

# Supplementary Material

## **Data processing:**

The Mie scattering images are processed in MATLAB using canny edge detection algorithm. This generates the flamefront contour with spatial resolution 0.07mm/pixel. The PIV measurements are processed using DaVis 8.0, generating velocity field with resolution 0.21mm/vector. Subsequently, the vectors were superimposed on flame edge points evaluated from Mie scattering.

After the flamefront contour is generated, we also investigated the derivative of the flamefront. The derivative is calculated using the central difference of the flamefront points. To assess the local flamefront curvature and local normal vector, we used a 31-points 5<sup>th</sup> order polynomial fitting of the edge points. A trail and error method showed 31-point 5<sup>th</sup> order fitting captures small scale structures and reduce the effect of individual pixels. This allows us to transform the velocities into normal component and tangential component on the flamefront.

Along the flamefront, we then perform the power spectral analysis for both normal and tangential velocities to obtain the 1-dimensional energy spectra. The Hanning window is applied to the PSD calculation to avoid the spectral leakage caused by non-periodicity. For each Peclet number, the calculated PSDs are averaged over all runs.

## **Uncertainty Analysis:**

The uncertainty of flamefront edge detection comes from the canny edge detection algorithm and the threshold. In combustion experiments, the seeding particles are consumed in the burned side, so there exists a sharp intensity gradient at the edge between burned and unburned sides. Therefore, the uncertainty comes from the threshold selection is very minor. In terms of the canny edge detection algorithm, the uncertainty is estimated to be less than +/-1% <sup>[S1]</sup>. As a result, the uncertainty of normal and tangential unit vector is estimated to be +/-2%. It is also the same for radial and azimuthal directions. When it comes to the derivative, the central difference algorithm generates uncertainty from the numerical error, which is estimated to be less than 1% given our spatial resolution of edge detection. Therefore, the uncertainty for the PSD of derivative of flamefront is negligible (< 2%).

The uncertainty of velocity measurement in PIV is estimated to be less than +/- 4% <sup>[S2]</sup>. Therefore, the uncertainty of normal and tangential velocity is less than +/-6%. From this, we could further derive the uncertainty of the power spectral density of velocity. The PSD is calculated through the Fast Fourier Transform algorithm in MATLAB. The discrete Fourier Transform is the linear combination of original velocity data, so the uncertainty remains the same with velocity measurement, and the PSD has uncertainty around +/-12%.

### Analytical and Numerical analysis of the Integral:

Here we show that in Eq. 3.8, the integral converges to a constant at large  $Pe$ .

Let's denote the integral in Eq. 3.8 as  $I = \int_{2\pi}^{2\pi Pe} \lambda^{1-\gamma} \cos(\lambda\theta + \psi_\lambda) d\lambda$ . Theoretically, for  $\gamma > 1$ , the oscillatory effect through *cosine* term in the integrand becomes weak at high  $Pe$ , as the amplitude (due to  $\lambda^{1-\gamma}$ ) decays. This forces the total integral converging to a constant for any given  $\theta$ . Numerically, we use MATLAB to calculate the absolute value of this integral  $|I|$  for different  $\psi$  at different  $\theta$ , as shown in Fig. S2. The value of  $\gamma$  is taken to be 2.25, which is consistent experimental results, however, the behavior does not change as long as  $\gamma > 1$ . The results show that the integral converges to a constant that only depends on  $\theta$ , and it converges very fast with  $Pe$ , which validates our analysis.

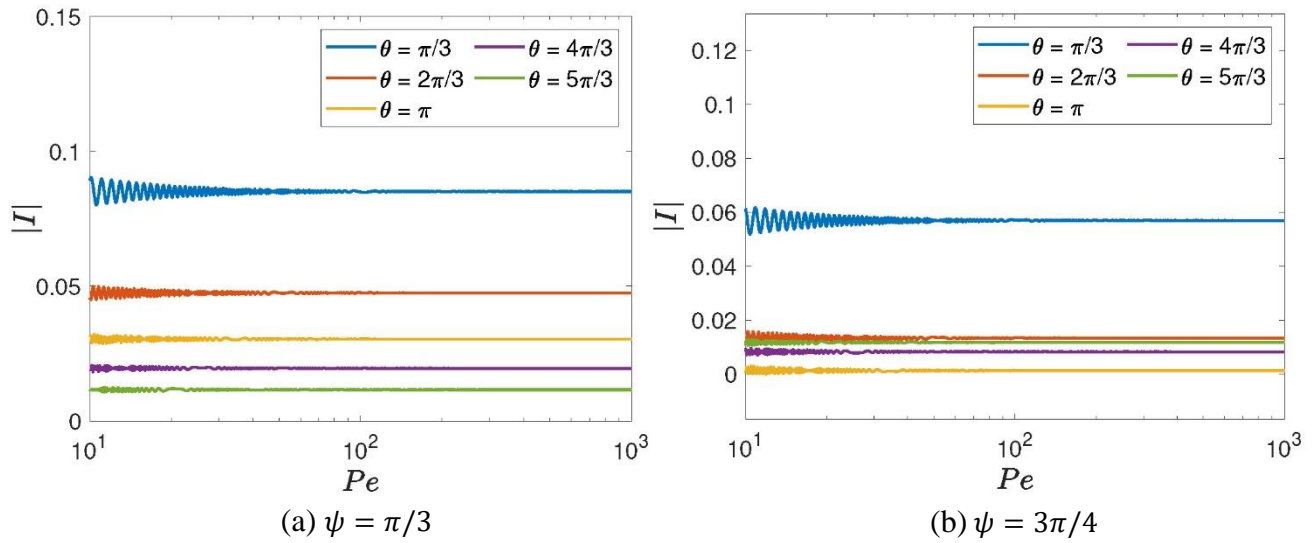


Figure S2. Absolute value of integration  $I$  with (a)  $\psi = \pi/3$  and (b)  $\psi = 3\pi/4$ .  $\gamma$  is taken to be 2.25. The magnitude of the integral is a constant which depends on  $\theta$  only.

### References:

- [S1] De Santo M, Liguori C, Pietrosanto A. Uncertainty characterization in image-based measurements: a preliminary discussion, IEEE Transactions on Instrumentation and Measurement, 2000, 49(5): 1101-1107.
- [S2] Bhattacharya S, Charonko J J, Vlachos P P. Particle image velocimetry (PIV) uncertainty quantification using moment of correlation (MC) plane, Measurement Science and Technology, 2018, 29(11): 115301.
- [S3] Chaudhuri S, Akkerman V, Law C K. Spectral formulation of turbulent flame speed with consideration of hydrodynamic instability, Physical Review E, 2011, 84(2): 026322.