

## Source of the Quasi-Periodic Trigger

The source of quasi-periodic trigger is identified by distinguishing the incommensurate frequency present in the system. The presence of two incommensurate frequencies at  $\kappa h = 1.70$  can be clearly seen from the frequency spectra of  $C_l$  presented in Fig. 1 of this document. It can be seen that the primary frequency ' $f_1$ ' corresponds to the flapping frequency and the secondary frequency ' $f_2$ ' takes a value of almost twice of ' $f_1$ ', but their ratio is not exactly 2 (irrational). It has also been observed that the other frequencies, present in the frequency spectra, are in a linear combination of ' $f_1$ ' and ' $f_2$ ' which establishes that ' $f_1$ ' and ' $f_2$ ' are incommensurate in nature, giving way to the quasi-periodic dynamics.

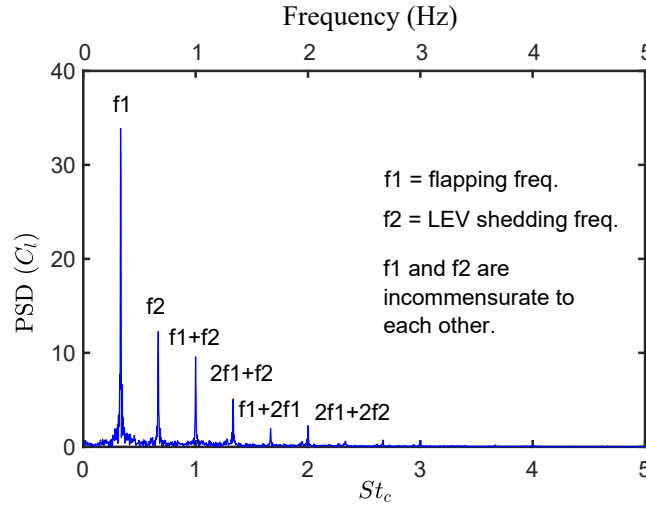


Figure 1: Frequency spectra of  $C_l$  time history in the quasi-periodic regime at  $\kappa h = 1.70$ .

On further investigation of the flow-field, it has been revealed that ' $f_2$ ' corresponds to the shedding frequency of the LEV which is not constant over different cycles. To demonstrate the same, we have chosen two different phases of a flapping cycle. The first is when the LEV sheds from the leading-edge (see Fig. 2(a) of this document) and the second is when the LEV shed from the trailing-edge (after getting reattached to the airfoil surface) along with the TEV (see Fig. 3(a) of this document). Fig. 2(b) demonstrates that the shedding of the LEV is associated with a phase lag from one cycle to another. This mutual delay gives rise to a second frequency  $f_2$ , which is incommensurate to  $f_1$ . As a result, the location of the LEV core differs slightly in the consecutive cycles, but remains in the neighborhood of the previous cycle; see Fig. 2(c). The deviation gradually increases, however, the LEV core comes back to the close neighborhood of its initial location after a few cycle, which is representative of a stroboscopic closure and indicates the presence of quasi-periodicity. Eventually, this discrepancy in the LEV shedding is propagated downstream and affects the shedding of the TEV; see Fig. 3(a). Figures 3(b) and 53(c) show the formation of similar stroboscopic closed loops by the LEV and TEV cores. This, in turn, makes the subsequent LEV-TEV interactions quasi-periodic. This discussion has now been added as a supplementary data.

