In[603]:=
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ClearAll["Global`*"]
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In[604]:=
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(*Code for evaluation of volume integral which gives to electro-viscoelastic lift
  Written by Akash Choudhary, 17/4/2020*)

(*shear and curvature in the flow eq.3.15*)
β = 4 (1 - 2 s); γ = -4 k;

(*Coefficients of the first reflection eq.3.36*)
Us = α + γ/3 + Ha (Zp - Zw) ; Ωs = β/2;
A1 = 3/4 (Us - α - 1/3 γ - Ha (Zp - Zw)) ;
B1 = -1/4 (Us - α - 3/5 γ - Ha (Zp - Zw)) + 1/2 Ha Zp ;
C1 = Ωs - β/2;
D1 = -5/2 β ; EE1 = -1/2 β; F1 = γ/3; G1 = -7/120 γ;
H1 = 1/8 γ;

(*First reflection of disturbance field eq.3.34*)
Rvec = {X, Y, Z};
eX = {1, 0, 0};
eZ = {0, 0, 1};
v01 = ( (*A1 not accounted because stokeslet=0*)
   + (B1) (- eX/R^3 + 3 X Rvec/R^5) + (*C1 not accounted because rotlet=0*) + D1 (Z X Rvec/R^5) +
    EE1 (Z eX + X eZ - 5 X Z Rvec/R^2) 1/R^5 + F1 (eX - (2 Z^2 eX + X Rvec)/R^2 + 2 X Z eZ/R^2) 1/R^3 +
    G1 (eX - 1/R^2 (5 Z^2 eX + 10 X Z eZ + 13 X Rvec) + 75 X Z^2 Rvec/R^4) 1/R^3 +
    H1 (eX - 1/R^2 (5 Z^2 eX + 10 X Z eZ + 5 X Rvec) + 35 Z^2 X Rvec/R^4) 1/R^5) /. {R → Sqrt[X^2 + Y^2 + Z^2]};


(*Test field Eq.B2*)
uT1 = (3/4 Z X/R^3 - 3/4 Z X/R^5) /. {R → Sqrt[X^2 + Y^2 + Z^2]};
vT1 = (3/4 Z Y/R^3 - 3/4 Z Y/R^5) /. {R → Sqrt[X^2 + Y^2 + Z^2]};
wT1 = (3/4 (1 + Z^2/R^2) 1/R + 1/4 (1 - 3 Z^2/R^2) 1/R^3) /. {R → Sqrt[X^2 + Y^2 + Z^2]};
u1 = {uT1, vT1, wT1};


(*Background flow field eq.3.39*)
V = {α + β Z + γ Z^2 - Ha Zw - (Us), 0, 0} // Simplify;
```

```
(*-----------------------------------------------*)
(*Stress Tensors (Eq.3.19-3.20)*)
E1 = Grad[V, {X, Y, Z}] + Transpose[Grad[V, {X, Y, Z}]];
e1 = Grad[v01, {X, Y, Z}] + Transpose[Grad[v01, {X, Y, Z}]];

e2 = Table[v01[[1]] ∂X (e1[[i, j]]) + v01[[2]] ∂Y (e1[[i, j]]) + v01[[3]] ∂Z (e1[[i, j]]),
   {i, 1, 3}, {j, 1, 3}] + e1.Grad[v01, {X, Y, Z}] + Transpose[Grad[v01, {X, Y, Z}]].e1;
(*The matrix multiplication in mathematica is different from dot
  product using Einstein notation. In manuscript: e1.(∇v)ᵀ= e1ikvk,j;
In mathematica: e1.(∇v). This maintains the consistency with Einstein's notation.*)
W1 = E1.e1 + e1.E1;
W2 = Table[V[[1]] ∂X (e1[[i, j]]) + V[[2]] ∂Y (e1[[i, j]]) + V[[3]] ∂Z (e1[[i, j]]),
    {i, 1, 3}, {j, 1, 3}] (*V.Transpose[Grad[e1,{X,Y,Z}]]*) +
   e1.Grad[V, {X, Y, Z}] + Transpose[Grad[V, {X, Y, Z}]].e1 +
   Table[v01[[1]] ∂X (E1[[i, j]]) + v01[[2]] ∂Y (E1[[i, j]]) + v01[[3]] ∂Z (E1[[i, j]]),
    {i, 1, 3}, {j, 1, 3}] (*v01.Transpose[Grad[E1,{X,Y,Z}]]*) +
   E1.Grad[v01, {X, Y, Z}] + Transpose[Grad[v01, {X, Y, Z}]].E1;
testGrad = (Grad[u1, {X, Y, Z}]) // Transpose;
(*Gradient of test field*)
(*-----------------------------------------------*)
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In[623]:=

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(*We now evaluate eq.3.41. Since the expression are quite lengthy,
we divide the integral in several parts.*)


(*-------------------------FIRST PART----------------------------*)
(*Contribution due to W1*)
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In[625]:=
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one1 = Total[Total[Table[ W1[[i]] testGrad[[i]] , {i, 1, 3}]]];
one1psiINT =
  ∫_0^(2 π) ( one1 r^2 Sin[θ] /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]}) ⅆϕ;
one1thetaINT = ∫_0^π (one1psiINT) ⅆθ;
one1radINT = ∫_1^∞ Simplify[one1thetaINT, r > 0] ⅆr;

