\Big[9(-20b^8x^5y^5(-5 + 14y^2) + b^7x^4y^4(235$$
$$+(235 + 313x^2)y^2 - 175x^2y^4) - b^6x^3y^3(1185 + (-605 + 1857x^2)y^2 + 3x^2(-645$$
$$+14x^2)y^4) + b^5x^2y^2(1650 + (-1175 + 3306x^2)y^2 - x^2(3695 + 99x^2)y^4$$
$$+x^4(-62 + 11x^2)y^6) - b^4xy(1155 + (-889 + 2796x^2)y^2 - x^2(3025$$
$$+501x^2)y^4 + x^4(692 + 169x^2)y^6 + x^6(-198 + x^2)y^8) + b^3(495 + 132(-3$$
$$+10x^2)y^2 - x^2(1346 + 387x^2)y^4 + 7x^4(163 + 177x^2)y^6 + 11x^6(-85$$
$$+3x^2)y^8 - 44x^8y^{10}) + (b(10 + (-8 + 55x^2)y^2 + 22x^2(-2 + 15x^2)y^4$$
$$+x^4(-119 + 1466x^2)y^6 + x^6(-1377 + 197x^2)y^8 + 2x^8(-74 + 33x^2)y^{10}$$
$$-44x^{10}y^{12}))/(x^2y^2) + (b^2(-110 + (88 - 495x^2)y^2 + (466x^2 - 93x^4)y^4$$
$$-11x^4(43 + 191x^2)y^6 + (1715x^6 - 306x^8)y^8 + x^8(220 - 3x^2)y^{10}$$
$$+4x^{10}y^{12}))/(xy) + xy^3(-35 + 11x^6y^4(-3 + 2y^2) + 7x^2(-78$$
$$+77y^2) + x^4(57y^2 - 38y^4) + x^8(-6y^6 + 4y^8)) + (2b(-5 + 4y^2)\vartheta^{11})$$
$$/(x^2y^2))/(40(1 - b^2)^{3/2}\vartheta^{11})\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[9(800b^9x^8y^8 + 8b^8x^7y^7(-70$$
$$+557x^2y^2) + b^7x^6y^6(-3055 - 18062x^2y^2 + 893x^4y^4)$$
$$+b^6x^5y^5(6435 + 28044x^2y^2 - 13389x^4y^4 + 94x^6y^6) + 3x^5y^5(2002$$
$$-2287x^2y^2 + 342x^4y^4 + 13x^6y^6 + 2x^8y^8) - b^5x^4y^4(5610$$
$$+21528x^2y^2 - 35445x^4y^4 + 3430x^6y^6 + 13x^8y^8) + b^3x^2y^2(-715$$
$$+495x^2y^2 + 22191x^4y^4 - 41354x^6y^6 + 606x^8y^8 - 39x^{10}y^{10})$$
$$+b^4x^3y^3(2145 + 8151x^2y^2 - 39819x^4y^4 + 21586x^6y^6 + 176x^8y^8$$
$$+x^{10}y^{10}) + b^2xy(130 + 715x^2y^2 - 7722x^4y^4 + 36356x^6y^6$$
$$-6529x^8y^8 + 471x^{10}y^{10} + 3x^{12}y^{12}) - b(10 + 65x^2y^2 + 2035x^4y^4$$
$$+14768x^6y^6 - 10301x^8y^8 + 929x^{10}y^{10} + 78x^{12}y^{12} - 10\vartheta^{13}))$$
$$/(200(1 - b^2)^{3/2}x^2y^2\vartheta^{13})\Big] \tag{S8c}$$

$$M_3^{ES,c,ij} = \Big[(9b(4 - 2y^2 + x^2(-2 + y^2 + b^2(-3 + 2y^2))))/(4(1 - b^2)^{3/2}x^2y^2)$$
$$+(9(4b^8x^7y^5(-18 + 13y^2) + b^7x^6y^4(252 + (-135 + 97x^2)y^2 - 3(5 + 29x^2)y^4)$$
$$+b^6x^5y^3(-318 + (246 - 422x^2)y^2 + (-26 + 241x^2 - 27x^4)y^4 + 3x^2(13$$
$$+9x^2)y^6) + b^5x^4y^2(189 + 9(-47 + 64x^2)y^2 + 3(52 - 180x^2 + 125x^4)y^4$$
$$+x^2(62 - 178x^2 + 9x^4)y^6 - 9(x^4 + x^6)y^8) + b^3x^2(6 + (-256$$
$$+153x^2)y^2 + 9(14 - 83x^2 + 47x^4)y^4 + 6x^2(53 - 120x^2 + 68x^4)y^6$$
$$+x^4(299 - 322x^2 + 81x^4)y^8 - 9x^6(-4 + 5x^2)y^{10}) + b^4x^3y(-54 + (456$$
$$-393x^2)y^2 + (-214 + 893x^2 - 606x^4)y^4 - 3x^2(100 - 174x^2 + 99x^4)y^6$$
$$-x^4(138 - 170x^2 + x^4)y^8 + (x^6 + x^8)y^{10}) - x^3y^5(4 + x^2(22$$
$$-26y^2) + x^6y^4(-3 + y^2) + x^4(-35 + 19y^2 + 6y^4)) + b^2xy$$
$$(-4x^8y^6(-3 + y^2)^2 - 36(-2 + y^2) + x^{10}y^8(-9 + 5y^2) - 2x^2(18 - 135y^2$$
$$+59y^4) + x^4(-129y^2 + 395y^4 - 204y^6) + x^6(-195y^4 + 248y^6 - 39y^8))$$
$$+b(4(-2 + y^2) + 6x^4y^2(3 - 9y^2 + 5y^4) + 2x^2(2 - 19y^2 + 9y^4)$$
$$+x^8y^6(-40 + 11y^2 + 9y^4) + x^6(9y^4 - 33y^6 - 10y^8))))/(8(1 - b^2)^{3/2}x^2y^2\vartheta^9)\Big]$$
$$-\lambda_{c_i}^2\Big[9(20b^8x^7y^7(-13 + 6y^2) + b^7x^6y^6(325 + (-91 + 463x^2)y^2$$
$$-393x^2y^4) + b^6x^5y^5(360 + (-286 + 183x^2)y^2 + (271x^2 - 27x^4)y^4$$

$$+27x^4y^6) + b^5x^4y^4(-990 + (548 - 2124x^2)y^2 + (602x^2 - 1581x^4)y^4$$
$$+x^4(1193 + 11x^2)y^6 - 11x^6y^8) + b^4x^3y^3(924 + (-490 + 2664x^2)y^2$$
$$+(-946x^2 + 3114x^4)y^4 + x^4(-2362 + 239x^2)y^6 - x^6(143 + x^2)y^8$$
$$+x^8y^{10}) + b^3x^2y^2(-396 - 66(-3 + 26x^2)y^2 + (752x^2 - 2271x^4)y^4$$
$$+7x^4(263 + 27x^2)y^6 - x^6(415 + 66x^2)y^8 + 44x^8y^{10}) + b^2xy(88$$
$$+44(-1 + 9x^2)y^2 + 2x^2(-71 + 492x^2)y^4 - x^4(946 + 721x^2)y^6 + x^6(923$$
$$+219x^2)y^8 + x^8(-103 + 6x^2)y^{10} - 4x^{10}y^{12}) - x^3y^5(28$$
$$+x^6y^4(-9 + y^2) + x^8y^6(-3 + y^2) - 6x^2(-52 + 55y^2) + 4x^4y^2(-81$$
$$+68y^2)) + b(22(x^2 + 3x^4)y^4 + (20x^4 + 374x^6)y^6 - x^6(504$$
$$+391x^2)y^8 + 11x^8(25 - 3x^2)y^{10} + 11x^{10}y^{12} + 8(-1 + \vartheta^{11}) - 4y^2(-1$$
$$+11x^2 + \vartheta^{11})))/(40(1 - b^2)^{3/2}x^2y^2\vartheta^{11})\Big] - \lambda_{c_j}^2\Big[9(288b^9x^8y^6$$
$$-4b^8x^7y^5(198 + (65 + 68x^2)y^2) + b^7x^6y^4(996 + (325 + 681x^2)y^2$$
$$+(463x^2 - 161x^4)y^4) - b^6x^5y^3(714 + 8(-45 + 143x^2)y^2 + x^2(-183$$
$$+451x^2)y^4 + 9x^4(3 + 7x^2)y^6) + b^5x^4y^2(297 + 165(-6 + 7x^2)y^2$$
$$+9x^2(-236 + 235x^2)y^4 + 3x^4(-527 + 614x^2)y^6 + 11(x^6 + x^8)y^8)$$
$$-b^4x^3y(66 + (-924 + 717x^2)y^2 + 3x^2(-888 + 781x^2)y^4 + (-3114x^4$$
$$+3477x^6)y^6 + x^6(-239 + 424x^2)y^8 + (x^8 + x^{10})y^{10}) + x^5y^5(-312$$
$$+3x^6y^6 + x^4y^2(-245 + 9y^2) + x^2(385 + 324y^2)) + b^2xy(88$$
$$-9x^{12}y^{10} + 3x^{10}y^8(-21 + 2y^2) + 44x^2(-1 + 9y^2) + x^8y^6(1145 + 219y^2)$$
$$+3x^4y^2(-73 + 328y^2) - x^6y^4(726 + 721y^2)) + 3b^3x^2(2 + 11(-12 + 7x^2)y^2$$
$$+(-572x^2 + 434x^4)y^4 + x^4(-757 + 759x^2)y^6 + (63x^6 - 77x^8)y^8 + 11x^8(-2$$
$$+3x^2)y^{10} - 2\vartheta^{11}) + b(x^{10}y^8(200 - 33y^2) + 17x^6y^4(-9 + 22y^2) - x^8y^6(741$$
$$+391y^2) + 22x^4(y^2 + 3y^4) + 8(-1 + \vartheta^{11}) - 4x^2(-1 + 11y^2 + \vartheta^{11})))/(40(1$$
$$-b^2)^{3/2}x^2y^2\vartheta^{11})\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[(9(2080b^9x^8y^8$$
$$-8b^8x^7y^7(650 + 467x^2y^2) + b^7x^6y^6(4225 + 3482x^2y^2 - 1883x^4y^4)$$
$$+b^6x^7y^7(7281 + 17178x^2y^2 - 139x^4y^4) + 3x^5y^5(-1144 + 2764x^2y^2$$
$$-699x^4y^4 + 14x^6y^6 + x^8y^8) + b^5x^4y^4(-1518 - 14742x^2y^2$$
$$-32673x^4y^4 + 4312x^6y^6 + 13x^8y^8) + b^4x^3y^3(1716 + 8580x^2y^2$$
$$+26490x^4y^4 - 19525x^6y^6 + 580x^8y^8 - x^{10}y^{10}) - b^3x^2y^2(572$$
$$+5544x^2y^2 + 7215x^4y^4 - 29420x^6y^6 + 5835x^8y^8 + 78x^{10}y^{10})$$
$$+b^2xy(104 + 572x^2y^2 + 5148x^4y^4 - 23135x^6y^6 + 13900x^8y^8$$
$$+141x^{10}y^{10} + 6x^{12}y^{12}) + b(-8 - 52x^2y^2 + 1342x^4y^4 + 4004x^6y^6$$
$$-12605x^8y^8 + 1118x^{10}y^{10} - 39x^{12}y^{12} + 8\vartheta^{13})))/(200(1 - b^2)^{3/2}x^2y^2\vartheta^{13})\Big] \tag{S8d}$$

$$M_4^{ES,c,ij} = \Big[(9(3 + (-3 - 4b^2 + 4b^4)x^2)(-1 + y^2))/(4(-1 + b^2)^2x^2y^2) + (3(-18(-1 + y^2)$$
$$+162bxy(-1 + y^2) - 27b^2(-15 + 7b^2 + 2b^4)x^{10}y^8(-1 + y^2) + 3b(-15 + 7b^2$$
$$+2b^4)x^{11}y^9(-1 + y^2) + 3bx^9y^7(67 - 78y^2 + 11y^4 + 63b^6(-1 + y^2)$$
$$+b^2(442 - 435y^2 - 7y^4) + 2b^4(-115 + 114y^2 + y^4)) + x^2(-18 + 94y^2$$
$$-78y^4 + b^2(-24 + 601y^2 - 573y^4) + b^4(24 - 29y^2 + 3y^4)) - 3bx^3y(-54$$
$$+232y^2 - 182y^4 + b^2(-72 + 409y^2 - 329y^4) + b^4(72 - 83y^2 + 7y^4))$$
$$+3bx^7y^5(90 - 155y^2 + 49y^4 + 20b^8(-6 + 7y^2) + b^4(-193 + 409y^2 - 212y^4)$$
$$+b^6(-277 + 174y^2 + 63y^4) + 4b^2(314 - 385y^2 + 79y^4)) + x^8y^6(3 - 5y^2$$
$$+b^8(285 - 315y^2) + b^2(-1395 + 1726y^2 - 297y^4) - 6b^6(-230 + 206y^2 + 9y^4)$$

