Analytic fits for the mean velocity and second moments of velocity near the wall for a turbulent flow in a plane channel: a supplement for the paper "Extension of QSQH theory of scale interaction in near-wall turbulence to all velocity components"

Sergei Chernyshenko

Department of Aeronautics, Imperial College London

s.chernyshenko@imperial.ac.uk

When making comparisons it is sometimes convenient to use analytic fits to the results of direct numerical simulations, even if only for plotting. Analytic expressions can also be differentiated, even though this involves a certain loss of accuracy. The analytic expressions were obtained by fitting the direct numerical simulation data for a plane channel flow with $Re_\tau = 180, 550, 950, 1000, 2000, 5200$. The DNS data were downloaded from the turbulence.oden.utexas.edu file server. The calculations are by Hoyas, S. & Jiménez [1] and Lee & Moser [2]. Fitting was done over the range of $0 \le y^+ \le 100$ using gnuplot command 'fit'.

The profiles are expressed via the following functions and their powers:

$$f(y,a,b,c,q,r,p) = y(ay^2+by+c)/(qy^2+ry+1) + p\log(y+1)$$

and

$$f_{15}(y,a,b,c,q,r,p) = y(ay^2+by+c)/(qy^2+ry+1) + p(\log(y+15) - \log(15)),$$

log is a natural logarithm, and a, b, c, q, r, p are the fitting parameters. These expressions in the text form suitable for cutting and pasting into a code are:

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f(x,a,b,c,q,r,p) = x*(a*x**2 + b*x+c)/(q*x**2+r*x+1)+p*log(x+1)
f15(x,a,b,c,q,r,p) = x*(a*x**2 + b*x+c)/(q*x**2+r*x+1)+p*(log(x+15)-log(15))
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The mean velocity $U^+(y^+)$, the root mean squares $u_{\mathrm{rms}} = \sqrt{\langle u'u' \rangle}$, $v_{\mathrm{rms}} = \sqrt{\langle v'v' \rangle}$, $w_{\mathrm{rms}} = \sqrt{\langle w'w' \rangle}$, and the Reynolds stress $\langle u'v' \rangle$ were fitted. All the variables are in wall units. The following pages give the representation of the fitted quantities in terms of f and/or f_{15}, plots with comparisons with DNS, and expressions in the text form suitable for cutting and pasting into a code. A text file with all these expressions is provided separately. The notation is intended to be self-explanatory. LM stands for Lee & Moser and HJ is for Hoyas & Jiménez, Re is Re_τ based on the channel half-width.

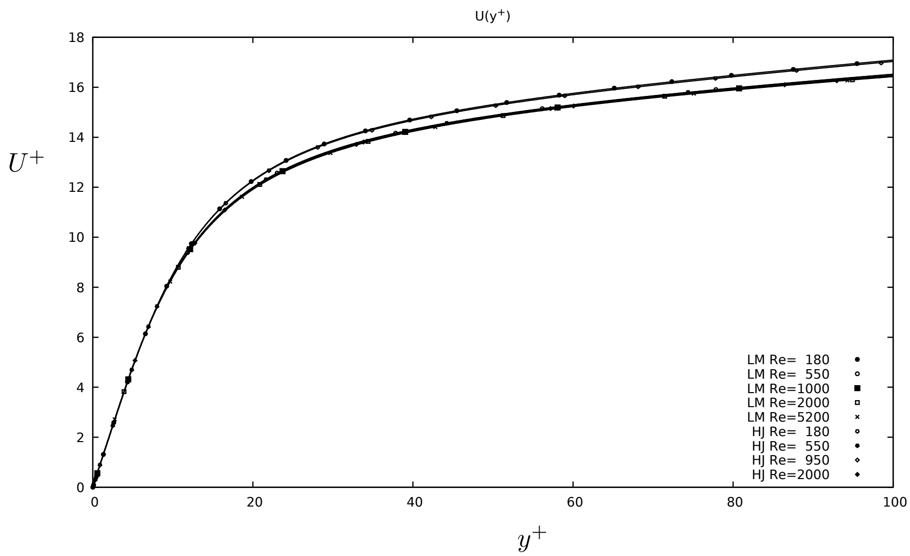

Figure 1: Mean velocity

$$U(y) = f(y, a, b, c, q, r, p) = y(ay^2 + by + c)/(qy^2 + ry + 1) + p\log(y + 1)$$

