1. Linearised Navier-Stokes equations

The explicit form of equation 2.25 is

Continuity ($\mathcal{L}_c\mathbf{q}_{s,b} = \mathcal{F}_c$):

$$\left(-i\omega + \mathcal{A}_1 - \frac{\mathcal{A}_7}{\bar{T}}\right)\frac{p_b}{\bar{P}} + \left(i\omega - \mathcal{A}_1 - \mathcal{C}_1 + \frac{\mathcal{A}_7}{\bar{T}}\right)\frac{\theta_b}{\bar{T}} + \frac{\bar{\rho}_x}{\bar{\rho}}u_b + \frac{\bar{\rho}_y}{\bar{\rho}}v_b + i\beta w_b +$$
$$u_{b,x} + v_{b,y} + \frac{\bar{U}}{\bar{P}}p_{b,x} + \frac{\bar{V}}{\bar{P}}p_{b,y} - \frac{\bar{U}}{\bar{T}}\theta_{b,x} - \frac{\bar{V}}{\bar{T}}\theta_{b,y} = \mathcal{F}_c(\bar{\mathbf{q}}, \mathbf{q}_w, \mathbf{q}_a)\ ,$$

X-momentum ($\mathcal{L}_x\mathbf{q}_{s,b} = \mathcal{F}_x$):

$$\left[\frac{\bar{\mu}_T\mathcal{C}_2}{R} + \frac{\bar{\rho}\mathcal{A}_2}{\bar{T}} + \frac{\bar{\mu}_{TT}}{R}\left(\bar{T}_x\mathcal{A}_4 + \bar{T}_y\mathcal{A}_3\right)\right]\theta_b + \frac{\bar{\mu}}{R}\left(du_{b,xx} + sv_{b,xy} + u_{b,yy}\right) - \bar{\rho}\bar{U}_y v_b +$$
$$\left(\frac{\bar{\mu}_T\bar{T}_y}{R} - \bar{\rho}\bar{V}\right)u_{b,y} - \frac{\bar{\rho}\mathcal{A}_2}{\bar{P}}p_b + \left(\frac{d\bar{\mu}_T\bar{T}_x}{R} - \bar{\rho}\bar{U}\right)u_{b,x} - \left(\bar{\rho}\bar{U}_x - \bar{\rho}i\omega + \frac{\beta^2\bar{\mu}}{R}\right)u_b - p_{b,x} +$$
$$\frac{is\beta\bar{\mu}}{R}w_{b,x} + \frac{\bar{\mu}_T}{R}\left[i\beta m\bar{T}_x w_b + \bar{T}_y v_{b,x} + \mathcal{A}_3\theta_{b,y} + \mathcal{A}_4\theta_{b,x} + m\bar{T}_x v_{b,y}\right] = \mathcal{F}_x(\bar{\mathbf{q}}, \mathbf{q}_w, \mathbf{q}_a)\ ,$$

Y-momentum ($\mathcal{L}_y\mathbf{q}_{s,b} = \mathcal{F}_y$):

$$\left[\frac{\bar{\mu}_T\mathcal{C}_3}{R} + \frac{\bar{\rho}\mathcal{A}_5}{\bar{T}} + \frac{\bar{\mu}_{TT}}{R}\left(\bar{T}_y\mathcal{A}_6 + \bar{T}_x\mathcal{A}_3\right)\right]\theta_b + \frac{\bar{\mu}}{R}\left(dv_{b,yy} + su_{b,xy} + v_{b,xx}\right) - \bar{\rho}\bar{V}_x u_b +$$
$$\left(\frac{\bar{\mu}_T\bar{T}_x}{R} - \bar{\rho}\bar{U}\right)v_{b,x} - \frac{\bar{\rho}\mathcal{A}_5}{\bar{P}}p_b + \left(\frac{d\bar{\mu}_T\bar{T}_y}{R} - \bar{\rho}\bar{V}\right)v_{b,y} - \left(\bar{\rho}\bar{V}_y - \bar{\rho}i\omega + \frac{\beta^2\bar{\mu}}{R}\right)v_b - p_{b,y} +$$
$$\frac{is\beta\bar{\mu}}{R}w_{b,y} + \frac{\bar{\mu}_T}{R}\left[i\beta m\bar{T}_y w_b + \bar{T}_x u_{b,y} + \mathcal{A}_3\theta_{b,x} + \mathcal{A}_6\theta_{b,y} + m\bar{T}_y u_{b,x}\right] = d\mathcal{F}_y(\bar{\mathbf{q}}, \mathbf{q}_w, \mathbf{q}_a)\ ,$$