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$$+b^4(-1785 + 1504y^2 + 189y^4)) - 3x^6y^4(-16 + 9y^2 - y^4 + 5b^8(-72 + 61y^2$$
$$+21y^4) - 4b^6(34 - 77y^2 + 76y^4) + b^2(751 - 1116y^2 + 349y^4) + b^4(517 - 766y^2$$
$$+355y^4)) - 3x^4y^2(-2(-13 + 9y^2 + 8y^4) + b^6(-252 + 226y^2 + 44y^4) + b^4(215$$
$$-344y^2 + 85y^4) + b^2(227 - 836y^2 + 643y^4)) + bx^5y^3(b^4(573 + 286y^2$$
$$-1227y^4) + 6(96 - 145y^2 + 33y^4) + b^6(-1290 + 997y^2 + 429y^4) + b^2(1653$$
$$-4193y^2 + 2868y^4))))/(8(-1 + b^2)^2x^2y^2\vartheta^9)\Big] - \lambda_{c_i}^2\Big[9(6 - 2x^2(14$$
$$+209x^2)y^4 + x^4(482 + 195x^2)y^6 + x^6(-131 + 3x^2)y^8 - 5x^8y^{10}$$
$$+140b^9x^7y^7(-5 + 6y^2) + b^8x^6y^6(1587 + (-1937 + 161x^2)y^2 - 231x^2y^4)$$
$$+b^7x^5y^5(-1277 + (1573 + 2208x^2)y^2 - x^2(2488 + 99x^2)y^4 + 99x^4y^6)$$
$$+2b^6x^4y^4(165 - 2(106 + 1257x^2)y^2 + (2992x^2 - 254x^4)y^4$$
$$+x^4(359 + 11x^2)y^6 - 11x^6y^8) + b^2x^2y^2(311 + (-307 + 2354x^2)y^2$$
$$-2x^2(1288 + 429x^2)y^4 + (986x^4 + 600x^6)y^6 + x^6(-526 + 121x^2)y^8$$
$$-121x^8y^{10}) + b^3x^3y^3(-799 + (775 - 6089x^2)y^2 + (6897x^2 - 584x^4)y^4$$
$$+(840x^4 - 503x^6)y^6 + x^6(503 + 7x^2)y^8 - 7x^8y^{10}) + b^5x^3y^3(53$$
$$+(-41 + 3574x^2)y^2 - 3x^2(1474 + 837x^2)y^4 + x^4(2523 + 334x^2)y^6$$
$$-2x^6(167 + x^2)y^8 + 2x^8y^{10}) + b^4x^2y^2(-7 + (5 - 286x^2)y^2 + (538x^2$$
$$+6876x^4)y^4 + (-7674x^4 + 734x^6)y^6 - 11x^6(86 + 7x^2)y^8$$
$$+77x^8y^{10}) + bxy(-66 + (66 - 244x^2)y^2 + 4x^2(64 + 255x^2)y^4$$
$$-x^4(1276 + 393x^2)y^6 + (265x^6 - 62x^8)y^8 + x^8(62 - 11x^2)y^{10}$$
$$+11x^{10}y^{12}) - 6\vartheta^{11} + y^2(-6 + 26x^2 + 6\vartheta^{11}))/(40(-1 + b^2)^2x^2y^2\vartheta^{11})\Big]$$

$$-\lambda_{c_j}^2\Big[9(6 - 66bxy + 11b^2(-15 + 7b^2 + 2b^4)x^{12}y^{10} + b(15 - 7b^2 - 2b^4)x^{13}y^{11}$$
$$+bx^{11}y^9(84 - 99b^6 - 11y^2 - 2b^4(178 + y^2) + b^2(701 + 7y^2)) + bx^3y(66$$
$$-244y^2 + b^2(88 - 799y^2) + b^4(-88 + 53y^2)) + x^{10}y^8(-5 + 281b^8 + b^6(766 + 22y^2)$$
$$-b^4(1258 + 77y^2) + b^2(-774 + 121y^2)) + bx^9y^7(367 + 80b^8 - 62y^2 + b^2(1558$$
$$-503y^2) - b^6(2686 + 99y^2) + b^4(2661 + 334y^2)) + x^8y^6(-131 + 480b^{10} + 3y^2$$
$$+b^6(5528 - 508y^2) + 23b^8(-39 + 7y^2) + 6b^2(59 + 100y^2) + b^4(-8106 + 734y^2))$$
$$-bx^7y^5(1122 + 393y^2 + 20b^8(66 + 35y^2) + b^2(-7755 + 584y^2) - 3b^6(561 + 736y^2)$$
$$+3b^4(1408 + 837y^2)) + x^4y^2(-28 - 418y^2 + 66b^6(6 + 5y^2) - b^4(347 + 286y^2) + b^2(-351$$
$$+2354y^2)) + bx^5y^3(326 + 1020y^2 + b^2(961 - 6089y^2) - b^6(994 + 1277y^2) + b^4(697$$
$$+3574y^2)) + x^6y^4(482 + 195y^2 - 2b^2(1520 + 429y^2) - 4b^6(322 + 1257y^2) + b^8(1520$$
$$+1587y^2) + b^4(346 + 6876y^2)) - 6\vartheta^{11} + x^2(-6 + 26y^2 + 6\vartheta^{11} + b^4(8 - 7y^2 - 8\vartheta^{11})$$
$$+b^2(-8 + 311y^2 + 8\vartheta^{11})))/(40(-1 + b^2)^2x^2y^2\vartheta^{11})\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[9(6 + 4x^2y^2$$
$$-4204x^4y^4 + 7969x^6y^6 - 1704x^8y^8 + 5600b^{10}x^8y^8 - 15x^{10}y^{10} + 8b^9x^7y^7$$
$$(-2106 + 371x^2y^2) + b^8x^6y^6(20631 - 29210x^2y^2 + 779x^4y^4)$$
$$-b^7x^5y^5(12727 - 67279x^2y^2 + 11241x^4y^4 + 143x^6y^6)$$
$$+2b^6x^4y^4(1697 - 34827x^2y^2 + 28352x^4y^4 - 1171x^6y^6 + 13x^8y^8)$$
$$+b^5x^3y^3(287 + 37375x^2y^2 - 104483x^4y^4 + 15115x^6y^6 + 492x^8y^8$$
$$-2x^{10}y^{10}) + b^3x^3y^3(-1861 - 45578x^2y^2 + 61651x^4y^4 - 12669x^6y^6$$
$$-750x^8y^8 + 7x^{10}y^{10}) - b^4x^2y^2(35 + 8847x^2y^2$$
$$-93106x^4y^4 + 44734x^6y^6 - 2761x^8y^8 + 91x^{10}y^{10}) + b^2x^2y^2(499$$
$$+13947x^2y^2 - 41756x^4y^4 + 21066x^6y^6 + 533x^8y^8 + 143x^{10}y^{10})$$
$$-bxy(78 + 142x^2y^2 - 13208x^4y^4 + 17895x^6y^6 - 1537x^8y^8$$

$$+67x^{10}y^{10} + 11x^{12}y^{12}) - 6\vartheta^{13})/(200(-1+b^2)^2x^2y^2\vartheta^{13})\Big] \tag{S8e}$$

$$
\begin{aligned}
M_5^{ES,c,ij} =\ & \Big[(-9((-2+x^2)(-2+y^2) + b^4x^2(2+y^2) + b^2(5-7y^2+x^2(-9+7y^2)))) \\
& /(8(-1+b^2)^2x^2y^2) - (3(x^8y^6(-27+y^2) + 12(-2+y^2) + 24b^9x^7y^5(6+5y^2) \\
& -3x^6y^4(-76+73y^2+9y^4) + x^2(12-119y^2+57y^4) + 3x^4y^2(19 \\
& -105y^2+76y^4) + 6b^8x^6y^4(-84-(39+79x^2)y^2+5(-7+5x^2)y^4) \\
& +4b^7x^5y^3(159+(139+141x^2)y^2+3(-13+37x^2+15x^4)y^4) \\
& -6b^6x^4y^2(63+(227-200x^2)y^2+(-263+452x^2-184x^4)y^4+x^2(-241 \\
& +321x^2+9x^4)y^6) + 2b^5x^3y(54+(831-1248x^2)y^2+(-1113 \\
& +2537x^2-2193x^4)y^4-3x^2(668-773x^2+70x^4)y^6+3x^4(-79 \\
& +143x^2+x^4)y^8) + b^4x^2(-12+2(-478+825x^2)y^2+3(442-1591x^2 \\
& +1607x^4)y^4+3x^2(1355-1895x^2+758x^4)y^6+x^4(1806-1727x^2 \\
& +135x^4)y^8-27x^6(-5+9x^2)y^{10}) + b^2(-81x^{10}y^8(-5+3y^2) \\
& +6(-5+7y^2)+3x^4y^2(205-766y^2+311y^4)+x^8y^6(1659-1520y^2 \\
& +351y^4) + 3x^6y^4(353-830y^2+507y^4)+x^2(54-923y^2+561y^4)) \\
& +b^3xy(54(5-7y^2)+3x^{10}y^8(-5+9y^2)-3x^8y^6(503 \\
& -320y^2+5y^4)-3x^6y^4(793-876y^2+431y^4)-3x^2(162-839y^2 \\
& +637y^4)+x^4(-2271y^2+4438y^4-1911y^6)) + 3bxy(-36(-2+y^2) \\
& +3x^{10}y^8(-5+3y^2)+x^2(-36+281y^2-133y^4)+x^8y^6(-65+60y^2 \\
& -13y^4)+x^4(-135y^2+424y^4-243y^6)+x^6(-249y^4+306y^6 \\
& -59y^8))))/(16(-1+b^2)^2x^2y^2\vartheta^9)\Big] + \lambda_{c_i}^2\Big[9((27x^2 \\
& -385x^4)y^4+x^4(437+821x^2)y^6-x^6(801+97x^2)y^8+45x^8y^{10} \\
& +40b^9x^7y^7(-5+14y^2)+2b^8x^6y^6(61-(351+313x^2)y^2+175x^2y^4) \\
& +4b^7x^5y^5(172+(-143+619x^2)y^2+3x^2(-291+7x^2)y^4)-2b^6x^4y^4(660 \\
& +(-883+2131x^2)y^2-6x^2(573+127x^2)y^4+x^4(257+11x^2)y^6) \\
& +2b^5x^3y^3(604+(-829+2189x^2)y^2-3x^2(1089+409x^2)y^4+(1083x^4 \\
& -47x^6)y^6+x^6(-123+x^2)y^8) + b^4x^2y^2(-502+(698-3421x^2)y^2 \\
& +x^2(3901+201x^2)y^4-x^4(1821+3511x^2)y^6+x^6(2927+33x^2)y^8 \\
& +55x^8y^{10}) + bxy(88+(-44+449x^2)y^2+x^2(-239+1134x^2)y^4-2x^4(605 \\
& +404x^2)y^6+2x^6(444+59x^2)y^8+x^8(-70+19x^2)y^{10} \\
& -13x^{10}y^{12}) + b^3xy(110+(-154+1313x^2)y^2+x^2(-1073+2116x^2)y^4 \\
& +6926x^6y^6+14x^6(-433+63x^2)y^8-x^8(674+3x^2)y^{10}-5x^{10}y^{12}) \\
& +y^2(4-51x^2-4\vartheta^{11})+8(-1+\vartheta^{11})+b^2((265x^2-814x^4)y^4-2x^4(82 \\
& +2599x^2)y^6+(4764x^6-260x^8)y^8+x^8(162-209x^2)y^{10} \\
& +143x^{10}y^{12}+y^2(14-437x^2-14\vartheta^{11})+10(-1+\vartheta^{11})))/(80(-1+b^2)^2x^2y^2\vartheta^{11})\Big] \\
& -\lambda_{c_j}^2\Big[9(8+192b^{10}x^8y^6-45x^{10}y^8+x^8y^6(801+97y^2)+x^4y^2(-27+385y^2) \\
& -x^6y^4(437+821y^2)-8b^9x^7y^5(66+(-25+116x^2)y^2)+2b^8x^6y^4(332 \\
& +(-61+647x^2)y^2+(313x^2-35x^4)y^4)-4b^7x^5y^3(119+(172 \\
& -66x^2)y^2+(619x^2-840x^4)y^4+3x^4(7+9x^2)y^6) \\
& +2b^6x^4y^2(99+(660-955x^2)y^2+(2131x^2-3516x^4)y^4+x^4(-762 \\
& +233x^2)y^6+11(x^6+x^8)y^8)-2b^5x^3y(22+(604-982x^2)y^2 \\
& +(2189x^2-3465x^4)y^4+3x^4(-409+345x^2)y^6-x^6(47+131x^2)y^8 \\
& +(x^8+x^{10})y^{10}) + bxy(-88+15x^{12}y^{10}+x^2(44-449y^2)
\end{aligned}
$$

