Supplementary material to the article

Onset of Darcy-Bénard convection in a horizontal layer of dual-permeability medium with isothermal boundaries

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In this supplementary material, we provide details of the numerical algorithm and the simulation results shown in figure 10. Also, the material includes the animated figures showing evolution of the flow parameters with time.

The thermal convection is simulated using the following governing equations:

$$\rho^j = \rho_0(1 + \eta(p^j - p_{ref}) - \beta(T^j - T_{ref})), \quad e^j = cT^j, \quad j = f, m, \tag{S.1}$$

$$\boldsymbol{u}^j = -\frac{K^j}{\mu}(\nabla p^j - \rho^j \boldsymbol{g}), \tag{S.2}$$

$$\partial_t\left(\phi\rho^j\right) + \boldsymbol{\nabla}\cdot\left(\rho^j\boldsymbol{u}^j\right) = -b^j\rho^{fm}q^{fm}, \quad q^{fm} = \sigma_1\frac{K^m}{\mu}(p^f - p^m), \tag{S.3}$$

$$\partial_t(\rho e)^j + \boldsymbol{\nabla}\cdot(\rho^j h^j \boldsymbol{u}^j) = \rho^j\boldsymbol{g}\cdot\boldsymbol{u}^j + \lambda^j\nabla^2 T^j - b^j q^{fm}\rho^{fm}h^{fm} - b^j\sigma_2\lambda^m(T^f - T^m), \tag{S.4}$$

where all notations are consistent with the article; e and $h = e + P/\rho$ are the specific internal energy and enthalpy of the fluid; η is the fluid compressibility; $\mu = \mathrm{const}$ is the dynamic viscosity; ρ_0, p_{ref}, and T_{ref} are constants; ρ^{fm} (and h^{fm}) are the upwind values of ρ (and h), i.e., if $p^f \geq p^m$ then $\rho^{fm} = \rho^f$ (and $h^{fm} = h^f$), and if $p^f < p^m$ then $\rho^{fm} = \rho^m$ (and $h^{fm} = h^m$); and

$$(\rho e)^f = \gamma\left(\phi\rho^f e^f + (1-\phi)\rho_r c_r T^f\right) = (\rho c)^f T^f,$$

$$(\rho e)^m = (1-\gamma)\left(\phi\rho^m e^m + (1-\phi)\rho_r c_r T^m\right) = (\rho c)^m T^m.$$

Equations (S.1) are the fluid equations of state, equation (S.2) is Darcy's law, and equations (S.3) and (S.4) are the mass and energy balance equations for each medium, respectively. In the Oberbeck-Boussinesq approximation, (S.1)–(S.4) reduce to equations (2.1)–(2.4) of the article.

The 2-D simulations are conducted using the MUFITS reservoir simulator (Afanasyev, 2017) in dimensional variables, and afterwards, their results are converted to the dimensionless variables. The domain in the $\{x, z\}$ plane is of height H and length $5H$. According to (2.9), the upper ($z = H$) and lower ($z = 0$) boundaries of the layer are impermeable and isothermal. The lateral boundaries $x = 0$ and $x = 5H$ are impermeable and adiabatic. According to (2.8), at $t = 0$, the temperature has linear distribution between $T = T_- = T_{ref}$ at $z = H$ and $T = T_+$ at $z = 0$. At $t = 0$, the pressure has close-to-hydrostatic distribution such that $p = p_{ref}$ at $z = H$. The layer thickness is $H = 30$ m. The following dimensional parameters used resemble the thermophysical properties of water at $p_{ref} = 20$ bar and $T_{ref} = 20°$C: $\rho_0 = 1000$ kg/m^3, $\eta = 5 \cdot 10^{-5}$ 1/bar, $\beta = 3 \cdot 10^{-4}$ 1/K, $c = 4.2$ kJ/kg·K, $\mu = 1$ cP. The compressibility η is so small that in the simulated flows the fluid density changes due to variations of T are much larger than those due to variations of p. The temperatures of $T_- = 20°$C and $T_+ = 32°$C are maintained at $z = H$ and $z = 0$, respectively. Other parameters are chosen to fit particular dimensionless quantities $\overline{Ra}$, κ, Λ, and B in table 1. Further results in this supplementary material are presented in dimensionless variables.