URe5200LM(x)=f(x,0.000155064859152652,0.124332316558984, 0.585852700238061,0.00960750534540338,0.0589324149643223, 0.493534445516441)
URe2000LM(x)=f(x,0.00015458793570163,0.121945362785562, 0.608802562811952,0.00934699868156456,0.058394426829014, 0.461629411942626)
URe1000LM(x)=f(x,0.000155568705758487,0.118289838303881, 0.619767033981213,0.00906452697385482,0.0562936331642849, 0.448968282677922)
URe0550LM(x)=f(x,0.000159454562845575,0.113267163433361, 0.636816719993512,0.0086861909085036, 0.0537966403282146, 0.428997666967737)
URe0180LM(x)=f(x,0.000164300321807816,0.0961432399193651,0.665040474947198,0.00728557305508607,0.044548055977009, 0.403728726538813)
URe2000HJ(x)=f(x,0.000155003439789658,0.121889353641462, 0.59668758158729, 0.00939945611981467,0.0576292141831145,0.480987959245497)
URe0950HJ(x)=f(x,0.000154946383447842,0.118568062133334, 0.613795545234793,0.00911155818726103,0.0561254847028229,0.457504875767387)
URe0550HJ(x)=f(x,0.000159480212600236,0.115284348323668, 0.619866802062511,0.00887866344547915,0.0541168724687089,0.451237458085645)
URe0180HJ(x)=f(x,0.000166302434061266,0.0963047516739797,0.65762882934244, 0.00736574280029303,0.0438959832544501,0.414636835720395)

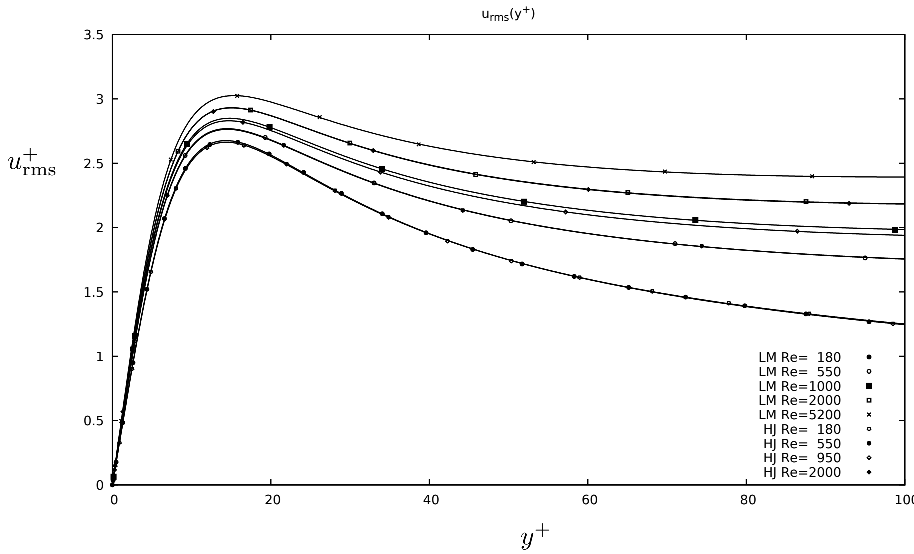

Figure 2: $u_{\rm rms}$

$$u_{\rm rms}(y) = f_{15}(y, a, b, c, q, r, p) = y(ay^2+by+c)/(qy^2+ry+1)+p(\log(y+15)-\log(15))$$

urmsRe0180LM(x)=f15(x,-2.96511771502127e-06,0.00713098516423203,0.368618519092697,0.00724292079273704,-0.00188104534382705,-0.0939004345943289)
urmsRe0550LM(x)=f15(x, 2.58534223375324e-05,0.0122455777205154, 0.404018861588927,0.00889216822122883, 0.00606957761017823,-0.161778516477361)
urmsRe1000LM(x)=f15(x, 2.78859635192058e-05,0.0128912190206297, 0.411914517490417,0.00914911302483312, 0.0072420062970093, -0.0682978880267656)
urmsRe2000LM(x)=f15(x, 2.62324052566284e-05,0.0134321561634876, 0.418549175660149,0.0093889429802691, 0.00778354559725009, 0.0338138600827896)
urmsRe5200LM(x)=f15(x, 2.42123399342968e-05,0.0137260007931729, 0.42570940965966, 0.00952531539247033, 0.00816310356056566, 0.142743764676396)
urmsRe0180HJ(x)=f15(x,-2.0232289518942e-06, 0.00733887166563185,0.367270461585974,0.007282396904903, -0.00154082404494924,-0.106440998116575)
urmsRe0550HJ(x)=f15(x, 2.60422226343772e-05,0.0121332976235026, 0.402982812604456,0.00885549199322238, 0.00613798778956479,-0.159540875042328)
urmsRe0950HJ(x)=f15(x, 2.4616787538755e-05, 0.012502990497006, 0.40893942174543, 0.00910233275841391, 0.00663208739941688,-0.0569507277015093)
urmsRe2000HJ(x)=f15(x, 2.66287209386734e-05,0.0133615203710217, 0.418719266671255,0.00937082552090975, 0.00775475024639174, 0.0324648258779921)