$$+x^{10}y^8(90-19y^2)-2x^8y^6(420+59y^2)+2x^6y^4(627+404y^2)$$
$$+x^4(253y^2-1134y^4))+b^3xy(-110+5x^{12}y^{10}+x^2(198-1313y^2)$$
$$+x^{10}y^8(746+3y^2)+x^4(1229y^2-2116y^4)+x^6(396y^4-6926y^6)$$
$$+x^8(6418y^6-882y^8))-8\vartheta^{11}+x^2(-4+51y^2+4\vartheta^{11})-b^4x^2((-502$$
$$+874x^2)y^2+x^2(-3421+4285x^2)y^4-3x^4(-67+479x^2)y^6+x^6(-3511$$
$$+3011x^2)y^8+11x^8(3+5x^2)y^{10}+4(-1+\vartheta^{11}))+b^2(-165x^{12}y^{10}$$
$$+4x^8y^6(-1252+65y^2)+x^{10}y^8(-310+209y^2)+x^4y^2(-287$$
$$+814y^2)+x^6(28y^4+5198y^6)-10(-1+\vartheta^{11})+x^2(-18+437y^2+18\vartheta^{11})))$$
$$/(80(-1+b^2)^2x^2y^2\vartheta^{11})\Big]-\lambda_{c_j}^2\lambda_{c_i}^2\Big[9^2(-8-87x^2y^2-2598x^4y^4+15678x^6y^6$$
$$-9038x^8y^8+1600b^{10}x^8y^8+485x^{10}y^{10}+16b^9x^7y^7(-218+557x^2y^2)$$
$$+2b^8x^6y^6(793-15774x^2y^2+893x^4y^4)+4b^7x^5y^5(572+11655x^2y^2-8506x^4y^4$$
$$+47x^6y^6)-2b^6x^4y^4(1108+19123x^2y^2-41949x^4y^4$$
$$+3289x^6y^6+13x^8y^8)+2b^5x^3y^3(1216+8177x^2y^2-42886x^4y^4$$
$$+28762x^6y^6-166x^8y^8+x^{10}y^{10})+b^4x^2y^2(-750-11751x^2y^2$$
$$+47086x^4y^4-116904x^6y^6+4136x^8y^8+39x^{10}y^{10})+bxy(104$$
$$+859x^2y^2+4199x^4y^4-27878x^6y^6+9062x^8y^8+83x^{10}y^{10}$$
$$+19x^{12}y^{12})+b^3(130xy+1857x^3y^3+325x^5y^5+101406x^7y^7$$
$$-28604x^9y^9+1465x^{11}y^{11}-3x^{13}y^{13})+8\vartheta^{13}-b^2(10+567x^2y^2$$
$$-3695x^4y^4+56992x^6y^6-48826x^8y^8+4977x^{10}y^{10}+247x^{12}y^{12}$$
$$-10\vartheta^{13}))/(400(-1+b^2)^2x^2y^2\vartheta^{13})\Big] \tag{S8f}$$

$$M_6^{ES,c,ij} = \Big[(-9b^2(-3+(1+2b^2)x^2)(-1+y^2))/(4(-1+b^2)^2x^2y^2)-(3(12b^9x^7y^5(-18$$
$$+13y^2)-3b^8x^6y^4(-192+(127-97x^2)y^2+3(5+29x^2)y^4)+b^7x^5y^3(-654$$
$$+(677-699x^2)y^2-3(53-234x^2+27x^4)y^4+9x^2(13+9x^2)y^6)$$
$$-3b^6x^4y^2(-126+(347-293x^2)y^2+(-239+529x^2-146x^4)y^4+(-104x^2$$
$$+167x^4-9x^6)y^6+9(x^4+x^6)y^8)+b^4x^2(12+(-574+240x^2)y^2$$
$$+3(188-569x^2+265x^4)y^4+3x^2(445-577x^2+334x^4)y^6+x^4(1254-1045x^2$$
$$+135x^4)y^8-135x^6(-1+x^2)y^{10})+b^5x^3y(-108-60(-17+10x^2)y^2-2(462$$
$$-1126x^2+507x^4)y^4-3x^2(428-509x^2+175x^4)y^6-3x^4(175-174x^2$$
$$+x^4)y^8+3(x^6+x^8)y^{10})-3bx^3y^3(11-7y^2+x^2(-15+16y^2$$
$$-33y^4)+2x^6y^4(2-3y^2+y^4)+x^4y^2(-51+19y^2+16y^4))$$
$$+b^3xy(-162(-1+y^2)+15x^{10}y^8(-1+y^2)+x^2(-54+687y^2-609y^4)$$
$$-15x^8y^6(2-3y^2+y^4)-12x^6y^4(41-49y^2+16y^4)$$
$$+x^4(-303y^2+899y^4-924y^6))+x^2y^2(5-3y^2+x^2(-3+45y^2$$
$$-66y^4)+6x^4y^2(-11+7y^4)+x^6(42y^4-40y^6))+b^2(18(-1+y^2)$$
$$+x^8y^6(-261+173y^2+54y^4)+x^2(6-97y^2+87y^4)+3x^4y^2(11$$
$$-71y^2+94y^4)+x^6(84y^4-81y^6-51y^8))))/(8(-1+b^2)^2x^2y^2\vartheta^9)\Big]$$
$$+\lambda_{c_i}^2\Big[9(20b^9x^7y^7(-13+6y^2)+b^8x^6y^6(333+(17+463x^2)y^2-393x^2y^4)$$
$$+b^7x^5y^5(287+(-583+156x^2)y^2+(124x^2-27x^4)y^4+27x^4y^6)$$
$$+x^2y^2(7+(-5+121x^2)y^2-37x^2(5+12x^2)y^4+4x^4(95$$
$$+18x^2)y^6-70x^6y^8)+b^6x^4y^4(-825+(919-1869x^2)y^2+(913x^2$$
$$-1637x^4)y^4+x^4(1427+11x^2)y^6-11x^6y^8)+bx^3y^3(-53+(41$$
$$-30x^2)y^2+x^2(286+711x^2)y^4-x^4(583+10x^2)y^6+2x^6(5$$

$$+x^2)y^8 - 2x^8y^{10}) + b^5x^3y^3(640 + 4(-163 + 542x^2)y^2 + 3x^2(-440$$
$$+987x^2)y^4 + x^4(-2973 + 176x^2)y^6 - x^6(176 + x^2)y^8 + x^8y^{10})$$
$$+b^4x^2y^2(-290 + (292 - 1001x^2)y^2 + (749x^2 - 1863x^4)y^4 + x^4(2661$$
$$+769x^2)y^6 - x^6(557 + 55x^2)y^8 + 55x^8y^{10}) + b^3xy(66 + (-66$$
$$+403x^2)y^2 + x^2(-379 + 347x^2)y^4 - x^4(1155 + 1588x^2)y^6 + x^6(1332$$
$$+191x^2)y^8 + x^8(-191 + 5x^2)y^{10} - 5x^{10}y^{12}) + b^2((43x^2$$
$$-275x^4)y^4 + 7x^4(71 + 153x^2)y^6 - x^6(1199 + 657x^2)y^8 + 11x^8(53$$
$$-2x^2)y^{10} + 22x^{10}y^{12} + y^2(6 - 47x^2 - 6\vartheta^{11}) + 6(-1 + \vartheta^{11})))/(40(-1$$
$$+b^2)^2x^2y^2\vartheta^{11})\Big] + \lambda_{c_j}^2\Big[9(288b^{10}x^8y^6 - 4b^9x^7y^5(198$$
$$+(65 + 68x^2)y^2) + b^8x^6y^4(856 + (333 + 465x^2)y^2 + (463x^2 - 161x^4)y^4)$$
$$+b^7x^5y^3(-518 + (287 - 165x^2)y^2 + 2x^2(78 + 29x^2)y^4 - 9x^4(3$$
$$+7x^2)y^6) + b^6x^4y^2(198 + (-825 + 199x^2)y^2 + x^2(-1869 + 613x^2)y^4$$
$$+x^4(-1637 + 1523x^2)y^6 + 11(x^6 + x^8)y^8) - b^5x^3y(44 + 20(-32$$
$$+11x^2)y^2 + 2x^2(-1084 + 759x^2)y^4 + 3x^4(-987 + 977x^2)y^6 + 2x^6(-88$$
$$+107x^2)y^8 + (x^8 + x^{10})y^{10}) + bx^3y^3(-53 + x^2(97 - 30y^2)$$
$$+2x^8y^6(6 + y^2) - x^6y^4(529 + 10y^2) + 9x^4y^2(44 + 79y^2))$$
$$+x^2y^2(7 - 70x^8y^6 - 37x^4y^2(5 + 12y^2) + 4x^6y^4(95 + 18y^2)$$
$$+x^2(-5 + 121y^2)) + b^3xy(66 - 5x^{12}y^{10} + 5x^{10}y^8(-13 + y^2)$$
$$+x^8y^6(1694 + 191y^2) + x^4y^2(-349 + 347y^2) + x^2(-22 + 403y^2)$$
$$-x^6y^4(693 + 1588y^2)) + b^4x^2(4 + 58(-5 + 2x^2)y^2 + x^2(-1001 + 941x^2)y^4$$
$$+3x^4(-621 + 871x^2)y^6 + (769x^6 - 785x^8)y^8 + 55x^8(-1 + x^2)y^{10}$$
$$-4\vartheta^{11}) + b^2(x^{10}y^8(483 - 22y^2) - 3x^8y^6(529 + 219y^2) + x^6y^4(169$$
$$+1071y^2) + x^4(21y^2 - 275y^4) + x^2(2 - 47y^2 - 2\vartheta^{11}) + 6(-1 + \vartheta^{11})))$$
$$/(40(-1 + b^2)^2x^2y^2\vartheta^{11})\Big]$$

$$-\lambda_{c_j}^2\lambda_{c_i}^2\Big[9(2080b^{10}x^8y^8 - 8b^9x^7y^7(654 + 467x^2y^2) + b^8x^6y^6$$
$$(4329 + 3626x^2y^2 - 1883x^4y^4) + b^7x^5y^5(-143 + 6659x^2y^2 + 18039x^4y^4$$
$$-139x^6y^6) + 5x^2y^2(7 + 194x^2y^2 - 1469x^4y^4 + 1044x^6y^6$$
$$-72x^8y^8) + b^6x^4y^4(-2305 - 13572x^2y^2 - 34696x^4y^4 + 5600x^6y^6$$
$$+13x^8y^8) + b^5x^3y^3(1000 + 10244x^2y^2 + 28019x^4y^4 - 26611x^6y^6$$
$$+549x^8y^8 - x^{10}y^{10}) + bx^3y^3(-287 + 949x^2y^2 + 12771x^4y^4$$
$$-6031x^6y^6 + 76x^8y^8 + 2x^{10}y^{10}) - b^4x^2y^2(394 + 855x^2y^2$$
$$+11206x^4y^4 - 42544x^6y^6 + 8056x^8y^8 + 65x^{10}y^{10}) + b^3xy(78$$
$$+1003x^2y^2 - 3328x^4y^4 - 31921x^6y^6 + 22629x^8y^8 - 18x^{10}y^{10}$$
$$+5x^{12}y^{12}) - b^2(6 + 109x^2y^2 + 2100x^4y^4 - 17498x^6y^6$$
$$+26496x^8y^8 - 2983x^{10}y^{10} + 26x^{12}y^{12} - 6\vartheta^{13}))$$
$$/(200(-1 + b^2)^2x^2y^2\vartheta^{13})\Big] \tag{S8g}$$

$$M_7^{ES,c,ij} = \Big[(9b(-8 + (7 + 5b^2)x^2)(-1 + y^2))/(4(1 - b^2)^{3/2}x^2y^2) + (9(20b^8x^7y^5(-6$$
$$+7y^2) - 5b^7x^6y^4(-84 + (69 - 19x^2)y^2 + 21(1 + x^2)y^4) + b^6x^5y^3(-530$$
$$+(610 - 730x^2)y^2 + (-70 + 617x^2 - 63x^4)y^4 + 63(x^2 + x^4)y^6)$$
$$-3b^5x^4y^2(-105 - 105(-3 + 4x^2)y^2 + (-210 + 579x^2 - 346x^4)y^4 - 3x^2(63$$
$$-110x^2 + 2x^4)y^6 + 6(x^4 + x^6)y^8) + x^5y^3(5 + (41 - 44x^2)y^2$$
$$+(-56 + 61x^2 - 18x^4)y^4 + (-27x^2 + 24x^4 - 4x^6)y^6 + (-6x^4$$

