Appendix. Theory of non-equilibrium and homogeneous condensation

Wegener & Mack (1958) and Hill (1966) through theoretical and experimental studies of moist air and steam flows through nozzles, developed a basic theory to describe a non-equilibrium and homogeneous condensation process in steam or water vapour-carrier gas mixture flows. A few important assumptions that the theory makes about the condensation process are: (a) the amount of liquid condensate formed is very small in comparison to the mass of a characteristic fluid element and therefore, the velocity of the liquid droplet is the same as gaseous phase and (b) the primary effect of condensation of water vapour on flow is heat release to the surrounding gaseous phase. Hill (1966) also suggested that the dependence of surface tension of the liquid droplets on curvature as well as the interaction of droplets with each other could be ignored. Wegener & Mack (1958) observed that condensation occurred at supersonic speeds, where the flow is in a highly supersaturated condition.

The nucleation rate, J, can be expressed as,

$$J = K_{nf}\ \exp\left(- \frac{W^*}{kT} \right). \tag{1}$$

In (1), K_{nf} is a nucleation factor, k is the boltzmann constant, T is flow temperature, and W^* is the energy expended to create a critical spherical liquid droplet reversibly and isothermally. The nucleation factor, K_{nf}, can be expressed as,

$$K_{nf} = \frac{\rho_v^2}{\rho_f} m^{-3/2} \sqrt{\frac{2\sigma(T)}{\pi}}. \tag{2}$$

In (2), $\sigma(T)$ is surface tension of water, $\rho_v(p, T)$ and $\rho_f(T)$ are water vapour density and liquid water density, respectively and m is the mass of one water molecule. W^* is the difference between the surface energy of a critical-sized droplet, $W_1 = 4\pi r^{*2}\sigma(T)$ and the energy required to expand the liquid droplet against pressure forces to a given volume, $W_2 = (4\pi/3)(p_l - p_g)r^{*3}$, i.e. $W^* = W_1 - W_2$. Here, p_l is the pressure of liquid inside droplet and $p_g(T)$ is the saturation pressure of vapour surrounding the droplet, they are related as $p_l - p_g = 2\sigma(T)/r^*$. Therefore, $W^* = (4\pi/3)r^{*2}\sigma(T)$. Critical radius r^* is given by the Thomson-Gibbs relation,

$$r^* = \frac{2\sigma(T)}{\rho_f R_v T\ \ln(S)}. \tag{3}$$

The supersaturation ratio (S) in (3) is:

$$S = \frac{p}{p_g(T)} = S_\infty \frac{\bar{p}}{\bar{p}_g(\bar{T})}. \tag{4}$$

Here, $\bar{p}_g(\bar{T}) = p_g(T)/p_g(T_\infty)$. Using (3) and noting that $R_v = k/m$, W^* can also be written as,

$$W^* = \frac{16\pi}{3} \left(\frac{m}{kT\rho_f\ \ln(S)} \right)^2 \sigma^3(T) \tag{5}$$

The non-dimensional nucleation rate ($\bar{J}_1$) may then be obtained as,

$$\bar{J}_1 = \frac{1}{\epsilon^{4/3}} \sqrt{\frac{27}{32\pi^3}} n_c^{3/2} \frac{\bar{\rho}^2}{\bar{\rho}_f} \sqrt{\bar{\sigma}(\bar{T})} (1-g)^2 \exp\left(- \frac{n_c \bar{\sigma}^3(\bar{T})}{2\bar{T}^3 [\ln(S)]^2} \right). \tag{6}$$

Here, $n_c = (4\pi/3)\rho_{f\infty} l_c^3/m$, $\bar{\rho}_f = \rho_f(T)/\rho_f(T_\infty)$ and $\bar{\sigma}(\bar{T}) = \sigma(T)/\sigma(T_\infty)$. If $S<1$, $\bar{J}_1$ is set to 0. The droplet growth rate is determined by the difference between condensation and evaporation rates (Hill 1966) and may be given by the Hertz-Knudsen model. Assuming that the liquid and water vapour in a fluid element have the same temperature,

$$\frac{dr}{dt} = \frac{p - p_g(T)}{\sqrt{2\pi R_v T}} \frac{\alpha_c(T)}{\rho_f}. \tag{7}$$

A non-dimensional form of droplet growth rate may be written as,

$$\frac{d\bar{r}}{d\bar{t}} = \frac{1}{u_c}\frac{dr}{dt} = \left(\bar{p} - \frac{\bar{p}_g(\bar{T})}{S_\infty}\right)\frac{\alpha_c(T)}{\sqrt{\bar{T}}}. \tag{8}$$

Here, $\alpha_c(T)$ is the condensation coefficient of water vapour defined as the fraction of water molecules that adhere to the liquid droplet out of those impinging on it. For calculations of J and dr/dt, empirical correlations provided by Schnerr & Dohrmann (1990) were used to obtain values of condensation coefficient ($\alpha_c(T)$) of water vapour and surface tension ($\sigma(T)$) of liquid water. Values of liquid water density ($\rho_f(T)$), specific latent heat of condensation of water ($h_{fg}(T)$) and saturation pressure ($p_g(T)$) were obtained from steam tables given by Moran *et al.* (2014). It is understood that uncertainties in the empirical correlations affect the numerical results of TSD theory. However, the current study is focused on the development of a transonic small-disturbance model of pure steam flow around a thin airfoil with real gas effects and non-equilibrium and homogeneous condensation. An uncertainty analysis of computed results is definitely required for relating results to practical applications and will be addressed in a future study.

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