Supplementary information

Accuracy of explicit Euler method used to track temporal evolution of the drop

For an equation, $\dot{y} = \lambda y$, where λ is a complex parameter, the exact solution is $y(t) = y_0 e^{\lambda t}$ stable if $Re(\lambda) < 0$. Numerically, the solution is bounded and stable if $|1 + h\lambda| \leq 1$. However the numerical solution need not be stable for it to be accurate locally. In our analysis, the equation is $\frac{d\mathbf{x}}{dt} = \frac{\mathbf{u}}{\mathbf{x}}\mathbf{x}$ the coefficient "λ" $= \frac{\mathbf{u}}{\mathbf{x}}$ varies for each iteration with change in local velocity and and therefore the comparison of our numerical solution with analytical exact result has to be done locally. Sample plots (for an arbitrary time $t = 0.249$) is shown comparing analytical exact solution with numerically obtained plot at specific node points '20' and '40' (figures 1 and 2). The initial conditions are $\epsilon = 1.85$, $Ca = 0.15$. Time step size and number of node points are, $\delta t = 0.00001$ and nd=150.

The following analyses will also show that the code is accurately optimized for both the number of node points and time step size.

Code optimization

A paramenter, normal electric field strength (E_n) is chosen to both check convergence of the code as well as to optimize number of node points and time step size δt. A test case is assumed with $\epsilon = 1.85$, to simulate the shape of the bridged drops, and E_n at the pole is plotted, one iteration before simulation ends (when singularity is reached) for 50 elements for varying δt for $Ca = 0.125$. The results are plotted in figure 3. The value of E_n becomes fairly constant when δt is taken as 10^{-3}. To make sure no inadvertant error creeps in, our simulations were carried out for $\delta t = 10^{-5}$.

Similar analysis is carried out to optimize number of node points. Again, E_n at the pole is plotted for varying number of node points for $Ca = 0.153$. δt is taken as 10^{-4}. Figure 4 shows that for number of nodes = 150 and above the value of E_n at the pole remains constant. As the increase in the number of elements does not improve much in the accuracy of the results but it significantly increases the computational time, we chose to carry out the numerical analysis with 150 elements.

The change in internal volume was also computed for all our simulations and it was noted that the value always remained below 0.1%. The figure 5 shows a test case for change in volume and change in area for 150 elements and $\delta t = 10^{-5}$.

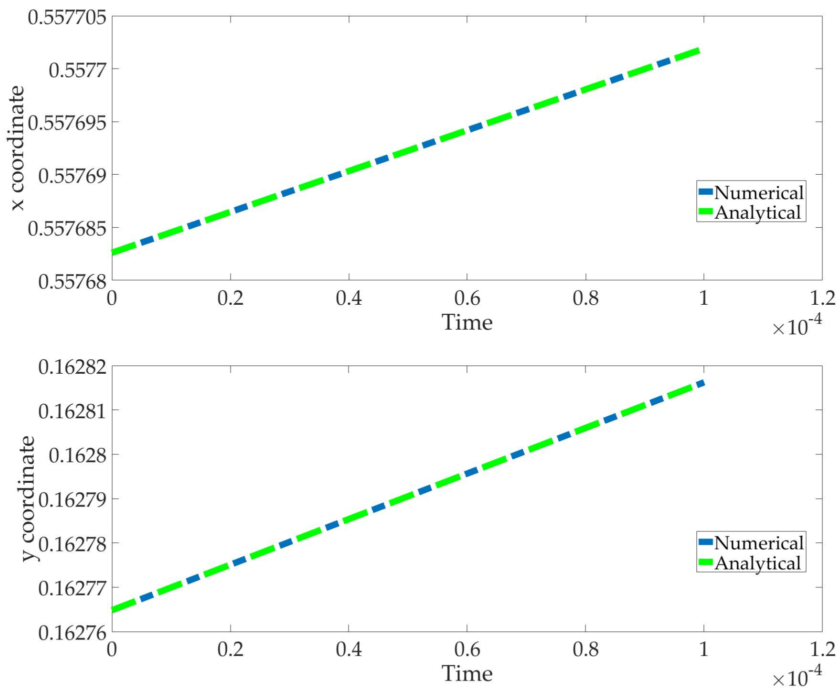

Figure 1: Comparison of analytical exact solution with numerical results of the temporal evolution of the droplet ($Ca = 0.15$) at 20^{th} node point).

