

Galerkin model (GK)_1D

In[739]:=

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Remove["Global`*"]

(* Derivatives of the coordinate transformation *)
sigmat = 1/H[x, t] * (D[2 * h[x, t], t] - Ht -  $\sigma$  * Ht);
sigmax = 1/H[x, t] * (D[2 h[x, t] - H[x, t], x] -  $\sigma$  * D[H[x, t], x]);
sigmaz = 2/H[x, t];
(*****)

(* Horizontal velocity assumption
   (Velocity unknowns are denoted as  $U_i$ ) *)
K = 2; (* K needs to be specified *)
Do[Ni = LegendreP[i,  $\sigma$ ], {i, 0, 5}];
Do[UCi = Ui[x, t] * Ni, {i, 0, K - 1, 1}];
u = Sum[UCi, {i, 0, K - 1}];
(*****)

(* Vertical velocity expression*)
ub = u /.  $\sigma \rightarrow -1$ ;
wb = -D[h[x, t], t] - ub * D[h[x, t], x];
w = wb - 1/sigmaz * Integrate[D[u, x] + D[u,  $\sigma$ ] * sigmax, { $\sigma$ , -1,  $\sigma$ ]];
(*****)

(* Depth-integaretd continuity equation *)
fluxx = Integrate[u, { $\sigma$ , -1, 1}]/sigmaz;
Ht = -D[fluxx, x];
(*****)
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In[752]:=

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Derivative[0, 1][H][x, t] = -D[fluxx, x];
Derivative[1, 1][H][x, t] = D[-D[fluxx, x], x];
Derivative[2, 1][H][x, t] = D[D[-D[fluxx, x], x], x];

(* Vertical momentum equation to get pressure field*)
wm = D[w, t] + D[w, σ] * sigmat + u * (D[w, x] + D[w, σ] * sigmax) + w * D[w, σ] * sigmaz;
ps = (-ρ * g - ρ * wm) / sigmaz;
cf = CoefficientList[wm, σ];
order = Length[cf];
pnh = Sum[Part[cf, i] * ((1 - σi) / i), {i, 1, order}];
p = (ρ * g * (1 - σ) + ρ * pnh) / sigmaz;    (* PRESSURE FIELD*)
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In[761]:= (* LHS of the horizontal momentum equation to form weighted residuals *)
LHS = D[u, t] + D[u, σ] * sigmat + u * (D[u, x] + D[u, σ] * sigmax) + w * D[u, σ] * sigmaz;
Do[lfi = LHS * Ni, {i, 0, K - 1}];
Do[lcfi = CoefficientList[lfi, σ], {i, 0, K - 1}];
Do[lorderi = Length[lcfi], {i, 0, K - 1}];
Do[MomLhsi = Sum[Part[lcfi, j] * ((1j - (-1)j) / j), {j, 1, lorderi}], {i, 0, K - 1}];
(*****)
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(* RHS of the horizontal momentum equation to form weighted residuals *)
RHS = -1 / ρ * (D[p, x] + ps * sigmax);
Do[fi = RHS * Ni, {i, 0, K - 1}];
Do[cfi = CoefficientList[fi, σ], {i, 0, K - 1}];
Do[orderi = Length[cfi], {i, 0, K - 1}];
Do[MomRhsi = Sum[Part[cfi, j] * ((1j - (-1)j) / j), {j, 1, orderi}], {i, 0, K - 1}];
(*****)
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(* Full equation *)
Do[XEQi+1 = MomLhsi - MomRhsi, {i, 0, K - 1}];
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(* Print out the full equation*)
(* 1. Continuity equation with ContinuityLHS=H_t *)
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In[772]:= Print[Ht]

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-U0[x, t] H(1,0)[x, t] - H[x, t] U0(1,0)[x, t]
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In[773]:=

(* 2. K number of weighted horizontal momentum equations *)

