CONTENTS OF SUPPLEMENTAL MATERIALS

This document lists and describes the contents of the Supplemental Materials.

LIST OF CONTENTS

The contents are as follows, organized by topic:

I. WALL SHEAR-STRESS FIELD AND STATISTICS: wallstress.tif, wallstressPDF.tif, lminwallstressPDF.tif, lmaxwallstressPDF.tif

II. LOCAL PLANAR vs. GLOBAL AVERAGES: U-glob-loc-eject.tif, Omz-glob-loc-eject.tif, U-glob-loc-sweep.tif, Omz-glob-loc-sweep.tif

III. PARTICLE ENSEMBLE STATISTICS: ymean_sweep.tif, dispersion_eject.tif, dispersion_sweep.tif

IV. MOVIES OF PARALLEL CAUCHY INVARIANT:
Movie1.avi, Movie2.avi

V. MOVIES OF NEGATIVE STREAMWISE VORTICITY SOURCE:
Movie3.avi, Movie4.avi

VI. COARSE-GRAINED CAUCHY INVARIANTS

a) *Plots Versus Time*:

wallpercent-coarse-eject.tif, parallelcauchy-coarse-eject.tif, perpcauchy-coarse-eject.tif, integraltime-coarse-eject.tif,

wallpercent-coarse-sweep.tif, parallelcauchy-coarse-sweep.tif, perpcauchy-coarse-sweep.tif, integraltime-coarse-sweep.tif

b) *Movies of Parallel Invariant*:

Movie5.avi, Movie6.avi

DESCRIPTION OF CONTENTS

Each of the figures/movies in the SM is discussed below in detail, with information provided on the exact content and the algorithms, parameter values, etc. used in their preparation.

I. WALL SHEAR-STRESS FIELD AND STATISTICS:

The values of $\partial u/\partial y$ were downloaded from the channel database over the entire top wall at the final stored time, using the *getVelocityGradient* function and looping over lines at constant x. The viscous wall shear-stress was then computed as $\tau_{xy}=\nu\partial u/\partial y$ and normalized by its mean value $\tau_*=u_*^2$ to obtain τ_{xy^+}. The local minima and maxima of the field τ_{xy^+} were found using the Matlab code *FastPeakFind.m* from

https://www.mathworks.com/matlabcentral/fileexchange/37388-fast-2d-peak-finder

The points identified by this software were found reliably to be the locations of local extrema, but many local extrema identifiable by eye were missed. Nevertheless, the statistics of the local extrema were found to be quite independent of the adjustable parameter *thresh* used in *FastPeakFind.m* and are thus included for the interest of readers. The set of plots are:

wallstress.tif: Plot of the field $\tau_{xy^+}(x,z)$, over the entire top wall of the channel at the final time in the database. Created with *imagesc* in Matlab.

wallstressPDF.tif: Plot of the PDF of τ_{xy}^+ values at the top wall of the channel at the final time in the database. Created with *histogram* in Matlab.

lminwallstressPDF.tif: Plot of the PDF of τ_{xy}^+ values at the points of local minimum stress. Created with output of *FastPeakFind.m* and *histogram* in Matlab.

lmaxwallstressPDF.tif: Plot of the PDF of τ_{xy}^+ values at the points of local maximum stress. Created with output of *FastPeakFind.m* and *histogram* in Matlab.

II. LOCAL PLANAR vs. GLOBAL AVERAGES:

The global mean value in the entire channel flow database for the streamwise velocity u^+, or U^+ as a function of y^+, was downloaded from

http://turbulence.pha.jhu.edu/docs/channel/profiles.txt

The global mean value of the spanwise vorticity $\Omega_z^+ = -\partial U^+/\partial y^+$ was then approximated by using 4th-order Lagrange interpolation. Local averages of u^+ and ω_z^+ over planes of constant y^+ were calculated for the two datacubes pictured in Fig.3a for the ejection and Fig.8a for the sweep. The sets of plots are:

U-glob-loc-eject.tif: Plot of global vs. local average for u^+ in the ejection.

Omz-glob-loc-eject.tif: Plot of global vs. local average for ω_z^+ in the ejection.

U-glob-loc-sweep.tif: Plot of global vs. local average for u^+ in the sweep.

Omz-glob-loc-sweep.tif: Plot of global vs. local average for ω_z^+ in the sweep.

III. PARTICLE ENSEMBLE STATISTICS: The variances of stochastic Lagrangian particle positions $x^{\sim +}_i(s)$ in the $i=x,y,z$ coordinate directions, were calculated as functions of $s<t$ with $N=10^7$ particles, for both the ejection and the sweep. Expressed in wall units, these were plotted in log-log versus δs^+. All coordinates exhibit a diffusive scaling $\sim 4\nu\,|\delta s|$ after the particles are first released, indicated by the dotted line in the log-log plots. A possible Corrsin scaling $\sim|\delta s|^3$ of the x-dispersion at longer times is indicated by the dot-dash line with slope -3, which fits well the numerical data for the ejection but which exceeds somewhat the observed slope for the sweep. In addition, the mean value of $y^{\sim +}(s)$ has been plotted versus δs^+ for the sweep case, with average over all particles and also over interior particles only. The sets of plots are:

dispersion_eject.tif: Log-log plot of $\mathrm{Var}[x^{\sim +}_i(s)]$ vs. δs^+ for $i=x,y,z$ in the ejection event.

dispersion_sweep.tif: Log-log plot of $\mathrm{Var}[x^{\sim +}_i(s)]$ vs. δs^+ for $i=x,y,z$ in the sweep event.

ymean_sweep.tif: Plot of $\mathbb{E}_C[y^{\sim +}(s)]$ vs. δs^+ in the sweep event, for C=A (all), I (interior)