$$+4\mathrm{x}^6)\mathrm{y}^8) + b^3\mathrm{x}^2(10 + (-514 + 486\mathrm{x}^2)\mathrm{y}^2 + 18(28 - 97\mathrm{x}^2 + 70\mathrm{x}^4)\mathrm{y}^4$$

$$+9\mathrm{x}^2(140 - 227\mathrm{x}^2 + 83\mathrm{x}^4)\mathrm{y}^6 + 3\mathrm{x}^4(231 - 295\mathrm{x}^2 + 54\mathrm{x}^4)\mathrm{y}^8$$

$$-54\mathrm{x}^6(-2 + 3\mathrm{x}^2)\mathrm{y}^{10}) + b^4\mathrm{x}^3\mathrm{y}(-90 - 15(-62 + 69\mathrm{x}^2)\mathrm{y}^2 - 3(280 - 811\mathrm{x}^2$$

$$+584\mathrm{x}^4)\mathrm{y}^4 - 3\mathrm{x}^2(476 - 717\mathrm{x}^2 + 192\mathrm{x}^4)\mathrm{y}^6 + (-369\mathrm{x}^4 + 574\mathrm{x}^6$$

$$-2\mathrm{x}^8)\mathrm{y}^8 + 2(\mathrm{x}^6 + \mathrm{x}^8)\mathrm{y}^{10}) + b(16(-1 + \mathrm{y}^2) - 36\mathrm{x}^{10}\mathrm{y}^8(-1 + \mathrm{y}^2)$$

$$+2\mathrm{x}^8\mathrm{y}^6(68 - 90\mathrm{y}^2 + 27\mathrm{y}^4) + 2\mathrm{x}^2(7 - 43\mathrm{y}^2 + 36\mathrm{y}^4) + 3\mathrm{x}^6\mathrm{y}^4(28$$

$$-81\mathrm{y}^2 + 63\mathrm{y}^4) + 63\mathrm{x}^4(\mathrm{y}^2 - 3\mathrm{y}^4 + 2\mathrm{y}^6)) + b^2\mathrm{xy}(-144(-1 + \mathrm{y}^2)$$

$$+18\mathrm{x}^{10}\mathrm{y}^8(-1 + \mathrm{y}^2) - 3\mathrm{x}^8\mathrm{y}^6(69 - 73\mathrm{y}^2 + 4\mathrm{y}^4) - 126\mathrm{x}^2(1 - 5\mathrm{y}^2$$

$$+4\mathrm{y}^4) - 6\mathrm{x}^4\mathrm{y}^2(76 - 158\mathrm{y}^2 + 77\mathrm{y}^4) + \mathrm{x}^6(-378\mathrm{y}^4 + 631\mathrm{y}^6$$

$$-243\mathrm{y}^8)))) / (8(1 - b^2)^{3/2}\mathrm{x}^2\mathrm{y}^2\vartheta^9)\Big] - \lambda_{c_i}^2\Big[9(140b^8\mathrm{x}^7\mathrm{y}^7(-5 + 6\mathrm{y}^2)$$

$$-7b^7\mathrm{x}^6\mathrm{y}^6(-125 + (155 - 23\mathrm{x}^2)\mathrm{y}^2 + 33\mathrm{x}^2\mathrm{y}^4) + b^6\mathrm{x}^5\mathrm{y}^5(840 + 7(-110$$

$$+393\mathrm{x}^2)\mathrm{y}^2 - \mathrm{x}^2(3101 + 99\mathrm{x}^2)\mathrm{y}^4 + 99\mathrm{x}^4\mathrm{y}^6) + b^5\mathrm{x}^4\mathrm{y}^4(-2310$$

$$+(2310 - 6783\mathrm{x}^2)\mathrm{y}^2 + (7413\mathrm{x}^2 - 1077\mathrm{x}^4)\mathrm{y}^4 + 11\mathrm{x}^4(117 + 2\mathrm{x}^2)\mathrm{y}^6$$

$$-22\mathrm{x}^6\mathrm{y}^8) - \mathrm{x}^5\mathrm{y}^5(-546 + (616 + 57\mathrm{x}^2)\mathrm{y}^2 + (13\mathrm{x}^2 - 33\mathrm{x}^4)\mathrm{y}^4$$

$$+(33\mathrm{x}^4 - 6\mathrm{x}^6)\mathrm{y}^6 + 6\mathrm{x}^6\mathrm{y}^8) + b^4\mathrm{x}^3\mathrm{y}^3(1848 + 42(-44 + 149\mathrm{x}^2)\mathrm{y}^2$$

$$+3\mathrm{x}^2(-2156 + 431\mathrm{x}^2)\mathrm{y}^4 + \mathrm{x}^4(-1083 + 583\mathrm{x}^2)\mathrm{y}^6 - \mathrm{x}^6(583 + 2\mathrm{x}^2)\mathrm{y}^8$$

$$+2\mathrm{x}^8\mathrm{y}^{10}) + 3b^3\mathrm{x}^2\mathrm{y}^2(-264 - 132(-2 + 7\mathrm{x}^2)\mathrm{y}^2 + (924\mathrm{x}^2 - 49\mathrm{x}^4)\mathrm{y}^4$$

$$-\mathrm{x}^4(161 + 499\mathrm{x}^2)\mathrm{y}^6 + (429\mathrm{x}^6 - 44\mathrm{x}^8)\mathrm{y}^8 + 44\mathrm{x}^8\mathrm{y}^{10}) + b^2\mathrm{xy}(176$$

$$+88(-2 + 9\mathrm{x}^2)\mathrm{y}^2 - 36\mathrm{x}^2(22 + 7\mathrm{x}^2)\mathrm{y}^4 + \mathrm{x}^4(462 + 1993\mathrm{x}^2)\mathrm{y}^6$$

$$+3\mathrm{x}^6(-641 + 121\mathrm{x}^2)\mathrm{y}^8 + 3\mathrm{x}^8(-121 + 4\mathrm{x}^2)\mathrm{y}^{10} - 12\mathrm{x}^{10}\mathrm{y}^{12})$$

$$+b(-22\mathrm{x}^2(-4 + 9\mathrm{x}^2)\mathrm{y}^4 + (198\mathrm{x}^4 - 1337\mathrm{x}^6)\mathrm{y}^6 + (1547\mathrm{x}^6 - 227\mathrm{x}^8)\mathrm{y}^8 - 33\mathrm{x}^8$$

$$(-9 + 2\mathrm{x}^2)\mathrm{y}^{10} + 66\mathrm{x}^{10}\mathrm{y}^{12} + 16(-1 + \vartheta^{11}) - 8\mathrm{y}^2(-2 + 11\mathrm{x}^2 + 2\vartheta^{11})))$$

$$/(40(1 - b^2)^{3/2}\mathrm{x}^2\mathrm{y}^2\vartheta^{11})\Big] - \lambda_{c_j}^2\Big[9(480b^9\mathrm{x}^8\mathrm{y}^6 + 20b^8\mathrm{x}^7\mathrm{y}^5(-66$$

$$+(-35 + 4\mathrm{x}^2)\mathrm{y}^2) + b^7\mathrm{x}^6\mathrm{y}^4(1660 + 125(7 + 3\mathrm{x}^2)\mathrm{y}^2 + \mathrm{x}^2(161 + 281\mathrm{x}^2)\mathrm{y}^4)$$

$$-b^6\mathrm{x}^5\mathrm{y}^3(1190 + 40(-21 + 55\mathrm{x}^2)\mathrm{y}^2 + \mathrm{x}^2(-2751 + 4139\mathrm{x}^2)\mathrm{y}^4 + 99(\mathrm{x}^4$$

$$+\mathrm{x}^6)\mathrm{y}^6) + \mathrm{x}^5\mathrm{y}^3(35 - 7(-78 + 77\mathrm{x}^2)\mathrm{y}^2 + 19\mathrm{x}^2(-3 + 2\mathrm{x}^2)\mathrm{y}^4 + (33\mathrm{x}^4$$

$$-22\mathrm{x}^6)\mathrm{y}^6 + (6\mathrm{x}^6 - 4\mathrm{x}^8)\mathrm{y}^8) + b^5\mathrm{x}^4\mathrm{y}^2(495 + 15(-154 + 207\mathrm{x}^2)\mathrm{y}^2$$

$$+(-6783\mathrm{x}^2 + 9162\mathrm{x}^4)\mathrm{y}^4 + 3\mathrm{x}^4(-359 + 676\mathrm{x}^2)\mathrm{y}^6 + 22(\mathrm{x}^6 + \mathrm{x}^8)\mathrm{y}^8)$$

$$-b^4\mathrm{x}^3\mathrm{y}(110 + 3(-616 + 669\mathrm{x}^2)\mathrm{y}^2 + (-6258\mathrm{x}^2 + 7887\mathrm{x}^4)\mathrm{y}^4 + 3\mathrm{x}^4(-431$$

$$+784\mathrm{x}^2)\mathrm{y}^6 + 11\mathrm{x}^6(-53 + 82\mathrm{x}^2)\mathrm{y}^8 + 2(\mathrm{x}^8 + \mathrm{x}^{10})\mathrm{y}^{10}) + b^2\mathrm{xy}(176$$

$$-18\mathrm{x}^{12}\mathrm{y}^{10} + 3\mathrm{x}^{10}\mathrm{y}^8(-99 + 4\mathrm{y}^2) - 42\mathrm{x}^4\mathrm{y}^2(19 + 6\mathrm{y}^2)$$

$$+22\mathrm{x}^2(-7 + 36\mathrm{y}^2) + \mathrm{x}^8\mathrm{y}^6(-1547 + 363\mathrm{y}^2) + \mathrm{x}^6\mathrm{y}^4(858 + 1993\mathrm{y}^2))$$

$$+b^3\mathrm{x}^2(44(-18 + 17\mathrm{x}^2)\mathrm{y}^2 + 6\mathrm{x}^2(-462 + 533\mathrm{x}^2)\mathrm{y}^4 - 3\mathrm{x}^4(49 + 39\mathrm{x}^2)\mathrm{y}^6$$

$$+3\mathrm{x}^6(-499 + 501\mathrm{x}^2)\mathrm{y}^8 + 66\mathrm{x}^8(-2 + 3\mathrm{x}^2)\mathrm{y}^{10} - 10(-1 + \vartheta^{11})) + b(44\mathrm{x}^{12}\mathrm{y}^{10}$$

$$+2\mathrm{x}^{10}\mathrm{y}^8(74 - 33\mathrm{y}^2) - \mathrm{x}^6\mathrm{y}^4(43 + 1337\mathrm{y}^2) + \mathrm{x}^4(77\mathrm{y}^2 - 198\mathrm{y}^4)$$

$$+\mathrm{x}^8(1188\mathrm{y}^6 - 227\mathrm{y}^8) + 16(-1 + \vartheta^{11}) - 2\mathrm{x}^2(-7 + 44\mathrm{y}^2 + 7\vartheta^{11})))$$

$$/(40(1 - b^2)^{3/2}\mathrm{x}^2\mathrm{y}^2\vartheta^{11})\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[9(5600b^9\mathrm{x}^8\mathrm{y}^8 + 56b^8\mathrm{x}^7\mathrm{y}^7(-250$$

$$+53\mathrm{x}^2\mathrm{y}^2) + b^6\mathrm{x}^7\mathrm{y}^7(59157 - 8214\mathrm{x}^2\mathrm{y}^2 - 143\mathrm{x}^4\mathrm{y}^4) + b^7\mathrm{x}^6\mathrm{y}^6(11375$$

$$-27146\mathrm{x}^2\mathrm{y}^2 + 779\mathrm{x}^4\mathrm{y}^4) + 3\mathrm{x}^5\mathrm{y}^5(2002 - 2287\mathrm{x}^2\mathrm{y}^2$$

$$+342\mathrm{x}^4\mathrm{y}^4 + 13\mathrm{x}^6\mathrm{y}^6 + 2\mathrm{x}^8\mathrm{y}^8) + b^5\mathrm{x}^4\mathrm{y}^4(-6006 - 58149\mathrm{x}^2\mathrm{y}^2$$

$$+32394\mathrm{x}^4\mathrm{y}^4 - 3481\mathrm{x}^6\mathrm{y}^6 + 26\mathrm{x}^8\mathrm{y}^8) + b^4\mathrm{x}^3\mathrm{y}^3(3432$$

$$+30030\mathrm{x}^2\mathrm{y}^2 - 48855\mathrm{x}^4\mathrm{y}^4 + 19990\mathrm{x}^6\mathrm{y}^6 + 845\mathrm{x}^8\mathrm{y}^8 - 2\mathrm{x}^{10}\mathrm{y}^{10})$$

$$-b^3\mathrm{x}^2\mathrm{y}^2(1144 + 5148\mathrm{x}^2\mathrm{y}^2 - 34125\mathrm{x}^4\mathrm{y}^4 + 40070\mathrm{x}^6\mathrm{y}^6 + 1095\mathrm{x}^8\mathrm{y}^8$$

$$+156x^{10}y^{10}) + b^2xy(208 + 1144x^2y^2 - 15444x^4y^4 + 38015x^6y^6$$
$$-4330x^8y^8 + 507x^{10}y^{10} + 12x^{12}y^{12}) - b(16 + 104x^2y^2$$
$$+286x^4y^4 + 14807x^6y^6 - 8630x^8y^8 + 779x^{10}y^{10} + 78x^{12}y^{12}$$
$$-16\vartheta^{13}))/(200(1-b^2)^{3/2}x^2y^2\vartheta^{13})\Big]$$