Do[Print[Collect[-XEQi, _[x, t], Simplify]], {i, 1, K, 1}]

$$\begin{aligned}
& 2 h^{(0,2)}[x, t] H^{(1,0)}[x, t] - \frac{U_1[x, t]^2 H^{(1,0)}[x, t]^3}{5 H[x, t]} - \\
& 2 U_0[x, t] U_0^{(1,0)}[x, t] - \frac{4}{3} U_1[x, t] U_1^{(1,0)}[x, t] + H^{(1,0)}[x, t]^2 \\
& \left(-U_1^{(0,1)}[x, t] + U_1[x, t] \left(3 U_0^{(1,0)}[x, t] - \frac{7}{5} U_1^{(1,0)}[x, t] \right) - U_0[x, t] U_1^{(1,0)}[x, t] \right) + \\
& h^{(1,0)}[x, t]^2 \left(-\frac{2 U_1[x, t]^2 H^{(1,0)}[x, t]}{3 H[x, t]} - 2 U_0[x, t] U_0^{(1,0)}[x, t] - \frac{4}{3} U_1[x, t] U_1^{(1,0)}[x, t] \right) + \\
& U_0^{(0,1)}[x, t] \left(-2 - 2 h^{(1,0)}[x, t]^2 + 2 h^{(1,0)}[x, t] H^{(1,0)}[x, t] + H[x, t] h^{(2,0)}[x, t] \right) + \\
& H^{(1,0)}[x, t] \left(-2 g - \frac{2 U_1[x, t]^2}{3 H[x, t]} + 4 U_0[x, t] h^{(1,1)}[x, t] + \right. \\
& 2 U_0[x, t]^2 h^{(2,0)}[x, t] + U_1[x, t] \left(\frac{4}{3} h^{(1,1)}[x, t] + \frac{4}{3} U_0[x, t] h^{(2,0)}[x, t] \right) + \\
& U_1[x, t]^2 \left(h^{(2,0)}[x, t] - H^{(2,0)}[x, t] \right) + H[x, t] \left(-2 U_0^{(1,0)}[x, t]^2 + \right. \\
& 2 U_0^{(1,0)}[x, t] U_1^{(1,0)}[x, t] - \frac{2}{5} U_1^{(1,0)}[x, t]^2 + 2 U_0^{(1,1)}[x, t] - U_1^{(1,1)}[x, t] + \\
& U_0[x, t] \left(2 U_0^{(2,0)}[x, t] - U_1^{(2,0)}[x, t] \right) + U_1[x, t] \left(4 U_0^{(2,0)}[x, t] - \frac{3}{5} U_1^{(2,0)}[x, t] \right) \left. \right) + \\
& h^{(1,0)}[x, t] \left(2 g - 2 h^{(0,2)}[x, t] + \frac{2 U_1[x, t]^2 H^{(1,0)}[x, t]^2}{3 H[x, t]} + \right. \\
& H^{(1,0)}[x, t] \left(\frac{4}{3} U_1^{(0,1)}[x, t] + U_0[x, t] \left(2 U_0^{(1,0)}[x, t] + \frac{4}{3} U_1^{(1,0)}[x, t] \right) + \right. \\
& U_1[x, t] \left(-\frac{4}{3} U_0^{(1,0)}[x, t] + \frac{8}{3} U_1^{(1,0)}[x, t] \right) \left. \right) - 4 U_0[x, t] h^{(1,1)}[x, t] - \\
& 2 U_0[x, t]^2 h^{(2,0)}[x, t] + U_1[x, t]^2 \left(-\frac{2}{3} h^{(2,0)}[x, t] + \frac{2}{3} H^{(2,0)}[x, t] \right) + \\
& H[x, t] \left(2 U_0^{(1,0)}[x, t]^2 + \frac{2}{3} U_1^{(1,0)}[x, t]^2 + \frac{2}{3} U_1^{(1,1)}[x, t] + \right. \\
& U_1[x, t] \left(-\frac{2}{3} U_0^{(2,0)}[x, t] + \frac{2}{3} U_1^{(2,0)}[x, t] \right) + \frac{2}{3} U_0[x, t] U_1^{(2,0)}[x, t] \left. \right) + \\
