## Supplementary Materials

## Dilatancy-driven secondary flows in dense granular materials

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## Supplementary figures

In figures 3, 4, 7 and 8 of the paper, the x (r in figure 8) and y axes were scaled by the width 2W ( $R_o$ ) and fill height H, respectively (see figure 2), so as to obtain figures of the same size. However, this does not readily convey the true aspect ratio of the shear cell and dimensions of the secondary vortices. Here, the same figures are presented but with the x (r) and y axes both scaled by 2W ( $R_o$ ). In the rectangular cell with side walls (figure S1), it is evident that the size of the larger (upper) secondary vortices are roughly the same for all three fill heights. In the cylindrical cell (figure S3), the vortices increase in size from the smallest to the intermediate fill height, but shrink substantially between the intermediate and largest fill heights.



FIGURE S1. Results of the DEM simulations for a rectangular shear cell (supplementing figures 3, 4 in the paper). The figures in each column are for the same fill height H. (a)-(c) The primary flow for fill heights H of 0.88 W, 1.13 W, and 1.38 W. The colours indicate the scaled velocity  $u_z/V_w$ , and the solid lines are contours of constant velocity in steps of 0.091. (d-f) Colour maps of the particle volume fraction  $\phi$ . (g)-(i) Streamlines of the secondary flow. The colours indicate the speed of the secondary flow  $s \equiv (u_x^2 + u_y^2)^{1/2}/V_w$  on a logarithmic scale.



FIGURE S2. Results of the DEM simulations for a rectangular shear cell with periodic side walls (supplementing figure 7 in the paper). The figures in each column are for the same fill height H. (a)-(c) The primary flow for fill heights H of 0.88 W, 1.13 W, and 1.38 W. Colours indicate the scaled velocity  $u_z/V_w$ , and the solid lines are contours of constant velocity in steps of 0.091. The dashed line in (b) is a contour for  $u_z/V_w = 0.6$ . (d-f) Colour maps of the particle volume fraction  $\phi$ . (g)-(i) Streamlines of the secondary flow. The colours indicate the speed of the secondary flow  $s \equiv (u_x^2 + u_y^2)^{1/2}/V_w$  on a logarithmic scale.



FIGURE S3. Results of the DEM simulations for the cylindrical shear cell (supplementing figure 8 in the paper). The figures in each column are for the same fill height H. (a)-(c) The primary flow for fill heights H of  $0.43 R_i$ ,  $0.73 R_i$ , and  $1.27 R_i$ . The colours indicate the scaled angular velocity  $\omega/\Omega_w$ , and the lines are contours of constant angular velocity in steps of 0.091. (d-f) Colour maps of the particle volume fraction  $\phi$ . (g)-(i) Streamlines of the secondary flow. The colours indicate the speed of the secondary flow  $s \equiv (u_r^2 + u_y^2)^{1/2}/V_w$  on a logarithmic scale.