<div align="right">(S8h)</div>

$$
\begin{aligned}
M_8^{ES,c,ij} =\ & \Big[(-9b(-2+(1+b^2)x^2))/(2(1-b^2)^{3/2}x^2y^2) - (9(4b^8x^7y^5(6+5y^2) \\
& +b^7x^6y^4(-144-(15+79x^2)y^2+5(-7+5x^2)y^4)+b^6x^5y^3(206 \\
& +5(-17+79x^2)y^2+(35-61x^2+30x^4)y^4)+x^5y^3(4+(22-26x^2)y^2 \\
& +(-35+19x^2+6x^4)y^4+x^4(-3+x^2)y^6)-3b^5x^4y^2(42+3(-35+57x^2)y^2 \\
& +x^2(-57+89x^2)y^4+x^2(-35+28x^2+3x^4)y^6)-b^3x^2(4+36(-7+4x^2)y^2 \\
& +18x^2(-35+17x^2)y^4+(-459x^4+321x^6)y^6+3x^4(35-43x^2+18x^4)y^8) \\
& +b^4x^3y(36+12(-35+29x^2)y^2+12x^2(-47+31x^2)y^4+3x^2(-35-9x^2y^6 \\
& +66x^4)+(x^6+x^8)y^8)+b(8-9x^{10}y^8+x^8y^6(-5+2y^2)+9x^4y^2(-2 \\
& +7y^2)+4x^2(-1+9y^2)+x^6y^4(-45+57y^2+35y^4))+b^2xy(-72 \\
& +6x^{10}y^8-6x^8y^6(-9+y^2)-36x^2(-1+7y^2)+3x^4y^2(38-127y^2 \\
& +35y^4)+x^6(243y^4-239y^6))))/(8(1-b^2)^{3/2}x^2y^2\vartheta^9)\Big] \\
& +\lambda_{c_i}^2\Big[9(20b^8x^7y^7(-5+14y^2)+b^7x^6y^6(485-(595+313x^2)y^2+175x^2y^4) \\
& +b^6x^5y^5(-1005+(385+327x^2)y^2+7x^2(-155+6x^2)y^4)-x^5y^5(-312 \\
& +(385+324x^2)y^2+x^2(-245+9x^2)y^4+3x^6y^6)-b^2xy(88 \\
& +396x^2y^2+204x^8y^8+6x^{10}y^{10}+7x^6y^6(-73+145y^2) \\
& -3x^4y^4(-543+385y^2))+b^4x^3y^3(-924-269x^6y^6+x^8y^8 \\
& -3x^2y^2(458+385y^2)+3x^4y^4(-1018+525y^2))+3b^3x^2y^2(132 \\
& +462x^2y^2+22x^8y^8+x^6y^6(17+175y^2)+x^4(947y^4-595y^6)) \\
& +b^5x^4y^4(1155-11x^6y^6+3x^2y^2(208+595y^2)+x^4(1236y^4 \\
& -525y^6))+b(8+44x^2y^2+99x^4y^4+33x^{10}y^{10}+x^8y^8(346-175y^2) \\
& +x^6y^6(-254+595y^2)-8\vartheta^{11}))/(40(1-b^2)^{3/2}x^2y^2\vartheta^{11})\Big] \\
& -\lambda_{c_j}^2\Big[9(96b^9x^8y^6-4b^8x^7y^5(66+(-25+116x^2)y^2) \\
& +b^7x^6y^4(472+(-485+1311x^2)y^2+(313x^2-35x^4)y^4)-b^6x^5y^3(434 \\
& +3(-335+693x^2)y^2+(327x^2-179x^4)y^4+6x^4(7+9x^2)y^6) \\
& -x^5y^3(28-6(-52+55x^2)y^2+4x^2(-81+68x^2)y^4+x^4(-9 \\
& +x^2)y^6+x^6(-3+x^2)y^8)+b^5x^4y^2(198+3(-385+529x^2)y^2+6x^2(-104 \\
& +57x^2)y^4+6x^4(-206+199x^2)y^6+11(x^6+x^8)y^8)-b^4x^3y(44 \\
& +12(-77+62x^2)y^2+6x^2(-229+44x^2)y^4+6x^4(-509+489x^2)y^6 \\
& +x^6(-269+289x^2)y^8+(x^8+x^{10})y^{10})+b^2xy(88-6x^{12}y^{10} \\
& +6x^{10}y^8(-16+y^2)+44x^2(-1+9y^2)+x^8y^6(851+204y^2) \\
& -x^6y^4(1419+511y^2)+3x^4y^2(-38+543y^2))+b^3x^2(4+44(-9 \\
& +5x^2)y^2+6x^2(-231+71x^2)y^4+3x^4(-947+797x^2)y^6-3x^6(17 \\
& +13x^2)y^8+66x^8(-1+x^2)y^{10}-4\vartheta^{11})+b(11x^{12}y^{10}+x^{10}y^8(200-33y^2) \\
& -2x^8y^6(222+173y^2)+x^6y^4(155+254y^2)+x^4(22y^2-99y^4)+8(-1+\vartheta^{11}) \\
& -4x^2(-1+11y^2+\vartheta^{11})))/(40(1-b^2)^{3/2}x^2y^2\vartheta^{11})\Big]-\lambda_{c_j}^2\lambda_{c_i}^2\Big[9(800b^9x^8y^8 \\
& +8b^8x^7y^7(-430+557x^2y^2)+b^7x^6y^6(6305-12302x^2y^2 \\
& +893x^4y^4)+b^6x^5y^5(-6435+13464x^2y^2-16323x^4y^4
\end{aligned}
$$

$$+94\mathrm{x}^6\mathrm{y}^6) + b^5\mathrm{x}^4\mathrm{y}^4(3003 - 6903\mathrm{x}^2\mathrm{y}^2 + 39828\mathrm{x}^4\mathrm{y}^4 - 2107\mathrm{x}^6\mathrm{y}^6$$
$$-13\mathrm{x}^8\mathrm{y}^8) - 3\mathrm{x}^5\mathrm{y}^5(-1144 + 2764\mathrm{x}^2\mathrm{y}^2 - 699\mathrm{x}^4\mathrm{y}^4 + 14\mathrm{x}^6\mathrm{y}^6$$
$$+\mathrm{x}^8\mathrm{y}^8) + b^4\mathrm{x}^3\mathrm{y}^3(-1716 + 4290\mathrm{x}^2\mathrm{y}^2 - 42060\mathrm{x}^4\mathrm{y}^4 + 15655\mathrm{x}^6\mathrm{y}^6$$
$$-490\mathrm{x}^8\mathrm{y}^8 + \mathrm{x}^{10}\mathrm{y}^{10}) + b^3\mathrm{x}^2\mathrm{y}^2(572 + 2574\mathrm{x}^2\mathrm{y}^2 + 18915\mathrm{x}^4\mathrm{y}^4$$
$$-28790\mathrm{x}^6\mathrm{y}^6 + 4395\mathrm{x}^8\mathrm{y}^8 + 78\mathrm{x}^{10}\mathrm{y}^{10}) - b^2\mathrm{xy}(104 + 572\mathrm{x}^2\mathrm{y}^2$$
$$+11583\mathrm{x}^4\mathrm{y}^4 - 26600\mathrm{x}^6\mathrm{y}^6 + 11605\mathrm{x}^8\mathrm{y}^8 + 186\mathrm{x}^{10}\mathrm{y}^{10}$$
$$+6\mathrm{x}^{12}\mathrm{y}^{12}) + b(8 + 52\mathrm{x}^2\mathrm{y}^2 + 143\mathrm{x}^4\mathrm{y}^4 - 4589\mathrm{x}^6\mathrm{y}^6 + 10760\mathrm{x}^8\mathrm{y}^8$$
$$\left.-893\mathrm{x}^{10}\mathrm{y}^{10} + 39\mathrm{x}^{12}\mathrm{y}^{12} - 8\vartheta^{13}))/(200(1 - b^2)^{3/2}\mathrm{x}^2\mathrm{y}^2\vartheta^{13})\right] \tag{S8i}$$