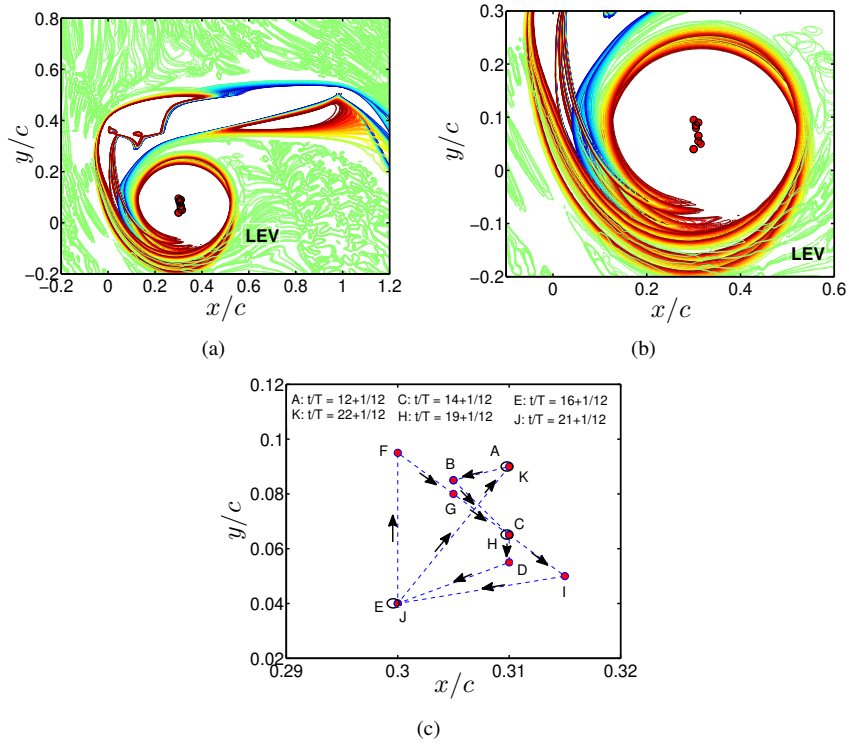


Figure 2: Deviations in the location of the LEV core when the LEV sheds from the lower surface of the leading-edge in the consecutive flapping cycles.

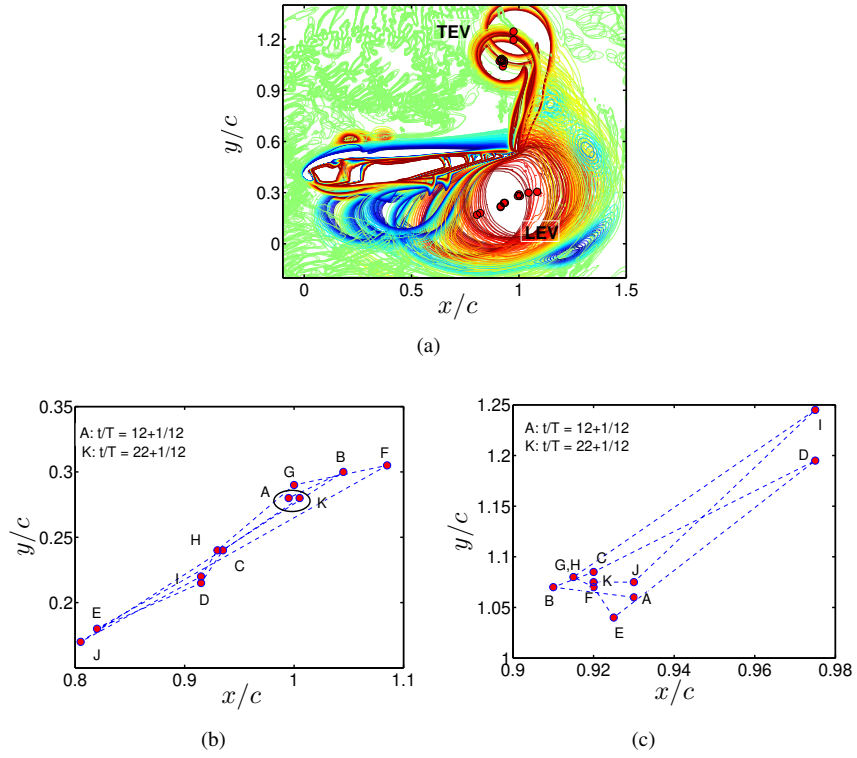


Figure 3: Deviations in the location of the LEV and TEV cores when the LEV sheds from the trailing-edge in the consecutive flapping cycles.