The simulations are conducted using a homogeneous rectilinear grid of 300×60 elements. The finite-difference scheme for equations (S.1)–(S.4) is constructed based on the finite-volume approach and the upwind approximation of fluxes. The fully-implicit method is applied (Aziz & Settari, 1979; Fanchi, 2006). The time step is restricted by the CFL condition to reduce the truncation error. The maximum Courant number is set to 0.5. A grid independence study has been performed using both twice denser and coarser grids. It has been found that the numerical solution does not depend on the considered grid resolutions. The usage of the rather coarse grid of 300×60 elements is justified by the near-critical $\overline{Ra}$ ($\overline{Ra} \leq 1.1\overline{Ra}_c$), which does not result in significant deviation of flow parameters from the basic solution (3.5).

To perturb the quiescent thermally stratified fluid at $t = 0$, we impose small perturbations of temperature on the initial distribution (3.5). A small quantity of absolute value less than 10^{-4} produced by a pseudo-random number generator is added to Θ^f and Θ^m in every grid block. Thus, all waves longer than the size of the grid blocks ($\sim 10^{-2}$) are perturbed. The perturbations of only particular wavelengths (with $\mathrm{Re}\Omega > 0$) grow in the simulation with rising t, leading to the onset of convection, although these wavelengths are not specifically distinguished by the numerical algorithm. The calculated patterns are presented in figure 10 at $t > 100$ when the quasi-steady state is reached.

Let us discuss in more detail the simulation results for the 4 selected scenarios, corresponding to the regimes F, M_z, C_f, and P shown in figure 10. The scenarios are also supplemented with the animated figures showing evolution of the parameters as the quasi-steady state is reached. All parameters drawn in the animations are dimensionless. The parameters in Φ^f and Φ^m are shown in the upper and lower panels of the animations, respectively. The red and blue curves are isotherms $\Theta^j = i/10$, $i = 1 \ldots 9$ in Φ^f and Φ^m, respectively. The arrows indicate the direction of fluid flow ($\boldsymbol{u}^j$). The thin grey curves, also shown in figure 10, are the streamlines calculated using the ParaView software (www.paraview.org). The points evenly distributed along the straight line $z = 1/2$ are chosen as the seed points for the streamlines. Therefore, some regions in the F_z, C_{fx}, C_{mx}, and M_z patterns are not covered with streamlines because not all of them intersect the straight line $z = 1/2$. The distribution of q^{fm}, i.e., the fluid flux from Φ^f into Φ^m, is shown in the upper panel. The distribution of $\Theta^f - \Theta^m$, which defines the conductive heat flux from Φ^f into Φ^m, is given in the lower panel.

As predicted by the analytical study, the co-rotating regime F develops in the case shown in figure 10(a). The velocities $\boldsymbol{u}^f$ and $\boldsymbol{u}^m$ at any $\boldsymbol{r}$ are pointing in the same direction, and the amplitude of temperature $\left|\delta\Theta^f\right|$ in Φ^f is larger than $|\delta\Theta^m|$ in Φ^m. The horizontal dimension of the convection cells is close to 1, which reflects that k_{xy} is close to π in the growing solutions to equation (4.9). As shown in the animated figure, the mode F_z is growing faster than the other modes during the initial transient processes. However, as the quasi-steady state is reached later on, the mode F becomes the dominant one.

The regime M_z develops in the case shown in figure 10(b). Here, the cellular flow pattern develops in Φ^m, whereas the vertical component of $\boldsymbol{u}^f$ is reverted. Thus, the convection cells in Φ^f are broken.

The counter-rotating regime C_f develops in the case shown in figure 10(c). Here, the fluid velocities in Φ^j are pointing in opposite directions at any position $\boldsymbol{r}$. As discussed in § 7, the inequality $\left|\delta\Theta^f\right| > |\delta\Theta^m|$ holds. The convection cells are stretched in the horizontal direction. This is supported by the theory because k_c in the corresponding solution to equation (4.9) is almost half of π (table 1).

The plane flow regime occurs in the case shown in figure 10(d). Here, the flow is directed upward in Φ^f and downward in Φ^m. Consequently, the isotherms in Φ^f and Φ^m are shifted upward and downward, respectively, as compared to the linear distribution in the basic state (3.5). As shown in the animated figure, the cellular convection develops during the initial transient processes. However, as the quasi-steady state is reached later on, the mode P becomes the dominant one.

References

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