$$M_9^{ES,c,ij} = \left[(-9(-2 + \mathrm{x}^2 + 3b^4\mathrm{x}^2 + 2b^2(-5 + 4\mathrm{x}^2))(-1 + \mathrm{y}^2))/(2(-1 + b^2)^2\mathrm{x}^2\mathrm{y}^2)\right.$$
$$-(9(-10\mathrm{x}^8\mathrm{y}^6 + 8(-1 + \mathrm{y}^2) + 20b^9\mathrm{x}^7\mathrm{y}^5(-6 + 7\mathrm{y}^2) + 9\mathrm{x}^4\mathrm{y}^2(2 - 9\mathrm{y}^2$$
$$+7\mathrm{y}^4) + 4\mathrm{x}^2(1 - 10\mathrm{y}^2 + 9\mathrm{y}^4) - 5b^8\mathrm{x}^6\mathrm{y}^4(-96 + (77 - 19\mathrm{x}^2)\mathrm{y}^2$$
$$+21(1 + \mathrm{x}^2)\mathrm{y}^4) + \mathrm{x}^6(39\mathrm{y}^4 - 49\mathrm{y}^6) + b^7\mathrm{x}^5\mathrm{y}^3(-630 - 15(-63 + 73\mathrm{x}^2)\mathrm{y}^2$$
$$-7(45 - 136\mathrm{x}^2 + 9\mathrm{x}^4)\mathrm{y}^4 + 63(\mathrm{x}^2 + \mathrm{x}^4)\mathrm{y}^6) + b^6\mathrm{x}^4\mathrm{y}^2(378 + 3(-651 + 725\mathrm{x}^2)\mathrm{y}^2$$
$$+(1575 - 3122\mathrm{x}^2 + 1573\mathrm{x}^4)\mathrm{y}^4 + 3\mathrm{x}^2(329 - 505\mathrm{x}^2 + 6\mathrm{x}^4)\mathrm{y}^6 - 18(\mathrm{x}^4$$
$$+\mathrm{x}^6)\mathrm{y}^8) - 3b^4\mathrm{x}^2(-4 + (424 - 354\mathrm{x}^2)\mathrm{y}^2 - 3(140 - 503\mathrm{x}^2 + 313\mathrm{x}^4)\mathrm{y}^4 +$$
$$(-1155\mathrm{x}^2 + 1694\mathrm{x}^4 - 589\mathrm{x}^6)\mathrm{y}^6 + (-735\mathrm{x}^4 + 669\mathrm{x}^6 - 84\mathrm{x}^8)\mathrm{y}^8 + 12\mathrm{x}^6(-5$$
$$+7\mathrm{x}^2)\mathrm{y}^{10}) + b^5\mathrm{x}^3\mathrm{y}(-108 - 6(-368 + 343\mathrm{x}^2)\mathrm{y}^2 - 3(700 - 1757\mathrm{x}^2$$
$$+1059\mathrm{x}^4)\mathrm{y}^4 + (-3213\mathrm{x}^2 + 3918\mathrm{x}^4 - 891\mathrm{x}^6)\mathrm{y}^6 + (-621\mathrm{x}^4 + 889\mathrm{x}^6$$
$$-2\mathrm{x}^8)\mathrm{y}^8 + 2(\mathrm{x}^6 + \mathrm{x}^8)\mathrm{y}^{10}) + b^2(40(-1 + \mathrm{y}^2) - 162\mathrm{x}^{10}\mathrm{y}^8(-1 + \mathrm{y}^2)$$
$$+135\mathrm{x}^4\mathrm{y}^2(2 - 9\mathrm{y}^2 + 7\mathrm{y}^4) + 16\mathrm{x}^2(2 - 29\mathrm{y}^2 + 27\mathrm{y}^4) + \mathrm{x}^8\mathrm{y}^6(607$$
$$-837\mathrm{y}^2 + 270\mathrm{y}^4) + \mathrm{x}^6\mathrm{y}^4(537 - 1442\mathrm{y}^2 + 945\mathrm{y}^4)) + b^3\mathrm{xy}(-360(-1 + \mathrm{y}^2)$$
$$+28\mathrm{x}^{10}\mathrm{y}^8(-1 + \mathrm{y}^2) + \mathrm{x}^8\mathrm{y}^6(-693 + 713\mathrm{y}^2 - 20\mathrm{y}^4) - 48\mathrm{x}^2(6 - 41\mathrm{y}^2$$
$$+35\mathrm{y}^4) - 21\mathrm{x}^4\mathrm{y}^2(58 - 163\mathrm{y}^2 + 105\mathrm{y}^4) + \mathrm{x}^6(-1473\mathrm{y}^4 + 2428\mathrm{y}^6$$
$$-1035\mathrm{y}^8)) + b\mathrm{xy}(-72(-1 + \mathrm{y}^2) + 18\mathrm{x}^{10}\mathrm{y}^8(-1 + \mathrm{y}^2) - 63\mathrm{x}^4\mathrm{y}^2(2 - 7\mathrm{y}^2$$
$$+5\mathrm{y}^4) - 36\mathrm{x}^2(1 - 8\mathrm{y}^2 + 7\mathrm{y}^4) - 3\mathrm{x}^8\mathrm{y}^6(27 - 37\mathrm{y}^2 + 10\mathrm{y}^4)$$
$$\left.+\mathrm{x}^6(-183\mathrm{y}^4 + 338\mathrm{y}^6 - 135\mathrm{y}^8))))/(8(-1 + b^2)^2\mathrm{x}^2\mathrm{y}^2\vartheta^9)\right]$$
$$+\lambda_{c_j}^2\left[9(480b^{10}\mathrm{x}^8\mathrm{y}^6 + 50\mathrm{x}^{10}\mathrm{y}^8 - \mathrm{x}^8\mathrm{y}^6(481 + 70\mathrm{y}^2) + \mathrm{x}^6\mathrm{y}^4(117 + 497\mathrm{y}^2)\right.$$
$$+20b^9\mathrm{x}^7\mathrm{y}^5(-66 + (-35 + 4\mathrm{x}^2)\mathrm{y}^2) + \mathrm{x}^4(22\mathrm{y}^2 - 99\mathrm{y}^4) + b^8\mathrm{x}^6\mathrm{y}^4(1800$$
$$+35(1 + 37\mathrm{x}^2)\mathrm{y}^2 + \mathrm{x}^2(161 + 281\mathrm{x}^2)\mathrm{y}^4) - b^7\mathrm{x}^5\mathrm{y}^3(1386 + 165(-21$$
$$+31\mathrm{x}^2)\mathrm{y}^2 + (-4186\mathrm{x}^2 + 5864\mathrm{x}^4)\mathrm{y}^4 + 99(\mathrm{x}^4 + \mathrm{x}^6)\mathrm{y}^6) + b^6\mathrm{x}^4\mathrm{y}^2(594$$
$$+(-5775 + 6429\mathrm{x}^2)\mathrm{y}^2 + 2\mathrm{x}^2(-5614 + 6791\mathrm{x}^2)\mathrm{y}^4 + \mathrm{x}^4(-1931 + 3133\mathrm{x}^2)\mathrm{y}^6$$
$$+22(\mathrm{x}^6 + \mathrm{x}^8)\mathrm{y}^8) - b^5\mathrm{x}^3\mathrm{y}(132 + 330(-14 + 13\mathrm{x}^2)\mathrm{y}^2 + 33\mathrm{x}^2(-371$$
$$+373\mathrm{x}^2)\mathrm{y}^4 + 6\mathrm{x}^4(-884 + 877\mathrm{x}^2)\mathrm{y}^6 + 11\mathrm{x}^6(-89 + 127\mathrm{x}^2)\mathrm{y}^8$$
$$+2(\mathrm{x}^8 + \mathrm{x}^{10})\mathrm{y}^{10}) + b\mathrm{xy}(88 - 18\mathrm{x}^{12}\mathrm{y}^{10} + 99\mathrm{x}^4\mathrm{y}^2(-2$$
$$+7\mathrm{y}^2) + 44\mathrm{x}^2(-1 + 9\mathrm{y}^2) + 3\mathrm{x}^{10}\mathrm{y}^8(-33 + 10\mathrm{y}^2) - \mathrm{x}^6\mathrm{y}^4(627 + 40\mathrm{y}^2)$$
$$+\mathrm{x}^8\mathrm{y}^6(146 + 165\mathrm{y}^2)) + b^3\mathrm{xy}(440 - 28\mathrm{x}^{12}\mathrm{y}^{10} + 5\mathrm{x}^{10}\mathrm{y}^8(-209$$
$$+4\mathrm{y}^2) + 88\mathrm{x}^2(-4 + 33\mathrm{y}^2) + 33\mathrm{x}^4\mathrm{y}^2(-62 + 175\mathrm{y}^2) + 5\mathrm{x}^8\mathrm{y}^6(-988 + 319\mathrm{y}^2)$$
$$+5\mathrm{x}^6\mathrm{y}^4(-561 + 1418\mathrm{y}^2)) + 8(-1 + \vartheta^{11}) - 4\mathrm{x}^2(-1 + 11\mathrm{y}^2 + \vartheta^{11})$$
$$+b^4\mathrm{x}^2(330(-6 + 5\mathrm{x}^2)\mathrm{y}^2 + 15\mathrm{x}^2(-539 + 445\mathrm{x}^2)\mathrm{y}^4 + 30\mathrm{x}^4(-224 + 117\mathrm{x}^2)\mathrm{y}^6$$
$$+15\mathrm{x}^6(-325 + 251\mathrm{x}^2)\mathrm{y}^8 + 44\mathrm{x}^8(-5 + 7\mathrm{x}^2)\mathrm{y}^{10} - 12(-1 + \vartheta^{11})) + b^2(198\mathrm{x}^{12}\mathrm{y}^{10}$$
$$+\mathrm{x}^{10}\mathrm{y}^8(691 - 330\mathrm{y}^2) - 5\mathrm{x}^8\mathrm{y}^6(-758 + 241\mathrm{y}^2) + \mathrm{x}^4(374\mathrm{y}^2$$
$$-1881\mathrm{y}^4) + \mathrm{x}^6(819\mathrm{y}^4 - 4760\mathrm{y}^6) + 40(-1 + \vartheta^{11}) - 8\mathrm{x}^2(-4 + 77\mathrm{y}^2$$

$$+4\vartheta^{11})))/(40(-1+b^2)^2x^2y^2\vartheta^{11})\Big] + \lambda_{c_i}^2\Big[9((44x^2-99x^4)y^4$$
$$+x^4(99+497x^2)y^6-7x^6(81+10x^2)y^8+140b^9x^7y^7(-5+6y^2)-7b^8x^6y^6(-5$$
$$+(15-23x^2)y^2+33x^2y^4)+b^7x^5y^5(3465+7(-495+598x^2)y^2$$
$$-3x^2(1582+33x^2)y^4+99x^4y^6)+b^6x^4y^4(-5775-7(-825$$
$$+1604x^2)y^2+(11508x^2-1931x^4)y^4+11x^4(201+2x^2)y^6$$
$$-22x^6y^8)+b^5x^3y^3(4620+231(-20+53x^2)y^2+3x^2(-4081+1768x^2)y^4$$
$$+x^4(-4464+979x^2)y^6-x^6(979+2x^2)y^8+2x^8y^{10})+5b^4x^2y^2(-396$$
$$-33(-12+49x^2)y^2-21x^2(-77+64x^2)y^4-15x^4(-84+65x^2)y^6$$
$$+(891x^6-44x^8)y^8+44x^8y^{10})+bxy(88+44(-2+9x^2)y^2$$
$$+99x^2(-4+7x^2)y^4-x^4(693+40x^2)y^6+15x^6(12+11x^2)y^8$$
$$+15x^8(-11+2x^2)y^{10}-30x^{10}y^{12})+b^3xy(440+88(-5+33x^2)y^2$$
$$+33x^2(-88+175x^2)y^4+(-5775x^4+7090x^6)y^6+5x^6(-1530+319x^2)y^8$$
$$+5x^8(-319+4x^2)y^{10}-20x^{10}y^{12})+y^2(8-44x^2-8\vartheta^{11})+8(-1+\vartheta^{11})$$
$$+b^2((616x^2-1881x^4)y^4+(1881x^4-4760x^6)y^6+(5040x^6-1205x^8)y^8$$
$$-165x^8(-9+2x^2)y^{10}+330x^{10}y^{12}+40(-1+\vartheta^{11})-8y^2(-5+77x^2$$
$$+5\vartheta^{11})))/(40(-1+b^2)^2x^2y^2\vartheta^{11})\Big] - \lambda_{c_j}^2\lambda_{c_i}^2\Big[9(-8$$
$$-52x^2y^2-143x^4y^4+7748x^6y^6-5663x^8y^8+5600b^{10}x^8y^8$$
$$+350x^{10}y^{10}+56b^9x^7y^7(-190+53x^2y^2)+b^8x^6y^6(455$$
$$-31066x^2y^2+779x^4y^4)+b^7x^5y^5(15015+69587x^2y^2$$
$$-8011x^4y^4-143x^6y^6)+b^6x^4y^4(-15015-70889x^2y^2$$
$$+25793x^4y^4-5547x^6y^6+26x^8y^8)+b^5x^3y^3(8580+33033x^2y^2$$
$$-28233x^4y^4+39973x^6y^6+1417x^8y^8-2x^{10}y^{10})-b^4x^2y^2(2860$$
$$+15873x^2y^2-17745x^4y^4+79115x^6y^6+5765x^8y^8+260x^{10}y^{10})$$
$$+b^3xy(520+4576x^2y^2+12441x^4y^4+61865x^6y^6-4865x^8y^8$$
$$+2275x^{10}y^{10}+20x^{12}y^{12})+bxy(104+572x^2y^2+1287x^4y^4$$
$$-10211x^6y^6+4255x^8y^8+195x^{10}y^{10}+30x^{12}y^{12})+8\vartheta^{13}-b^2(40$$
$$+832x^2y^2+3289x^4y^4+37427x^6y^6-22675x^8y^8+3545x^{10}y^{10}$$
$$+390x^{12}y^{12}-40\vartheta^{13}))/(200(-1+b^2)^2x^2y^2\vartheta^{13})\Big] \tag{S8j}$$

$$M_{10}^{ES,c,ij} = \Big[(9(b^4x^2+(-1+x^2)(-2+y^2)+b^2(1-2y^2+2x^2(-1+y^2))))/(8(-1$$
$$+b^2)^2x^2y^2)-(3(12b^7x^5y^3(-6+y^2)-6(-2+y^2)+x^6y^4(-3+5y^2)$$
$$+x^2(-12+53y^2-33y^4)+x^4y^2(-45+46y^2+3y^4)+2b^6x^4y^2(48$$
$$+(4+69x^2)y^2+(-39+36x^2)y^4)-2b^5x^3y(21+(46-12x^2)y^2+(-114$$
$$+115x^2+78x^4)y^4+27x^2(-1+x^2)y^6)+b^4x^2(6-2(-55+87x^2)y^2$$
$$-3(74-143x^2+93x^4)y^4+x^2(-261+260x^2+63x^4)y^6+21x^4(-1$$
$$+x^2)y^8)+bxy(42(-2+y^2)-3x^8y^6(-9+5y^2)+3x^6y^4(32-25y^2+5y^4)$$
$$+x^4y^2(195-227y^2+54y^4)+x^2(84-239y^2+123y^4))+b^3xy(-42$$
$$+84y^2-3x^8y^6(3+y^2)+3x^6y^4(146-89y^2+y^4)$$
$$+x^4y^2(483-563y^2+270y^4)+x^2(84-425y^2+279y^4))+b^2(6-12y^2$$
$$+21x^8y^6(-9+5y^2)+x^2(-12+233y^2-123y^4)-3x^4y^2(85$$
$$-175y^2+98y^4)+x^6(-486y^4+419y^6-105y^8))))/(16(-1+b^2)^2x^2y^2\vartheta^7)\Big]$$
$$-\lambda_{c_j}^2\Big[9(-5x^8y^6-x^6y^4(126+17y^2)+x^4y^2(23+102y^2)+8b^7x^5y^3(5$$

