

Parameters of the POD-based reduced order model

This document provides the values of the parameters of the reduced order model derived in section 4 of the paper, so that the results shown can be reproduced.

We recall the general form of the model

$$\frac{da_n(t)}{dt} = (L_{nm}^T - \alpha_{nm} r^2) a_m(t) + Q_{nmp} a_m(t) a_p(t)$$

where $r^2 = \sum_{n=2}^7 |a_n(t)|^2$ and L_{nm}^T , α_{nm} and Q_{nmp} are coefficients to be determined, associated with linear, cubic and quadratic contributions respectively. The linear coefficients are themselves split in buoyancy, molecular diffusion and closure contributions according to $L_{nm}^T = L_{nm}^B + L_{nm}^D + L_{nm}^A$.

The coefficients L_{nm}^B , L_{nm}^D and Q_{nmp} are directly computed from the POD-modes (equations (4.2)-(4.4) of the paper) and the values obtained are reported in tables 1-2 for the first seven modes. Values associated with the first mode are given although this mode is taken as a constant in the model.

The coefficients L_{nm}^T and α_{nm} are then adjusted according to the procedure detailed in section 4.1 and appendix A of the paper. The values used for model integration are reported in table 3.

L_{nm}^D	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	L_{nm}^B	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$
$n=1$	0.15							$n=1$	0.05						
$n=2$		-0.09			0.03			$n=2$		0.12			-0.08		
$n=3$			-0.09			0.02		$n=3$			0.12			-0.08	
$n=4$				-0.14				$n=4$				0.06			
$n=5$		0.03			-0.09			$n=5$		0.02			0.08		
$n=6$			0.03			-0.09		$n=6$			0.02			0.08	
$n=7$							-0.17	$n=7$							0.12

TABLE 1. Linear diffusion matrix (left) and linear buoyancy matrix (right). Entries below 10^{-5} are left empty.

Q_{1mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	Q_{2mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
$m=1$								$m=1$		0.29			0.01		
$m=2$		-0.58			-0.12			$m=2$	0.29						0.33
$m=3$			-0.58			-0.12		$m=3$				-0.53			
$m=4$				-0.95				$m=4$			-0.53			0.005	
$m=5$		-0.12			-0.79			$m=5$	0.01						0.18
$m=6$			-0.12			-0.79		$m=6$				0.005			
$m=7$							1.17	$m=7$		0.33			0.18		

Q_{3mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	Q_{4mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
$m=1$			0.29			0.01		$m=1$				0.48			
$m=2$				-0.53				$m=2$			1.06			1.98	
$m=3$	0.29						-0.33	$m=3$		1.06			1.98		
$m=4$		-0.53			0.005			$m=4$	0.48					1.10	
$m=5$				0.005				$m=5$			1.98				
$m=6$	0.01					-0.18	-0.18	$m=6$		1.98			1.10		
$m=7$			-0.33				-0.18	$m=7$							

Q_{5mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	Q_{6mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
$m=1$		0.11			0.40			$m=1$			0.11			0.40	
$m=2$	0.11						0.09	$m=2$				-1.98			-0.09
$m=3$				-1.98				$m=3$	0.11						
$m=4$			-1.98			-0.55		$m=4$		-1.98			-0.55		
$m=5$	0.40						0.20	$m=5$				-0.55			
$m=6$				-0.55				$m=6$	0.40						-0.20
$m=7$		0.09			0.20			$m=7$			-0.09			-0.20	

Q_{7mp}	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$
$m=1$							0.58
$m=2$		-0.67			-0.28		
$m=3$			0.67			0.28	
$m=4$							
$m=5$		-0.28			-0.41		
$m=6$			0.28			0.41	
$m=7$	0.58						

TABLE 2. Quadratic interaction matrices (in symmetric form i.e. corresponding to $Q_{nmp} + Q_{npm}$ for $p \neq m$ and Q_{nmp} for $p = m$). Entries below 10^{-5} are left empty.

L_{nm}^T	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	α_{nm}	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$
$n=2$	-0.002			-0.02			$n=2$	2.28			-3.00		
$n=3$		-0.002			-0.02		$n=3$		2.28			-3.00	
$n=4$			-0.17				$n=4$			8.20			
$n=5$	0.03			-0.11			$n=5$	1.00			4.00		
$n=6$		0.03			-0.11		$n=6$		1.00			4.00	
$n=7$						-0.08	$n=7$						3.00

TABLE 3. Parameters L_{nm}^T and α_{nm} of the seven-mode model.