Z-momentum ($\mathcal{L}_z\mathbf{q}_{s,b} = \mathcal{F}_z$):

$$\frac{i\beta\bar{\mu}_T}{R}\left[m\mathcal{A}_1\theta_b + \bar{T}_y v_b + \bar{T}_x u_b\right] + \frac{\bar{\mu}}{R}\left(w_{b,xx} + w_{b,yy}\right) - \left(\frac{d\beta^2\bar{\mu}}{R} - i\omega\bar{\rho}\right)w_b - i\beta p_b +$$
$$\frac{is\beta\bar{\mu}}{R}\left(u_{b,x} + v_{b,y}\right) + \left(\frac{\bar{\mu}_T\bar{T}_y}{R} - \bar{\rho}\bar{V}\right)w_{b,y} + \left(\frac{\bar{\mu}_T\bar{T}_x}{R} - \bar{\rho}\bar{U}\right)w_{b,x} = \mathcal{F}_z(\bar{\mathbf{q}}, \mathbf{q}_w, \mathbf{q}_a)\ ,$$

Energy ($\mathcal{L}_e\mathbf{q}_{s,b} = \mathcal{F}_e$):

$$\left(\Gamma\bar{P}_x - \sigma\bar{\rho}\bar{T}_x\right)u_b + \left(\Gamma\bar{P}_y - \sigma\bar{\rho}\bar{T}_y\right)v_b - \left[\frac{\sigma\bar{\rho}\mathcal{A}_7}{\bar{P}} + i\omega\Gamma\right]p_b + \Gamma\bar{U}p_{b,x} + \Gamma\bar{V}p_{b,y} +$$
$$\frac{d\Gamma\bar{\mu}}{R}\left(\mathcal{A}_8 u_{b,x} - \mathcal{A}_9 v_{b,y}\right) + \frac{2\Gamma\bar{\mu}\mathcal{A}_3}{R}\left(v_{b,x} + u_{b,y}\right) + \frac{\bar{\mu}}{R}\left(\theta_{b,xx} + \theta_{b,yy}\right) - \left(\sigma\bar{\rho}\bar{U} - \frac{2\bar{\mu}_T\bar{T}_x}{R}\right)\theta_{b,x} -$$
$$\left(\sigma\bar{\rho}\bar{V} - \frac{2\bar{\mu}_T\bar{T}_y}{R}\right)\theta_{b,y} - \frac{\Gamma di\beta\bar{\mu}\mathcal{A}_1}{R}w_b + \left[\frac{\bar{\mu}_{TT}}{R}\left(\bar{T}_x^2 + \bar{T}_y^2\right) + \sigma i\omega\bar{\rho} + \frac{\sigma\bar{\rho}\mathcal{A}_7}{\bar{T}} - \frac{\beta^2\bar{\mu}}{R} +\right.$$
$$\left.\frac{\bar{\mu}_T}{R}\left[\bar{T}_{xx} + \bar{T}_{yy} + \Gamma\left(d\bar{U}_x^2 - d\bar{U}_x\bar{V}_y + d\bar{V}_y^2 + \bar{V}_x^2 + 2\bar{V}_x\bar{U}_y + \bar{U}_y^2\right)\right]\right]\theta_b = \mathcal{F}_e(\bar{\mathbf{q}}, \mathbf{q}_w, \mathbf{q}_a)\ ,$$

whereas the bilinear forcing terms are given by

$$\bar{\rho}\mathcal{F}_c = \left[\left(-i\omega + \mathcal{A}_1\right)\mathcal{B}_1 + \bar{U}\mathcal{B}_{1,x} + \bar{V}\mathcal{B}_{1,y}\right](p\theta)_{w,a} + \bar{V}\mathcal{B}_1\left[(p_y\theta)_{w,a} + (\theta_y p)_{w,a}\right] -$$
$$2\left[\left(-i\omega + \mathcal{A}_1\right)\mathcal{B}_2 + \bar{U}\mathcal{B}_{2,x} + \bar{V}\mathcal{B}_{2,y}\right]\theta_a\theta_w + \bar{U}\mathcal{B}_1\left[(p_x\theta)_{w,a} + (\theta_x p)_{w,a}\right] -$$
$$\mathcal{B}_3\left[(up_x)_{w,a} + (vp_y)_{w,a} + (pu_x)_{w,a} + (pv_y)_{w,a} + i\beta w_w p_a\right] + \mathcal{B}_{4,y}(v\theta)_{w,a} +$$
$$\mathcal{B}_4\left[(u\theta_x)_{w,a} + (v\theta_y)_{w,a} + (\theta u_x)_{w,a} + (\theta v_y)_{w,a} + i\beta w_w\theta_a\right] - \mathcal{B}_{3,y}(vp)_{w,a} -$$
$$2\mathcal{B}_2\left[\bar{U}(\theta_x\theta)_{w,a} + \bar{V}(\theta_y\theta)_{w,a}\right] - \mathcal{B}_{3,x}(up)_{w,a} + \mathcal{B}_{4,x}(u\theta)_{w,a} \ ,$$