IV. MOVIES OF PARALLEL CAUCHY INVARIANT: Corresponding to Figs.17/21 in the main text, we made movies of the parallel Cauchy invariant contributions at the wall for the ejection and sweep, respectively. The frames of the movies correspond exactly to the panels of the two figures, except that the movies are made with $N=10^7$ particles rather than $N=10^6$ and the movie frames are separated by t_ν (viscous time) rather than 25 t_ν as for the figure panels. The sets of movies are:

Movie1.avi: Movie of $\omega^{\sim +}_{\parallel}(\boldsymbol{x},t)$ values for particles hitting the wall, corresponding to Fig.17 for the ejection in the main text

Movie2.avi: Movie of $\omega^{\sim +}_{\parallel}(\boldsymbol{x},t)$ values for particles hitting the wall, corresponding to Fig.21 for the sweep in the main text

V. MOVIES OF NEGATIVE STREAMWISE VORTICITY SOURCE: Corresponding to Figs.18/22 in the main text, we made movies of the negative streamwise vorticity source $-\sigma^{+}_{z} = \partial\omega^{+}_{z}/\partial y^{+}$ together with locations of particles hitting the wall, for both ejection & sweep, respectively. The frames of the movies correspond exactly to the panels of the two figures, except that: (1) the movies are made with N=60,000 particles rather than N=40,000, (2) the movie frames are separated by t_v (viscous time) rather than 5 t_v as for the figure panels, and (3) the light grey contour levels in the figures have been omitted in the movies. The sets of movies are:

Movie3.avi: Movie of $-\sigma^{+}_{z}(\mathbf{x},t)$ field and locations of particles hitting the wall, corresponding to Fig.18 for the ejection in the main text

Movie4.avi: Movie of $-\sigma^{+}_{z}(\mathbf{x},t)$ field and locations of particles hitting the wall, corresponding to Fig.22 for the sweep in the main text

VI. COARSE-GRAINED CAUCHY INVARIANTS: As discussed in the text, the stochastic Cauchy invariant was coarse-grained with a box filter over (n_x,n_y,n_z)=(4,0,4) grid spaces in the three coordinate directions. Results analogous to those for the fine-grained invariant were obtained using the same Monte Carlo Lagrangian method, with $N=10^7$ particles and time-step $\Delta s=10^{-3}$.

a) *Plots Versus Time*: Plots for the coarse-grained invariants were prepared analogous to Figs.15abc/19abc in the main text for fine-grained fields. The sets of plots are:

wallpercent-coarse-eject.tif: Fraction of particles at the wall in the ejection, analogous to Fig.15a for the fine-grained invariant.

parallelcauchy-coarse-eject.tif: Partial means of the parallel component of the stochastic Cauchy invariant in the ejection, analogous to Fig.15b for the fine-grained invariant.

perpcauchy-coarse-eject.tif: Partial means of the perpendicular component of the stochastic Cauchy invariant in the ejection, analogous to Fig.15c for the fine-grained invariant.

wallpercent-coarse-sweep.tif: Fraction of particles at the wall in the sweep, analogous to Fig.19a for the fine-grained invariant.

parallelcauchy-coarse-sweep.tif: Partial means of the parallel component of the stochastic Cauchy invariant in the sweep, analogous to Fig.19b for the fine-grained invariant.

perpcauchy-coarse-sweep.tif: Partial means of the parallel component of the stochastic Cauchy invariant in the sweep, analogous to Fig.19c for the fine-grained invariant.

Plots for the coarse-grained invariants were prepared analogous to Figs.16/20 in the main text for fine-grained fields. The sets of plots are:

integraltime-coarse-eject.tif: Fractional contribution of the coarse-grained parallel vorticity arising from interior particles, in the ejection, analogous to Fig.16 for the fine-grained invariant

integraltime-coarse-sweep.tif: Fractional contribution of the coarse-grained parallel vorticity arising from interior particles, in the sweep, analogous to Fig.20 for the fine-grained invariant.

The integral formation time for the parallel component defined by Eq.(4.2) in the main text, when evaluated with data for the coarse-grained invariant in the above two plots, is:

Ejection: $T^{+}_{\|}= 38.5$
Sweep: $T^{+}_{\|} = 25.4$

For the ejection, the formation time of the coarse-grained parallel component is considerably larger than the formation time $T^{+}_{\|}= 20.7$ reported in the text for the fine-grained parallel component. This is presumably due to the spanwise coarse-graining, since particles further from the center of the ejection should approach the wall less quickly backward in time. For the sweep, the formation time of the coarse-grained parallel component is only slightly larger than the formation time $T^{+}_{\|}= 24.6$ reported in the text for the fine-grained parallel component. This is presumably due to the greater spanwise width of the sweep than of the ejection.

b) *Movies of Parallel Invariant*: Analogous to item IV in the Contents for the fine-grained Cauchy invariants, we made movies also of the coarse-grained parallel Cauchy invariants for both the ejection and sweep, again with $N=10^7$ particles and time separation t_v between movie frames. The sets of movies are:

Movie5.avi: Movie of $\tilde{\omega}^{+}_{\|}(\boldsymbol{x},t)$ values coarse-grained over position $\boldsymbol{x}$ for particles hitting the wall, analogous to Fig.17 for the ejection in the main text.

Movie6.avi: Movie of $\tilde{\omega}^{+}_{\|}(\boldsymbol{x},t)$ values coarse-grained over position $\boldsymbol{x}$ for particles hitting the wall, analogous to Fig.21 for the sweep in the main text.

The analogous movies for the fine-grained and coarse-grained Cauchy invariants are clearly quite closely similar.