& H[x, t] \left(2 U_0^{(1,0)}[x, t] h^{(1,1)}[x, t] + \frac{2}{3} U_1^{(1,0)}[x, t] h^{(1,1)}[x, t] + h^{(1,2)}[x, t] + \right. \\
& U_1^{(0,1)}[x, t] \left(\frac{1}{3} h^{(2,0)}[x, t] - \frac{1}{2} H^{(2,0)}[x, t] \right) + U_0[x, t] \\
& \left(3 U_0^{(1,0)}[x, t] h^{(2,0)}[x, t] + U_1^{(1,0)}[x, t] \left(h^{(2,0)}[x, t] - \frac{1}{2} H^{(2,0)}[x, t] \right) + 2 h^{(2,1)}[x, t] \right) + \\
& U_0[x, t]^2 h^{(3,0)}[x, t] + U_1[x, t] \left(U_1^{(1,0)}[x, t] \left(\frac{4}{3} h^{(2,0)}[x, t] - \frac{11}{10} H^{(2,0)}[x, t] \right) + \right. \\
& U_0^{(1,0)}[x, t] \left(h^{(2,0)}[x, t] + \frac{3}{2} H^{(2,0)}[x, t] \right) + \frac{2}{3} h^{(2,1)}[x, t] + \frac{2}{3} U_0[x, t] h^{(3,0)}[x, t] \left. \right) + \\
& U_1[x, t]^2 \left(\frac{1}{3} h^{(3,0)}[x, t] - \frac{3}{10} H^{(3,0)}[x, t] \right) + \\
& H[x, t]^2 \left(U_1^{(1,0)}[x, t] \left(\frac{7}{6} U_0^{(2,0)}[x, t] + \frac{1}{30} U_1^{(2,0)}[x, t] \right) + \right. \\
& U_0^{(1,0)}[x, t] \left(-\frac{2}{3} U_0^{(2,0)}[x, t] + \frac{1}{6} U_1^{(2,0)}[x, t] \right) + \frac{2}{3} U_0^{(2,1)}[x, t] - \frac{1}{6} U_1^{(2,1)}[x, t] + \\
& U_0[x, t] \left(\frac{2}{3} U_0^{(3,0)}[x, t] - \frac{1}{6} U_1^{(3,0)}[x, t] \right) + U_1[x, t] \left(\frac{5}{6} U_0^{(3,0)}[x, t] - \frac{1}{30} U_1^{(3,0)}[x, t] \right) \left. \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{U_1[x, t]^2 H^{(1,0)}[x, t]^3}{15 H[x, t]} - \frac{2}{3} U_1[x, t] U_0^{(1,0)}[x, t] - \frac{2}{3} U_0[x, t] U_1^{(1,0)}[x, t] + \\
& h^{(1,0)}[x, t]^2 \left(-\frac{2}{3} U_1[x, t] U_0^{(1,0)}[x, t] - \frac{2}{3} U_0[x, t] U_1^{(1,0)}[x, t] \right) + H^{(1,0)}[x, t]^2 \\
& \left(U_1[x, t] \left(\frac{3}{5} U_0^{(1,0)}[x, t] - \frac{1}{15} U_1^{(1,0)}[x, t] \right) - \frac{1}{5} U_0[x, t] U_1^{(1,0)}[x, t] \right) + U_1^{(0,1)}[x, t] \\
& \left(-\frac{2}{3} - \frac{2}{3} h^{(1,0)}[x, t]^2 + \frac{2}{3} h^{(1,0)}[x, t] H^{(1,0)}[x, t] - \frac{1}{5} H^{(1,0)}[x, t]^2 + \frac{1}{10} H[x, t] H^{(2,0)}[x, t] \right) + \\
& h^{(1,0)}[x, t] \left(\frac{2 U_1[x, t]^2 H^{(1,0)}[x, t]^2}{15 H[x, t]} + \right. \\