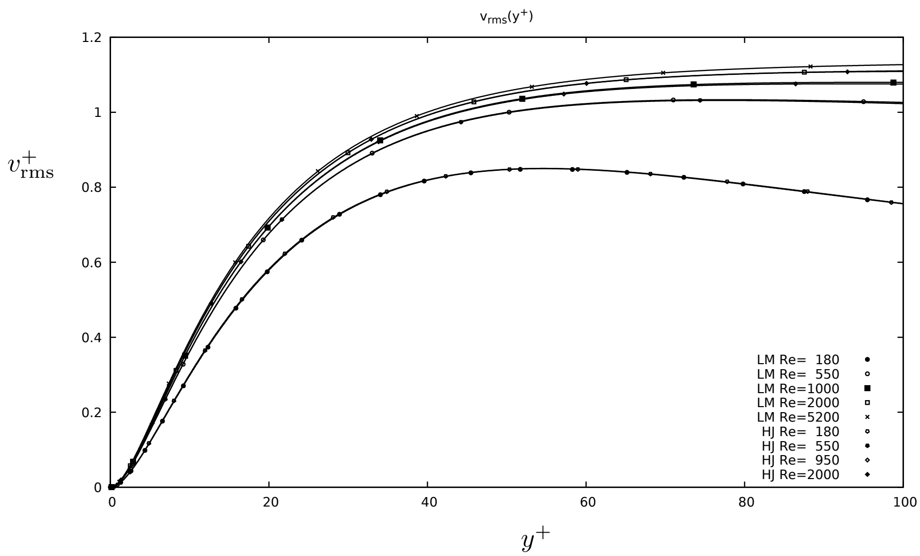

Figure 3: v_{rms}

$$v_{\text{rms}}(y) = f_{15}^2(y, a, b, c, q, r, p) = \left(y(ay^2 + by + c)/(qy^2 + ry + 1) + p(\log(y + 15) - \log(15))\right)^2$$

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vsqrtrmsRe0180LM(x)=f15(x, 2.41938530630219e-06,-0.00589206856212848,-0.00776762262782838,0.000828781770821384,0.154590693259966,1.56314045596651)
vsqrtrmsRe0550LM(x)=f15(x,-3.62370968972199e-06,-0.00771444345752978,-0.0132743698870516, 0.0013922374768463, 0.147054321405927,1.85771815751584)
vsqrtrmsRe1000LM(x)=f15(x,-4.1893276442043e-06, -0.00799556103055885,-0.0117451244065584, 0.00146866281868965, 0.147654082043006,1.8856633345211)
vsqrtrmsRe2000LM(x)=f15(x,-4.63204035488413e-06,-0.00823521098799527,-0.00900585799466542,0.00153937507562706, 0.149967447907337,1.8918597191896)
vsqrtrmsRe5200LM(x)=f15(x,-5.54815570335504e-06,-0.0091658705452141, -0.0115209267452397, 0.00167001292415056, 0.157343089533074,1.96548649536376)
vrmsRe0180LM(x)=vsqrtrmsRe0180LM(x)**2
vrmsRe0550LM(x)=vsqrtrmsRe0550LM(x)**2
vrmsRe1000LM(x)=vsqrtrmsRe1000LM(x)**2
vrmsRe2000LM(x)=vsqrtrmsRe2000LM(x)**2
vrmsRe5200LM(x)=vsqrtrmsRe5200LM(x)**2
vsqrtrmsRe0180HJ(x)=f15(x,9.65439923792012e-07,-0.00709909883501314,-0.0127371958888107,0.00105054100367451,0.169102448901133,1.65886763757801)
vsqrtrmsRe0550HJ(x)=f15(x,-4.78060080164494e-06,-0.00864264739332554,-0.0176231572361471,0.00152284981682245,0.155121745948214,1.93455363238752)
vsqrtrmsRe0950HJ(x)=f15(x,-4.46099073773041e-06,-0.00816966657663943,-0.0123845666354246,0.0014956398243829,0.149357021643905,1.89801856396605)
vsqrtrmsRe2000HJ(x)=f15(x,-1.21797163605269e-05,-0.0157105582660346,-0.0416069555393806,0.0023632813495,0.202766339807415,2.41307665699363)
vrmsRe2000HJ(x)=vsqrtrmsRe2000HJ(x)**2
vrmsRe0950HJ(x)=vsqrtrmsRe0950HJ(x)**2
vrmsRe0550HJ(x)=vsqrtrmsRe0550HJ(x)**2
vrmsRe0180HJ(x)=vsqrtrmsRe0180HJ(x)**2
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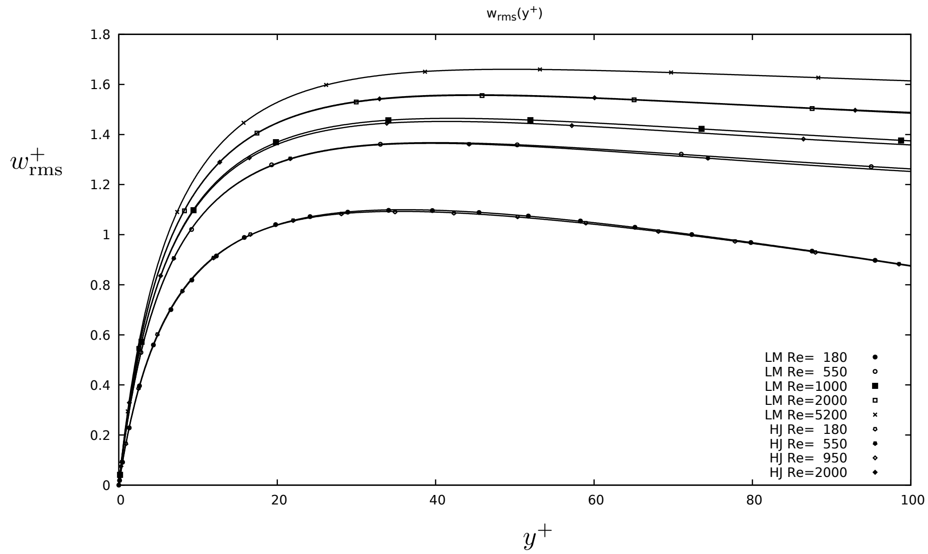