$$\mathcal{F}_x = \bar{\rho}(uu_x)_{w,a} + \bar{\rho}\left(i\omega - \bar{U}_x\right)\left(\frac{u_a\theta_w}{\bar{T}} - \frac{p_w u_a}{\bar{P}}\right) +$$
$$\bar{\rho}\bar{U}_x\left(\frac{p_a u_w}{\bar{P}} - \frac{\theta_a u_w}{\bar{T}}\right) + \frac{\bar{\rho}\bar{U}}{\bar{P}}(pu_x)_{w,a} - \left(\frac{\bar{\rho}\bar{U}}{\bar{T}} + \frac{d\bar{\mu}_{TT}\bar{T}_x}{R}\right)(\theta u_x)_{w,a} - \mathcal{B}_1\mathcal{A}_2(p\theta)_{w,a} +$$
$$\left[\frac{2\bar{\rho}\mathcal{A}_2}{\bar{T}^2} - \frac{\bar{\mu}_{TTT}}{R}\left(\mathcal{A}_3\bar{T}_y + \mathcal{A}_4\bar{T}_x\right) - \frac{\bar{\mu}_{TT}\mathcal{C}_2}{R}\right]\theta_a\theta_w + \bar{\rho}(vu_y)_{w,a} +$$
$$\bar{\rho}\bar{U}_y\left(\frac{(pv)_{w,a}}{\bar{P}} - \frac{(\theta v)_{w,a}}{\bar{T}}\right) + \frac{\bar{\rho}\bar{V}}{\bar{P}}(pu_y)_{w,a} - \left(\frac{\bar{\rho}\bar{V}}{\bar{T}} + \frac{\bar{\mu}_{TT}\bar{T}_y}{R}\right)(\theta u_y)_{w,a} -$$
$$\frac{\bar{\mu}_{TT}}{R}\left[\mathcal{A}_4(\theta\theta_x)_{w,a} + m\bar{T}_x(\theta v_y)_{w,a} + \mathcal{A}_3(\theta_y\theta)_{w,a} + \bar{T}_y(\theta v_x)_{w,a} + i\beta m\bar{T}_x w_w\theta_a\right] -$$
$$\frac{\bar{\mu}_T}{R}\left[i\beta m w_w\theta_{a,x} + d(\theta u_{xx})_{w,a} + (\theta u_{yy})_{w,a} + s(\theta v_{xy})_{w,a} + i\beta s\theta_a w_{w,x} - \beta^2\theta_a u_w\right] -$$
$$\frac{\bar{\mu}_T}{R}\left[d(u_x\theta_x)_{w,a} + m(v_y\theta_x)_{w,a} + (u_y\theta_y)_{w,a} + (v_x\theta_y)_{w,a}\right] \ ,$$

$$d\mathcal{F}_y = \bar{\rho}(vv_y)_{w,a} + \bar{\rho}\left(i\omega - \bar{V}_y\right)\left(\frac{v_a\theta_w}{\bar{T}} - \frac{p_w v_a}{\bar{P}}\right) +$$
$$\bar{\rho}\bar{V}_y\left(\frac{p_a v_w}{\bar{P}} - \frac{\theta_a v_w}{\bar{T}}\right) + \frac{\bar{\rho}\bar{V}}{\bar{P}}(pv_y)_{w,a} - \left(\frac{\bar{\rho}\bar{V}}{\bar{T}} + \frac{d\bar{\mu}_{TT}}{R}\bar{T}_y\right)(\theta v_y)_{w,a} - \mathcal{B}_1\mathcal{A}_5(p\theta)_{w,a} +$$
$$\left[\frac{2\bar{\rho}\mathcal{A}_5}{\bar{T}^2} - \frac{\bar{\mu}_{TTT}}{R}\left(\mathcal{A}_3\bar{T}_x + \mathcal{A}_6\bar{T}_y\right) - \frac{\bar{\mu}_{TT}\mathcal{C}_3}{R}\right]\theta_a\theta_w + \bar{\rho}(uv_x)_{w,a} +$$
$$\bar{\rho}\bar{V}_x\left(\frac{(pu)_{w,a}}{\bar{P}} - \frac{(\theta u)_{w,a}}{\bar{T}}\right) + \frac{\bar{\rho}\bar{U}}{\bar{P}}(pv_x)_{w,a} - \left(\frac{\bar{\rho}\bar{U}}{\bar{T}} + \frac{\bar{\mu}_{TT}\bar{T}_x}{R}\right)(\theta v_x)_{w,a} -$$
$$\frac{\bar{\mu}_{TT}}{R}\left[\mathcal{A}_6(\theta\theta_y)_{w,a} + m\bar{T}_y(\theta u_x)_{w,a} + \mathcal{A}_3(\theta_x\theta)_{w,a} + \bar{T}_x(\theta u_y)_{w,a} + i\beta m\bar{T}_y w_w\theta_a\right] -$$
$$\frac{\bar{\mu}_T}{R}\left[i\beta m w_w\theta_{a,y} + d(\theta v_{yy})_{w,a} + (\theta v_{xx})_{w,a} + s(\theta u_{xy})_{w,a} + i\beta s\theta_a w_{w,y} - \beta^2\theta_a v_w\right] -$$
$$\frac{\bar{\mu}_T}{R}\left[d(v_y\theta_y)_{w,a} + m(u_x\theta_y)_{w,a} + (v_x\theta_x)_{w,a} + (u_y\theta_x)_{w,a}\right] \ ,$$