& H^{(1,0)}[x, t] \left(U_1[x, t] \left(-\frac{2}{3} U_0^{(1,0)}[x, t] + \frac{2}{15} U_1^{(1,0)}[x, t] \right) + \frac{2}{3} U_0[x, t] U_1^{(1,0)}[x, t] \right) + \\
& U_1[x, t] \left(-\frac{4}{3} h^{(1,1)}[x, t] - \frac{4}{3} U_0[x, t] h^{(2,0)}[x, t] \right) + \\
& \frac{2}{15} U_1[x, t]^2 H^{(2,0)}[x, t] + H[x, t] \left(-\frac{4}{15} U_1^{(1,0)}[x, t]^2 - \frac{2}{3} U_0^{(1,1)}[x, t] - \right. \\
& \left. \frac{2}{3} U_0[x, t] U_0^{(2,0)}[x, t] + U_1[x, t] \left(-\frac{2}{3} U_0^{(2,0)}[x, t] - \frac{2}{15} U_1^{(2,0)}[x, t] \right) \right) \Bigg) + \\
& H^{(1,0)}[x, t] \left(U_1[x, t] \left(\frac{2}{3} h^{(1,1)}[x, t] + \frac{2}{3} U_0[x, t] h^{(2,0)}[x, t] \right) - \frac{1}{15} U_1[x, t]^2 H^{(2,0)}[x, t] + \right. \\
& H[x, t] \left(-\frac{2}{5} U_0^{(1,0)}[x, t] U_1^{(1,0)}[x, t] + \frac{2}{15} U_1^{(1,0)}[x, t]^2 + \right. \\
& \left. \frac{1}{5} U_1^{(1,1)}[x, t] + \frac{1}{5} U_0[x, t] U_1^{(2,0)}[x, t] + \frac{1}{15} U_1[x, t] U_1^{(2,0)}[x, t] \right) \Bigg) + \\
& H[x, t] \left(-\frac{1}{3} h^{(1,2)}[x, t] - \frac{1}{3} U_0^{(0,1)}[x, t] h^{(2,0)}[x, t] + \right. \\
& U_0^{(1,0)}[x, t] \left(-\frac{2}{3} h^{(1,1)}[x, t] - U_0[x, t] h^{(2,0)}[x, t] \right) + \\
& U_1[x, t] \left(U_1^{(1,0)}[x, t] \left(-\frac{1}{3} h^{(2,0)}[x, t] + \frac{1}{6} H^{(2,0)}[x, t] \right) - \frac{3}{10} U_0^{(1,0)}[x, t] H^{(2,0)}[x, t] \right) + \\
& U_0[x, t] \left(\frac{1}{10} U_1^{(1,0)}[x, t] H^{(2,0)}[x, t] - \frac{2}{3} h^{(2,1)}[x, t] \right) - \\
& \frac{1}{3} U_0[x, t]^2 h^{(3,0)}[x, t] + U_1[x, t]^2 \left(-\frac{1}{15} h^{(3,0)}[x, t] + \frac{1}{30} H^{(3,0)}[x, t] \right) \Bigg) + \\
& H[x, t]^2 \left(-\frac{4}{15} U_1^{(1,0)}[x, t] U_0^{(2,0)}[x, t] + U_0^{(1,0)}[x, t] \left(\frac{1}{6} U_0^{(2,0)}[x, t] - \frac{1}{15} U_1^{(2,0)}[x, t] \right) - \right. \\
& \frac{1}{6} U_0^{(2,1)}[x, t] + \frac{1}{15} U_1^{(2,1)}[x, t] - \frac{2}{15} U_1[x, t] U_0^{(3,0)}[x, t] + \\
& U_0[x, t] \left(-\frac{1}{6} U_0^{(3,0)}[x, t] + \frac{1}{15} U_1^{(3,0)}[x, t] \right) \Bigg)
\end{aligned}$$

(* ***** THE END *****)