Supplementary Material for Intermediate Scaling and Logarithmic Invariance in Turbulent Pipe Flow

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Indicator function plots for the mean velocity and variance

It was mentioned in the main text that the range of y/y_m used for determining log fits for the mean velocity and variance (in Overlap Layer II) is $1.2 \leqslant (y/y_m) \leqslant 13$. The log fits were obtained for the two highest Re_τ, i.e., 68370 and 98190. It was mentioned that the extent of the logarthmic region decreases with decrease in Re_τ. In this document, we plot the data in the form of an "indicator function" (McKeon *et al.* 2004) to show this behaviour more clearly. The indicator function for the mean velocity is defined as $\mathrm{IF1} = (U - U_m)/u_m - [1/\kappa_m \ln(y/y_m) + A_m]$ (see equation 2.5 in the main text) and is plotted in figure 1. The presence of logarithmic variation is indicated by the flat portion of the indicator-function profiles around IF1 = 0.

As clearly seen in figure 1, the range $1.2 \leqslant (y/y_m) \leqslant 13$ for the log fit works well for the two highest Re_τ. For $Re_\tau = 98190$ this translates into $4.2\sqrt{Re_\tau} \leqslant y^+ \leqslant 0.145Re_\tau$ (see the main text). With a decrease in Reynolds number, the profiles peel off from the flat region at progressively lower y/y_m, and therefore the extent of the log region continuously decreases (figure 1). At $Re_\tau = 20250$, an approximately flat portion of the profile can be identified in the range $1.2 \leqslant (y/y_m) \leqslant 5.5$, although the profile starts departing from the IF1=0 line earlier than that. This translates into $4.2\sqrt{Re_\tau} \leqslant y^+ \leqslant 0.135Re_\tau$ for this Re_τ. For $Re_\tau = 10480$, there is a narrow range, $1.2 \leqslant (y/y_m) \leqslant 3$, where a log region may be identified in figure 1, which results in $4.2\sqrt{Re_\tau} \leqslant y^+ \leqslant 0.103Re_\tau$. For $Re_\tau < 10^4$, there is no perceptible flat region in the indicator-function profiles near IF1=0 and therefore a log law, in terms of intermediate variables, is untenable. This observation provides an additional support to our conclusion in the main text (figures 3b and d) that Overlap Layer II for the mean velocity is governed by a power law for low Reynolds numbers and the log law emerges only when $Re_\tau > 10^4$. (This is the reason why the Kármán constant obtained for $Re_\tau < 10^4$ shows much higher values than reported in the literature, indicated by crosses in figure 3(d) in the main text.) The present exercise also suggests that the outer limit of the classical log law is variable and changes from about $0.1Re_\tau$ at $Re_\tau \approx 10^4$ to $0.145Re_\tau$ at $Re_\tau \approx 10^6$. The latter limit matches well with the outer limit of $0.15Re_\tau$ used in the analysis of Marusic *et al.* (2013), who used very-high Reynolds number wall turbulence data from different facilities to determine the log-law constants.

Figure 2 shows the indicator function for the streamwise variance, $\mathrm{IF2} = (\overline{u^2}/u_m^2) - [B_1^m - A_1^m \ln(y/y_m)]$, as a function of y/y_m. Just as for the mean velocity, the range

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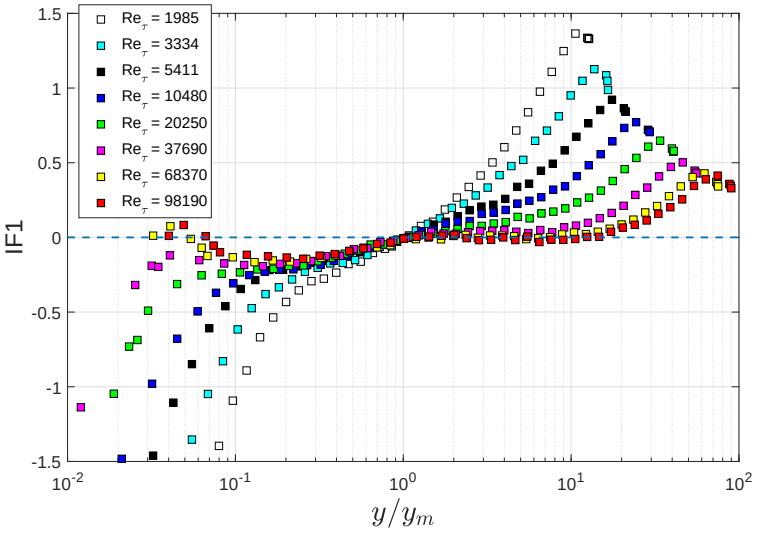

FIGURE 1. Indicator function for the mean velocity, IF1, as a function of y/y_m for the pipe. IF1 $= (U - U_m)/u_m - [1/\kappa_m \ln(y/y_m) + A_m]$.

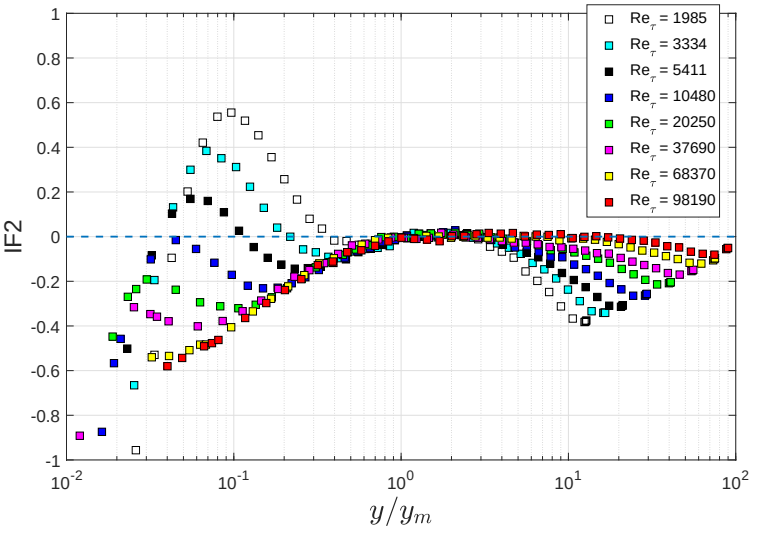

FIGURE 2. Indicator function for the variance, IF2, as a function of y/y_m for the pipe. IF2 $=(\overline{u^2}/u_m^2) - [B_1^m - A_1^m \ln(y/y_m)]$.

$1.2 \leqslant (y/y_m) \leqslant 13$ represents log variation for the two highest Re_τ, and as Re_τ decreases the extent of the log region goes on decreasing. Interestingly, for the variance, there seems to exist a log region even for $Re_\tau < 10^4$ (figure 2) in contrast to the mean velocity (figure 1). It is for this reason that we have included the values of A_1 and B_1 (log-law coefficients for the variance) in table 1 in the main text for $Re_\tau < 10^4$, but not for κ and A (log-law coefficients for the mean velocity).

REFERENCES

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