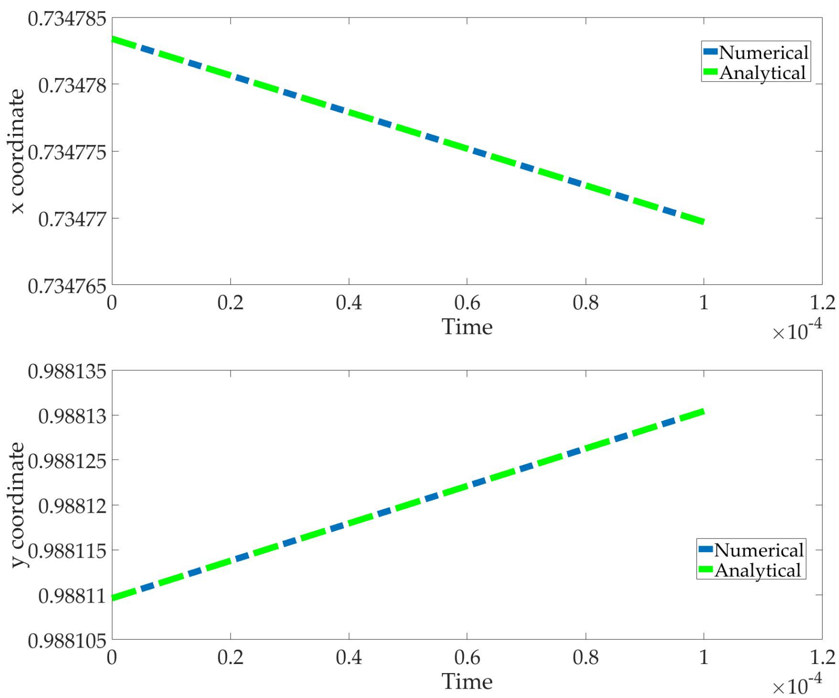

Figure 2: Comparison of analytical exact solution with numerical results of the temporal evolution of the droplet ($Ca = 0.15$) at 40^{th} node point).

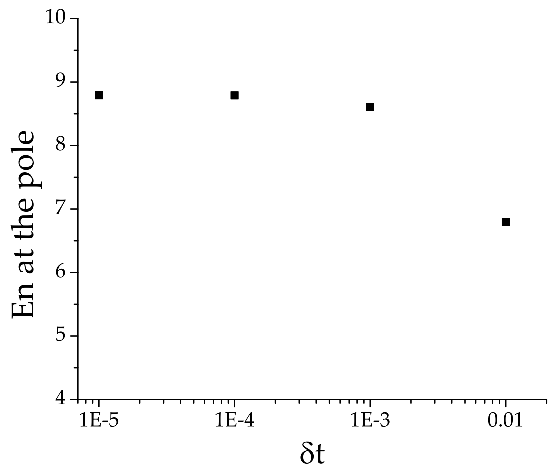

Figure 3: E_n at the pole for varying δt, with 50 elements for $Ca = 0.125$

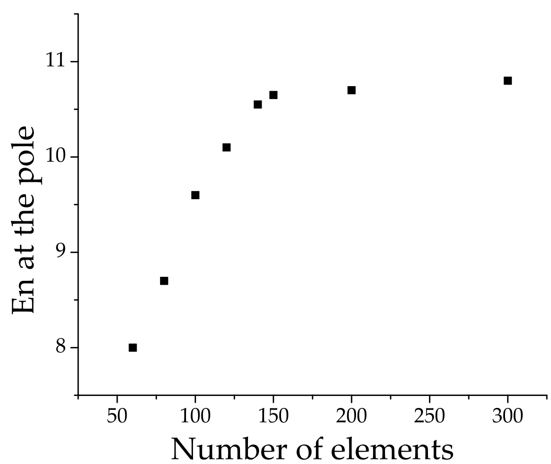

Figure 4: E_n at the pole for varying number of node points, for $Ca = 0.153$, for $\delta t = 10^{-4}$

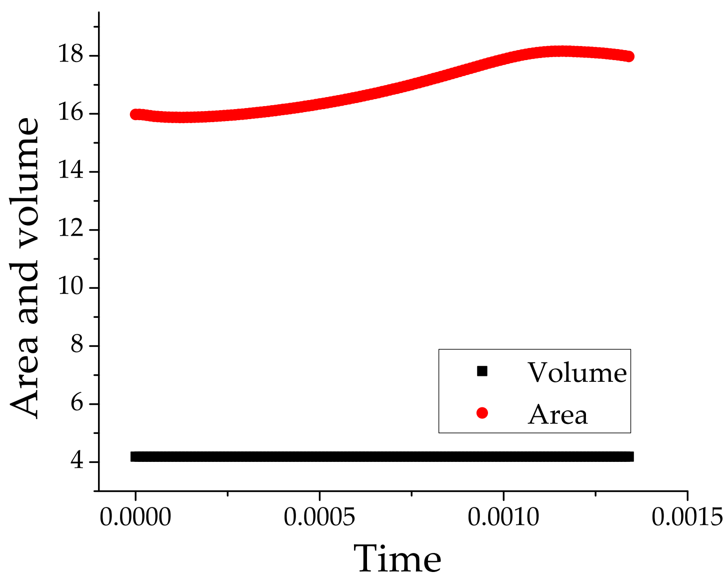

Figure 5: Volume and area with time, for $Ca = 0.153$, and $\delta t = 10^{-5}$