Figure 4: $w_{\rm rms}$

$$w_{\rm rms}(y) = f_{15}(y, a, b, c, q, r, p) = y(ay^2{+}by{+}c)/(qy^2{+}rx{+}1){+}p(\log(y{+}15){-}\log(15))$$

wrmsRe0180LM(x)=f15(x,-2.04534705125964e-05,-0.00638020344624595,0.121205632571833,0.00231725044396721,0.16939693949413, 1.29171921662097)
wrmsRe0550LM(x)=f15(x,-5.85344373638104e-06,-0.00789945389929343,0.173618817273645,0.00204585414584547,0.179950441311365,1.45966054095059)
wrmsRe1000LM(x)=f15(x,-5.90837356791285e-06,-0.00743709757663359,0.190362109612082,0.00204567529079099,0.176653687437557,1.44339839948606)
wrmsRe2000LM(x)=f15(x,-6.70256935487831e-06,-0.00725894289215538,0.200140297679313,0.00208929203393882,0.172450786312379,1.4726031977162)
wrmsRe5200LM(x)=f15(x,-8.94631764983125e-06,-0.0084598574019162,0.191311317447557,0.00218061920820277,0.166057798726084,1.71943819382951)
wrmsRe0180HJ(x)=f15(x,-4.0412927558585e-05,-0.0170065932647172,0.0495404717376413,0.00304789096947113, 0.189338120863009, 2.43086522096585)
wrmsRe0550HJ(x)=f15(x, 3.32805132121153e-05,0.0272744795173471,0.365649546944411, 0.0062019719528945, 0.198186594343618,-1.41550156068556)
wrmsRe0950HJ(x)=f15(x, 3.20298512131681e-05,0.0265164208411655,0.372681804636301, 0.0062630619696859, 0.195930883247131,-1.30612633893608)
wrmsRe2000HJ(x)=f15(x, 3.10318707015377e-05,0.0273777055381056,0.371979111530335,0.00702168956398879,0.19985554051313, -1.11038827797596)