$$+2(1+x^2)y^2) - 2b^6x^4y^2(24 + 9(3 + 7x^2)y^2 + x^2(-47 + 93x^2)y^4)$$
$$+2b^5x^3y(9 + (36 + 11x^2)y^2 + x^2(-13 + 128x^2)y^4 + 3x^4(-5$$
$$+16x^2)y^6) + bxy(36 - 9x^{10}y^8 + 33x^6y^4(3 + y^2) + x^8y^6(-39$$
$$+7y^2) - 3x^4y^2(29 + 19y^2) + x^2(-36 + 93y^2)) + b^3xy(18$$
$$+3x^{10}y^8 - x^8y^6(273 + y^2) + x^6y^4(-1127 + 213y^2) + x^2(-36 + 339y^2)$$
$$+x^4y^2(-479 + 823y^2)) + x^2(4 - 25y^2 - 4\vartheta^9) + 4(-1 + \vartheta^9) + b^4x^2((-70$$
$$+92x^2)y^2 + 21x^2(-11 + 15x^2)y^4 + (-436x^4 + 430x^6)y^6 + (9x^6$$
$$-27x^8)y^8 + 2(-1 + \vartheta^9)) + b^2(81x^{10}y^8 + x^4(149y^2 - 573y^4)$$
$$+x^6(693y^4 - 145y^6) + x^8(265y^6 - 63y^8) + x^2(4 - 121y^2 - 4\vartheta^9)$$
$$+2(-1 + \vartheta^9)))/(80(-1 + b^2)^2x^2y^2\vartheta^9)\Big] + \lambda_{c_i}^2\Big[9(4 - x^2(29 + 102x^2)y^4 + x^4(108$$
$$+17x^2)y^6 - 5x^6y^8 + 8b^7x^5y^5(-2 + 13y^2) - 2b^6x^4y^4(-27 + (117 + 47x^2)y^2$$
$$+7x^2y^4) + 2b^5x^3y^3(-36 + (118 + 13x^2)y^2 + x^2(7 + 15x^2)y^4$$
$$+15x^4y^6) - b^4x^2y^2(-70 + (146 - 231x^2)y^2 + (117x^2 - 436x^4)y^4$$
$$+x^4(376 + 9x^2)y^6 + 9x^6y^8) + b^3xy(-18 + (36 - 339x^2)y^2$$
$$+(179x^2 - 823x^4)y^4 + (731x^4 - 213x^6)y^6 + x^6(165 + x^2)y^8$$
$$+x^8y^{10}) + bxy(-36 + (18 - 93x^2)y^2 + x^2(89 + 57x^2)y^4$$
$$-3x^4(31 + 11x^2)y^6 - 7x^6(-3 + x^2)y^8 + 5x^8y^{10}) - 4\vartheta^9$$
$$+y^2(-2 + 25x^2 + 2\vartheta^9) + b^2(2 + x^2(-41 + 573x^2)y^4 + x^4(-513 + 145x^2)y^6$$
$$+x^6(-109 + 63x^2)y^8 - 45x^8y^{10} - 2\vartheta^9 + y^2(-4 + 121x^2$$
$$+4\vartheta^9)))/(80(-1 + b^2)^2x^2y^2\vartheta^9)\Big] - \lambda_{c_j}^2\lambda_{c_i}^2\Big[9(4 + 57x^2y^2$$
$$-1243x^4y^4 + 1139x^6y^6 + 64b^8x^6y^6 - 85x^8y^8 + 16b^7x^5y^5(-11 + 54x^2y^2)$$
$$+2b^6x^4y^4(99 - 906x^2y^2 + 55x^4y^4) + 2b^5x^3y^3(28 + 539x^2y^2$$
$$-1610x^4y^4 + 27x^6y^6) + b^4x^2y^2(134 - 187x^2y^2 + 6881x^4y^4$$
$$+223x^6y^6 - 11x^8y^8) + b^3xy(-22 - 1135x^2y^2 - 5324x^4y^4$$
$$+1406x^6y^6 - 350x^8y^8 + x^{10}y^{10}) - bxy(44 - 89x^2y^2$$
$$-1650x^4y^4 + 1030x^6y^6 + 34x^8y^8 + 7x^{10}y^{10}) - 4\vartheta^{11}$$
$$+b^2(2 + 139x^2y^2 + 3212x^4y^4 - 3500x^6y^6 + 742x^8y^8$$
$$+77x^{10}y^{10} - 2\vartheta^{11}))/(400(-1 + b^2)^2x^2y^2\vartheta^{11})\Big] \tag{S8k}$$

$$M_{11}^{ES,c,ij} = \Big[(3(12(-1 + y^2) + 12b^7x^5y^3(-6 + 5y^2) - x^6y^4(3 + 5y^2) + x^4y^2(33 - 46y^2 + 3y^4)$$
$$+x^2(6 - 53y^2 + 45y^4) + 2b^6x^4y^2(48 + (-178 + 237x^2)y^2 - 3(-43$$
$$+78x^2)y^4) + 2b^5x^3y(-21 + (400 - 477x^2)y^2 + (-375 + 631x^2 - 237x^4)y^4$$
$$+3x^2(-44 + 79x^2)y^6) + b^4x^2(6 + (-746 + 786x^2)y^2 + 3(246 - 583x^2$$
$$+313x^4)y^4 + x^2(957 - 1064x^2 + 189x^4)y^6 - 21x^4(-5 + 9x^2)y^8)$$
$$+b^2(42(-1 + y^2) - 189x^8y^6(-1 + y^2) + 3x^4y^2(73 - 291y^2 + 224y^4)$$
$$+x^6y^4(480 - 731y^2 + 273y^4) + x^2(42 - 389y^2 + 351y^4))$$
$$+b^3xy(-294(-1 + y^2) + 27x^8y^6(-1 + y^2) + x^2(-294 + 1193y^2$$
$$-915y^4) - 3x^6y^4(188 - 193y^2 + 5y^4) + x^4(-711y^2 + 1391y^4 - 732y^6))$$
$$+bxy(-84(-1 + y^2) + 27x^8y^6(-1 + y^2) + x^2(-42 + 275y^2 - 225y^4)$$
$$-3x^6y^4(32 - 45y^2 + 13y^4) + x^4(-153y^2 + 311y^4 - 138y^6))))$$
$$/(16(-1 + b^2)^2x^2y^2\vartheta^7) + (9(-2 + x^2 + b^4x^2 + 7b^2(-1 + x^2))(-1 + y^2)(1$$
$$-2bxy + x^2y^2)^3)/(8(-1 + b^2)^2x^2y^2\vartheta^6)\Big] + \lambda_{c_i}^2\Big[9(4 - x^2(23$$
$$+102x^2)y^4 + x^4(126 + 17x^2)y^6 + 5x^6y^8 + 8b^7x^5y^5(-38 + 43y^2)$$

$$+2b^6x^4y^4(381+(-387+127x^2)y^2-137x^2y^4)+2b^5x^3y^3(-387$$
$$+(383-311x^2)y^2+(247x^2-78x^4)y^4+78x^4y^6)+b^4x^2y^2(448$$
$$+(-446+705x^2)y^2+x^2(-657+874x^2)y^4+x^4(-812+45x^2)y^6$$
$$-45x^6y^8)+b^3xy(-126+(126-573x^2)y^2+(589x^2-1309x^4)y^4$$
$$-5x^4(-289+87x^2)y^6-5x^6(-87+x^2)y^8+5x^8y^{10})+bxy(-36$$
$$+(36-165x^2)y^2+(157x^2-33x^4)y^4-3x^4(5+19x^2)y^6+(57x^6$$
$$-13x^8)y^8+13x^8y^{10})-4\vartheta^9+y^2(-4+25x^2+4\vartheta^9)+b^2(x^2(-179$$
$$+903x^2)y^4+x^4(-963+367x^2)y^6+x^6(-431+117x^2)y^8$$
$$-117x^8y^{10}-14(-1+\vartheta^9)+7y^2(-2+25x^2+2\vartheta^9)))/(80(-1+b^2)^2x^2y^2\vartheta^9)\Big]$$

$$+\lambda_{c_j}^2\Big[9(4-5x^8y^6+x^6y^4(108+17y^2)-x^4y^2(29+102y^2)+8b^7x^5y^3(5$$
$$+(-38+58x^2)y^2)-2b^6x^4y^2(24+(-381+567x^2)y^2+x^2(-127+237x^2)y^4)$$
$$+2b^5x^3y(9+(-387+512x^2)y^2+x^2(-311+382x^2)y^4+3x^4(-26$$
$$+47x^2)y^6)+bxy(-36+9x^{10}y^8-13x^8y^6(-3+y^2)$$
$$-3x^6y^4(3+19y^2)-3x^2(-6+55y^2)+x^4(159y^2-33y^4))+b^3xy(-126$$
$$+9x^{10}y^8+x^2(126-573y^2)-17x^4y^2(-17+77y^2)+x^6(1049y^4$$
$$-435y^6)+x^8(327y^6-5y^8))-4\vartheta^9+x^2(-2+25y^2+2\vartheta^9)+b^4x^2((448$$
$$-500x^2)y^2+(705x^2-459x^4)y^4+(874x^4-758x^6)y^6$$
$$-9x^6(-5+9x^2)y^8+2(-1+\vartheta^9))+b^2(-81x^{10}y^8+x^8y^6(-275$$
$$+117y^2)+x^6y^4(-783+367y^2)+x^4y^2(-71+903y^2)-14(-1+\vartheta^9)$$
$$+7x^2(-2+25y^2+2\vartheta^9)))/(80(-1+b^2)^2x^2y^2\vartheta^9)\Big]-\lambda_{c_j}^2\lambda_{c_i}^2\Big[9(4$$
$$+57x^2y^2-1243x^4y^4+1139x^6y^6+1216b^8x^6y^6-85x^8y^8$$
$$+16b^7x^5y^5(-209+6x^2y^2)+2b^6x^4y^4(1881-534x^2y^2$$
$$+301x^4y^4)-2b^5x^3y^3(917-836x^2y^2+2489x^4y^4+126x^6y^6)$$
$$+b^4x^2y^2(728-1771x^2y^2+10253x^4y^4+1615x^6y^6+55x^8y^8)$$
$$-b^3xy(154+721x^2y^2+8096x^4y^4+298x^6y^6+662x^8y^8$$
$$+5x^{10}y^{10})-bxy(44+415x^2y^2-1452x^4y^4+760x^6y^6$$
$$+76x^8y^8+13x^{10}y^{10})-4\vartheta^{11}+b^2(14+205x^2y^2+5192x^4y^4$$
$$-3224x^6y^6+838x^8y^8+143x^{10}y^{10}-14\vartheta^{11}))/(400(-1+b^2)^2x^2y^2\vartheta^{11})\Big]$$
$$\text{(S8l)}$$

$$M_{12}^{ES,c,ij} = \Big[(-9(2b^6x^5y^3(-6+5y^2)+b^5x^4y^2(16+(-48+71x^2)y^2+(35-70x^2)y^4)$$
$$+b^4x^3y(-7+(86-126x^2)y^2-2(40-82x^2+35x^4)y^4+35x^2(-1$$
$$+2x^2)y^6)+b^3x^2(1+(-71+94x^2)y^2+(70-170x^2+103x^4)y^4$$
$$+7x^2(10-17x^2+4x^4)y^6+(14x^4-28x^6)y^8)+b(4(-1+y^2)$$
$$-14x^8y^6(-1+y^2)+x^6y^4(36-49y^2+14y^4)+x^2(5-19y^2$$
$$+14y^4)+x^4y^2(16-48y^2+35y^4))+x^3y^3(9-10y^2+2x^6y^4(-1$$
$$+y^2)+x^2(-10+16y^2-7y^4)+x^4(-7y^2+9y^4-2y^6))$$
$$+b^2xy(-28(-1+y^2)+4x^8y^6(-1+y^2)+x^2(-35+87y^2-50y^4)$$
$$-2x^4y^2(31-52y^2+21y^4)+x^6(-49y^4+51y^6-2y^8))))/(8(1$$
$$-b^2)^{3/2}x^2y^2\vartheta^7)-(9b(-4+(5+b^2)x^2)(-1+y^2)(1-2bxy$$
$$+x^2y^2)^3)/(8(1-b^2)^{3/2}x^2y^2\vartheta^6)\Big]-\lambda_{c_i}^2\Big[9(20b^6x^5y^5(-6+7y^2)$$
$$-5b^5x^4y^4(-19+21y^2)(3+x^2y^2)+b^4x^3y^3(-270-5(-56+27x^2)y^2$$
$$-21x^2(-5+3x^2)y^4+63x^4y^6)+x^3y^3(-60+(70+3x^2)y^2$$