(*Contribution due to e1.e1*)
e1e1 = e1.e1 // Simplify;
one2 = Total[Total[Table[ e1e1[[i]] testGrad[[i]] , {i, 1, 3}]]];
one2sph =
  Simplify[one2 /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]}, r > 0];
one2psiINT = ∫_0^(2 π) ( one2sph r^2 Sin[θ]) ⅆϕ;
one2thetaINT = ∫_0^π (one2psiINT) ⅆθ;
one2radINT = ∫_1^∞ Simplify[one2thetaINT, r > 0] ⅆr;
(*-----------------------------------------------------------------*)
```

In[635]:=

In[636]:=
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(*-------------------------SECOND PART----------------------------*)
(*Contribution due to W2*)
two1 = Total[Total[Table[ W2[[i]] testGrad[[i]],  {i, 1, 3}]]];
two1angINT =
  ∫_0^π (∫_0^(2 π) ( two1 r^2 Sin[θ] /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]})
        ⅆϕ) ⅆθ;
two1radINT = ∫_1^∞ Simplify[two1angINT, r > 0] ⅆr
```

Out[638]= $-160\, k\, \pi\, (-1 + 2\, s)$

In[639]:=
```
(*Contribution due to e2-part1*)
e21 = Table[v01[[1]] ∂_X (e1[[i, j]]) +
    v01[[2]] ∂_Y (e1[[i, j]]) + v01[[3]] ∂_Z (e1[[i, j]]), {i, 1, 3}, {j, 1, 3}];
two21 = Total[Total[Table[ e21[[i]] testGrad[[i]] , {i, 1, 3}]]];
two21sph =
  Simplify[two21 /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]}, r > 0];
two21angINT = ∫_0^π (∫_0^(2 π) ( two21sph r^2 Sin[θ]) ⅆϕ) ⅆθ;
two21radINT = ∫_1^∞ Simplify[two21angINT, r > 0] ⅆr
```

Out[643]= $\frac{1}{7}\, \pi\, (-1 + 2\, s)\, (32\, k - 63\, \mathrm{Ha}\, \mathrm{Zp})$

In[644]:=
```
(*Contribution due to e2-part2*)
e22 = e1.Grad[v01, {X, Y, Z}];
two22 = Total[Total[Table[ e22[[i]] testGrad[[i]] , {i, 1, 3}]]];
two22sph =
  Simplify[two22 /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]}, r > 0];
two22psiINT = ∫_0^(2 π) ( two22sph r^2 Sin[θ]) dϕ;
two22thetaINT = ∫_0^π ( two22psiINT ) dθ;
two22radINT = ∫_1^∞ Simplify[two22thetaINT, r > 0] dr
```

Out[649]= $-\frac{1}{7}\pi\left(-1+2\,s\right)\left(16\,k-21\,Ha\,Zp\right)$

In[650]:=
```
(*Contribution due to e2-part3(i)*)
e23i = Transpose[Grad[v01, {X, Y, Z}]].(Grad[v01, {X, Y, Z}]);
two23i = Total[Total[Table[ e23i[[i]] testGrad[[i]] , {i, 1, 3}]]];
two23sph =
  Simplify[two23i /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]}, r > 0];
two23psiINTi = ∫_0^(2 π) (two23sph r^2 Sin[θ]) dϕ;
two23thetaINTi = ∫_0^π ( two23psiINTi ) dθ;
two23radINTi = ∫_1^∞ Simplify[two23thetaINTi, r > 0] dr
```

Out[655]= $-\frac{1}{7}\pi\left(-1+2\,s\right)\left(16\,k-21\,Ha\,Zp\right)$

In[656]:=
```
(*Contribution due to e2-part3(ii)*)
e23ii = Transpose[Grad[v01, {X, Y, Z}]].Transpose[(Grad[v01, {X, Y, Z}])];
two23ii = Total[Total[Table[ e23ii[[i]] testGrad[[i]] , {i, 1, 3}]]];
two23iisph =
  Simplify[two23ii /. {X → r Sin[θ] Cos[ϕ], Y → r Sin[θ] Sin[ϕ], Z → r Cos[θ]}, r > 0];
two23psiINTii = ∫_0^(2 π) ( two23iisph r^2 Sin[θ]) dϕ;
two23thetaINTii = ∫_0^π ( two23psiINTii) dθ;
two23radINTii = ∫_1^∞ Simplify[two23thetaINTii, r > 0] dr
```

Out[661]= $-3\,Ha\,\pi\left(-1+2\,s\right)Zp$

```
(*All terms multiplied by -1 because the integral(eq.3.41) has -
 1 coefficient in front of it. Here the second part is multiplied with  δ,
which is consistent with eq. 3.19*)
TOTAL = -( one2radINT   +   one1radINT) -
   δ (two1radINT + two21radINT + two22radINT + two23radINTi + two23radINTii );

(*Components from electrophoresis: Electro-Viscoelastic migration*)
EVM = beta * HaZp * Coefficient[TOTAL/β, Ha Zp] // Simplify
(*Remaining terms: Ho and Leal (1974), Viscoelastic migration*)
VM = beta * gamma * (TOTAL - (Ha Zp) Coefficient[TOTAL, Ha Zp])/(β γ) // Simplify // Factor
```

Out[663]= $-\frac{3}{2}\,\text{beta}\,\text{HaZp}\,\pi\,(1+\delta)$

Out[664]= $\frac{10}{3}\,\text{beta}\,\text{gamma}\,\pi\,(1+3\,\delta)$