Supplementary Material B. Pycnocline thickness variation for constant energy injected

We present here analogous results to the ones obtained in §4.1, when instead of verifying that the waves arriving at x = L have same energy $E_{k,0}$, we inject waves with the same energy, matching the case $\delta = 0.2$ m presented in §3, without considering the evolution of the wave until it reaches the slope. This approach may be more relevant for experimental studies as it may not be possible to measure directly how much energy is present at the breaking location. This strategy results in comparatively less energy being injected at the broader pycnocline thicknesses and more energy injected for the narrow pycnoclines. The resulting (non-monotonic) S_b and D_b trends with δ are presented in figure S1.

As in the case where energy is constant at the breaking location, the size of the bolus has a maximum value at an intermediate pycnocline thickness. This peak occurs for a narrower thickness between $\delta = 0.1 \,\mathrm{m}$ and $0.15 \,\mathrm{m}$, while the peak for the constant energy at the breaking location study is at $\delta = 0.15 \,\mathrm{m}$. Due to the relatively higher energies for the narrower pycnoclines, the maximum bolus size is also larger than those observed in §4.1. The distance traveled upslope also has a similar overall trend in that the displacement upslope increases as δ increases. However, unlike the constant energy at the breaking point example, there is ultimately a maximum displacement upslope reached at $\delta = 0.3 \,\mathrm{m}$. The shorter travel upslope for broader pycnoclines is due to the relatively lower energy in the generated bolus.



Figure S1: (a) Bolus size S_b and (b) maximum displacement upslope D_b as a function of the pycnocline thickness, δ , for mode-1 waves breaking with constant energy. Different markers are used to represent bolus shape categories: ball (circle), hook (triangle) and sliver (square).