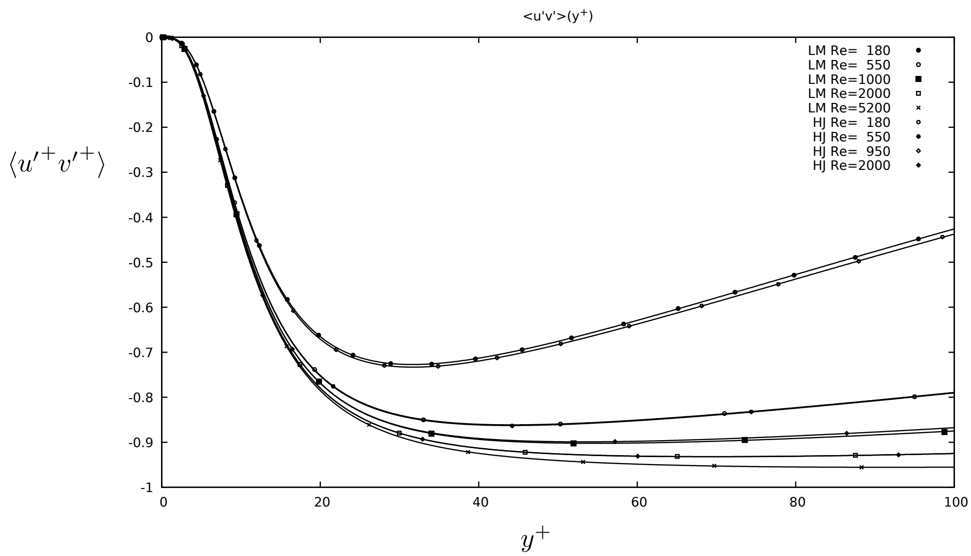

Figure 5: Reynolds stress

$$\langle u'v' \rangle(y) = -f_{15}^3(y,a,b,c,q,r,p) = -\left(y(ay^2+by+c)/(qy^2+ry+1)+p(\log(y+15)-\log(15))\right)^3$$

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uvmeanonethridRe5200LM(x)=-f15(x,-6.00235084251445e-06,0.0110393880134619, 0.0971494514578099,0.0130740607504558, 0.0560872281771557,0.0756599507715887)
uvmeanonethridRe2000LM(x)=-f15(x,-7.84036407745564e-06,0.0104994462335006, 0.0954735210603693,0.0126525035622012, 0.0529466282944463,0.0839618555498294)
uvmeanonethridRe1000LM(x)=-f15(x,-1.1095556379209e-05, 0.00992872527830186,0.0930050150094885,0.0122329354646834, 0.0494819001584414,0.0964466884083661)
uvmeanonethridRe0550LM(x)=-f15(x,-1.59595147229951e-05,0.0093134863610134, 0.0903362030849532,0.0117460658495685, 0.0462169807046629,0.109839166662012)
uvmeanonethridRe0180LM(x)=-f15(x,-4.21477317909565e-05,0.00628540745070078,0.0746623203329355,0.00944055500852679,0.0314695416519397,0.229052223299449)
uvmeanonethridRe2000HJ(x)=-f15(x,-7.98298357378521e-06,0.0105404260114485, 0.0953981402353763,0.0126992843177695, 0.0529534563160199,0.0843797100585562)
uvmeanonethridRe0950HJ(x)=-f15(x,-1.14314892385872e-05,0.00993283249037142,0.0929962794981221,0.0122221513424288, 0.0495100179460231,0.0959626510640031)
uvmeanonethridRe0550HJ(x)=-f15(x,-1.58957890152675e-05,0.00935492568149067,0.0904621607225798,0.0117681062374765, 0.0463835645296073,0.108764477729061)
uvmeanonethridRe0180HJ(x)=-f15(x,-4.13115872958601e-05,0.00634117288335139,0.0750839076181533,0.00949281371640684,0.0311579741625425,0.225915980134867)
uvmeanRe5200LM(x)=uvmeanonethridRe5200LM(x)**3
uvmeanRe2000LM(x)=uvmeanonethridRe2000LM(x)**3
uvmeanRe1000LM(x)=uvmeanonethridRe1000LM(x)**3
uvmeanRe0550LM(x)=uvmeanonethridRe0550LM(x)**3
uvmeanRe0180LM(x)=uvmeanonethridRe0180LM(x)**3
uvmeanRe2000HJ(x)=uvmeanonethridRe2000HJ(x)**3
uvmeanRe0950HJ(x)=uvmeanonethridRe0950HJ(x)**3
uvmeanRe0550HJ(x)=uvmeanonethridRe0550HJ(x)**3
uvmeanRe0180HJ(x)=uvmeanonethridRe0180HJ(x)**3
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References

1. Hoyas, S. & Jiménez, J. 2006 Scaling of the velocity fluctuations in turbulent channels up to $Re_\tau = 2003$. *Phys. Fluids* **18** (011702).

2. Lee, M. & Moser, R. D. 2015 Direct numerical simulation of turbulent channel flow up to $Re_\tau = 5200$. *J. Fluid Mech.* **774**.