$$+(7x^2 - 9x^4)y^4 + (9x^4 - 2x^6)y^6 + 2x^6y^8) - 2b^3x^2y^2(-63$$
$$+(63 - 30x^2)y^2 - 94x^4y^4 + (84x^4 - 9x^6)y^6 + 9x^6y^8)$$
$$+2b^2xy(-18 - 3(-6 + x^2)y^2 - 7(x^2 + 18x^4)y^4 - 18x^4(-7 + 2x^2)y^6$$
$$-x^6(-36 + x^2)y^8 + x^8y^{10}) + b(4 + 3x^2(-6 + 53x^2)y^4 + x^4$$
$$(-189 + 53x^2)y^6 + 9x^6(-7 + 2x^2)y^8 - 18x^8y^{10} - 4\vartheta^9 + 2y^2(-2 + 9x^2$$
$$+2\vartheta^9)))/(40(1 - b^2)^{3/2}x^2y^2\vartheta^9)\Big] - \lambda_{c_j}^2\Big[9(20b^6x^5y^3(1$$
$$+2(-3 + 5x^2)y^2) + b^5x^4y^2(-24 + (285 - 495x^2)y^2 + (95x^2 - 205x^4)y^4)$$
$$+3b^4x^3y(3 + (-90 + 143x^2)y^2 + 5x^2(-9 + 20x^2)y^4 + 21x^4(-1$$
$$+2x^2)y^6) + x^3y^3(-60 + 2x^8y^6 + x^2(70 + 3y^2) + x^4(7y^2$$
$$-9y^4) + x^6(9y^4 - 2y^6)) + b^2xy(-36 + 4x^{10}y^8 + x^2(45 - 6y^2)$$
$$-3x^4y^2(5 + 84y^2) + x^6(249y^4 - 72y^6) + x^8(81y^6 - 2y^8))$$
$$+b^3x^2(-1 - 3(-42 + 59x^2)y^2 + (60x^2 - 81x^4)y^4 + (188x^4$$
$$-241x^6)y^6 + (18x^6 - 36x^8)y^8 + \vartheta^9) + b(4 - 18x^{10}y^8 + 2x^8y^6(-29$$
$$+9y^2) + x^6y^4(-180 + 53y^2) + 3x^4y^2(-5 + 53y^2) - 4\vartheta^9$$
$$+x^2(-5 + 18y^2 + 5\vartheta^9)))/(40(1 - b^2)^{3/2}x^2y^2\vartheta^9)\Big] + \lambda_{c_j}^2\lambda_{c_i}^2\Big[9(480b^7x^6y^6$$
$$+40b^6x^5y^5(-33 + 2x^2y^2) + b^5x^4y^4(1485 - 790x^2y^2 + 281x^4y^4)$$
$$-b^4x^3y^3(882 - 1485x^2y^2 + 1504x^4y^4 + 99x^6y^6) - x^3y^3(420$$
$$-693x^2y^2 + 156x^4y^4 + 11x^6y^6 + 2x^8y^8) + 2b^3x^2y^2(99$$
$$-561x^2y^2 + 1456x^4y^4 + 115x^6y^6 + 11x^8y^8) - 2b^2xy(22$$
$$-321x^2y^2 + 1353x^4y^4 - 130x^6y^6 + 55x^8y^8 + x^{10}y^{10})$$
$$+b(4 + 22x^2y^2 + 957x^4y^4 - 754x^6y^6 + 149x^8y^8 + 22x^{10}y^{10}$$
$$-4\vartheta^{11}))/(200(1 - b^2)^{3/2}x^2y^2\vartheta^{11})\Big] \tag{S8m}$$

$$
\begin{aligned}
M_{13}^{ES,c,ij} = {}& \Big[(-9(2b^6x^5y^3(-6 + y^2) + b^5x^4y^2(16 + (-10 + 31x^2)y^2 + (-5 + 4x^2)y^4) \\
& + b^4x^3y(-7 + (32 - 41x^2)y^2 + (5 + 8x^2 - 35x^4)y^4) + b^3x^2(1 + (-35 \\
& + 24x^2)y^2 + x^2(-50 + 43x^2)y^4 + x^2(10 - 15x^2 + 14x^4)y^6) \\
& + x^3y^3(-3 + 5y^2 + x^4y^4 + x^2(5 - 2y^2)) - b(2 + x^6y^4(4 + 3y^2) \\
& + x^2(-1 + 7y^2) + x^4y^2(-2 + 10y^2 + 5y^4)) + b^2xy(14 - 2x^8y^6 \\
& + x^6y^4(-7 + y^2) + x^2(-7 + 41y^2 - 10y^4) + x^4(-22y^2 \\
& + 34y^4))))/(8(1 - b^2)^{3/2}x^2y^2\vartheta^7) + (9b(-2 + (1 + b^2)x^2)(1 - 2bxy \\
& + x^2y^2)^3)/(8(1 - b^2)^{3/2}x^2y^2\vartheta^6)\Big] + \lambda_{c_i}^2\Big[9(4b^6x^5y^5(6 \\
& + 5y^2) + b^5x^4y^4(-69 - (45 + 79x^2)y^2 + 25x^2y^4) + b^4x^3y^3(81 \\
& + 7(5 + 27x^2)y^2 + 15x^2(-5 + 2x^2)y^4) + x^3y^3(-24 + (35 + 48x^2)y^2 \\
& + x^2(-35 + 3x^2)y^4 + x^6y^6) - b^3x^2y^2(63 + 9x^6y^6 + x^4y^4(31 \\
& + 50y^2) + x^2(177y^2 - 90y^4)) + b^2xy(18 + 39x^6y^6 + x^8y^8 \\
& + x^2(111y^2 - 70y^4) + x^4(-9y^4 + 90y^6)) - b(2 + 9x^2y^2 \\
& + 9x^8y^8 + x^4(6y^4 + 45y^6) + x^6(58y^6 - 25y^8) - 2\vartheta^9))/(40(1 \\
& - b^2)^{3/2}x^2y^2\vartheta^9)\Big] - \lambda_{c_j}^2\Big[9(4b^6x^5y^3(5 + 2(-3 + 5x^2)y^2) \\
& + b^5x^4y^2(-24 + (69 - 135x^2)y^2 + (79x^2 - 125x^4)y^4) + 3b^4x^3y(3 \\
& + (-27 + 38x^2)y^2 + 9x^2(-7 + 10x^2)y^4 + x^4(-10 + 21x^2)y^6) \\
& - x^3y^3(-24 + x^6y^6 + x^4y^2(-35 + 3y^2) + x^2(35 + 48y^2)) \\
& + b^2xy(-18 + 2x^{10}y^8 + x^2(9 - 111y^2) - x^8y^6(-9 + y^2)
\end{aligned}
$$

$$-3x^6y^4(31 + 13y^2) + x^4(69y^2 + 9y^4)) + b^3x^2(-1 + (63 - 51x^2)y^2$$
$$-3x^2(-59 + 57x^2)y^4 + (31x^4 - 23x^6)y^6 + (9x^6 - 18x^8)y^8$$
$$+\vartheta^9) + b(2 + x^8y^6(-20 + 9y^2) + 3x^4(y^2 + 2y^4) + x^6(54y^4$$
$$+58y^6) - 2\vartheta^9 + x^2(-1 + 9y^2 + \vartheta^9)))/(40(1 - b^2)^{3/2}x^2y^2\vartheta^9)\Big]$$

$$+\lambda_{c_j}^2\lambda_{c_i}^2\Big[9(96b^7x^6y^6 - 8b^6x^5y^5(33 + 58x^2y^2) - b^4x^3y^3(63$$
$$+1188x^2y^2 - 625x^4y^4 + 54x^6y^6) + b^5(297x^4y^4 + 1162x^6y^6$$
$$-35x^8y^8) - x^3y^3(-168 + 792x^2y^2 - 291x^4y^4 + 10x^6y^6$$
$$+x^8y^8) + b^3x^2y^2(99 + 330x^2y^2 - 1226x^4y^4 + 466x^6y^6$$
$$+11x^8y^8) - b^2xy(22 + 435x^2y^2 - 1320x^4y^4 + 1112x^6y^6$$
$$+46x^8y^8 + x^{10}y^{10}) + b(2 + 11x^2y^2 + 33x^4y^4 + 892x^6y^6$$
$$-101x^8y^8 + 11x^{10}y^{10} - 2\vartheta^{11}))/(200(1 - b^2)^{3/2}x^2y^2\vartheta^{11})\Big]$$

$$\text{(S8n)}$$

$$M_{14}^{ES,c,ij} = \Big[(-9(1 + 2b^2)(-1 + x^2)(-1 + y^2))/(8(-1 + b^2)^2x^2y^2) - (3(6(-1 + y^2)$$
$$+x^4(3y^2 - 5y^4) + x^2(6 - 11y^2 + 3y^4) + 4b^5x^3y^3(7 - 6y^2$$
$$+6x^2(-1 + y^2)) + b^3xy(-60(-1 + y^2) + 3x^6y^4(-1 + y^2)$$
$$+x^2(-60 + 169y^2 - 117y^4) - 3x^4y^2(39 - 40y^2 + y^4)) + bxy(-30(-1$$
$$+y^2) + 15x^6y^4(-1 + y^2) + x^2(-30 + 73y^2 - 39y^4) - 3x^4y^2(13$$
$$-18y^2 + 5y^4)) + b^4x^2y^2(-80 + 78y^2 - 15x^4y^2(-1 + y^2) + x^2(78$$
$$-95y^2 + 15y^4)) + b^2(12(-1 + y^2) - 75x^6y^4(-1 + y^2) + x^4y^2(99$$
$$-170y^2 + 75y^4) + x^2(12 - 107y^2 + 99y^4))))/(16(-1 + b^2)^2x^2y^2\vartheta^5)\Big]$$

$$+\lambda_{c_i}^2\Big[9(-(x^2(13 + x^2)y^4) - 5x^4y^6 + 32b^6x^4y^4(-1$$
$$+y^2) - 4b^5x^3y^3(-17 + (14 - 4x^2)y^2 + 4x^2y^4) + b^4x^2y^2(-56$$
$$+(50 - 77x^2)y^2 + (71x^2 - 7x^4)y^4 + 7x^4y^6) + bxy(14$$
$$+(-14 + 33x^2)y^2 + x^2(-21 + 16x^2)y^4 + x^4(-16 + 5x^2)y^6 - 5x^6y^8)$$
$$+b^3xy(28 + (-28 + 109x^2)y^2 + x^2(-133 + 94x^2)y^4 + x^4(-94$$
$$+x^2)y^6 - x^6y^8) + y^2(2 + 7x^2 - 2\vartheta^7) + 2(-1 + \vartheta^7) + b^2((89x^2$$
$$-100x^4)y^4 - 7x^4(-16 + 5x^2)y^6 + 35x^6y^8 + y^2(4 - 77x^2$$
$$-4\vartheta^7) + 4(-1 + \vartheta^7)))/(80(-1 + b^2)^2x^2y^2\vartheta^7)\Big] + \lambda_{c_j}^2\Big[9(-5x^6y^4$$
$$+32b^6x^4(-1 + x^2)y^4 - x^4y^2(13 + y^2) - 4b^5x^3y^3(-17 + 4x^4y^2 + x^2(14$$
$$-4y^2)) + bxy(14 - 5x^8y^6 + x^6y^4(-16 + 5y^2) + x^4y^2(-21$$
$$+16y^2) + x^2(-14 + 33y^2)) + b^3xy(28 - x^8y^6 + x^6y^4(-94 + y^2)$$
$$+x^4y^2(-133 + 94y^2) + x^2(-28 + 109y^2)) + b^4x^2y^2(-56 + 7x^6y^4$$
$$+x^2(50 - 77y^2) + x^4(71y^2 - 7y^4)) + x^2(2 + 7y^2 - 2\vartheta^7) + 2(-1 + \vartheta^7)$$
$$+b^2(35x^8y^6 - 7x^6y^4(-16 + 5y^2) + x^4(89y^2 - 100y^4) + x^2(4$$
$$-77y^2 - 4\vartheta^7) + 4(-1 + \vartheta^7)))/(80(-1 + b^2)^2x^2y^2\vartheta^7)\Big] - \lambda_{c_j}^2\lambda_{c_i}^2\Big[9^2(-2$$
$$+61x^2y^2 - 72x^4y^4 + 128b^7x^5y^5 + 5x^6y^6 + 32b^6x^4y^4(-9 + x^2y^2)$$
$$+4b^5x^3y^3(63 + 2x^2y^2 + 9x^4y^4) - b^4x^2y^2(56 + 189x^2y^2$$
$$+458x^4y^4 + 9x^6y^6) + b^3xy(36 + 147x^2y^2 + 671x^4y^4$$
$$+153x^6y^6 + x^8y^8) + bxy(18 + 105x^2y^2 - 51x^4y^4$$
$$+27x^6y^6 + 5x^8y^8) + 2\vartheta^9 - b^2(4 + 221x^2y^2 + 207x^4y^4$$
$$+83x^6y^6 + 45x^8y^8 - 4\vartheta^9))/(400(-1 + b^2)^2x^2y^2\vartheta^9)\Big]$$

$$\text{(S8o)}$$

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