$$\mathcal{F}_z = \bar{\rho} v_a w_{w,y} + \frac{\bar{\rho}\bar{V}}{\bar{P}} p_a w_{w,y} - \left(\frac{\bar{\rho}\bar{V}}{\bar{T}} + \frac{\bar{\mu}_{TT}\bar{T}_y}{R} \right) \theta_a w_{w,y} - \left(\frac{\bar{\mu}_{TT}\bar{T}_x}{R} + \frac{\bar{\rho}\bar{U}}{\bar{T}} \right) w_{w,x}\theta_a +$$
$$\bar{\rho} w_{w,x} u_a + \frac{\bar{\rho}\bar{U}}{\bar{P}} w_{w,x} p_a - \frac{\bar{\mu}_T}{R}\theta_a \left[w_{w,yy} + w_{w,xx} + si\beta \left(v_{w,y} + u_{w,x} \right) - d\beta^2 w_w \right] -$$
$$\frac{\bar{\mu}_T}{R} \left[i\beta m v_{a,y}\theta_w + w_{w,y}\theta_{a,y} + i\beta v_w \theta_{a,y} + i\beta m \theta_w u_{a,x} + \theta_{a,x} w_{w,x} + i\beta \theta_{a,x} u_w \right] -$$
$$\frac{\bar{\mu}_{TT}}{R} \left[i\beta \bar{T}_y \theta_a v_w + i\beta \bar{T}_x \theta_a u_w + i\beta m \mathcal{A}_1 \theta_w \theta_a \right] ,$$

$$\mathcal{F}_e = \left[\frac{\sigma i\omega\bar{\rho}}{\bar{T}} + \frac{\beta^2\bar{\mu}_T}{R} + \frac{2\sigma\bar{\rho}}{\bar{T}^2}\mathcal{A}_7 - \frac{\bar{\mu}_{TTT}}{R}\left(\bar{T}_x^2 + \bar{T}_y^2 \right) - \frac{\bar{\mu}_{TT}}{R}\left(\bar{T}_{xx} + \bar{T}_{yy} \right) + \right.$$
$$\left. \frac{\Gamma\bar{\mu}_{TT}}{R} \left(2m\left(\bar{U}_x^2 - \bar{U}_x\bar{V}_y + \bar{V}_y^2 \right) - \bar{V}_x^2 - 2\bar{V}_x\bar{U}_y - \bar{U}_y^2 \right) \right] \theta_a\theta_w -$$
$$\sigma\mathcal{B}_1\mathcal{A}_7 \left(p\theta \right)_{w,a} - \frac{\sigma i\omega\bar{\rho}}{\bar{P}} p_w\theta_a - \frac{\sigma\bar{\rho}\bar{T}_y}{\bar{T}} \left(\theta v \right)_{w,a} + \bar{\rho}\sigma \left(u\theta_x + v\theta_y \right)_{w,a} +$$
$$\frac{\sigma\bar{\rho}\bar{U}}{\bar{P}} \left(p\theta_x \right)_{w,a} + \frac{\sigma\bar{\rho}\bar{V}}{\bar{P}} \left(p\theta_y \right)_{w,a} - \left(\frac{\sigma\bar{\rho}\bar{U}}{\bar{T}} + \frac{2\bar{\mu}_{TT}\bar{T}_x}{R} \right) \left(\theta\theta_x \right)_{w,a} -$$
$$\left(\frac{\sigma\bar{\rho}\bar{V}}{\bar{T}} + \frac{2\bar{\mu}_{TT}\bar{T}_y}{R} \right) \left(\theta\theta_y \right) + \frac{\sigma\bar{\rho}\bar{T}_x}{\bar{P}} \left(pu \right)_{w,a} + \frac{\sigma\bar{\rho}\bar{T}_y}{\bar{P}} \left(pv \right)_{w,a} - \frac{\sigma\bar{\rho}\bar{T}_x}{\bar{T}} \left(\theta u \right)_{w,a} -$$
$$\Gamma \left[\left(up_x \right)_{w,a} + \left(vp_y \right)_{w,a} \right] - \frac{2\Gamma\bar{\mu}}{R} \left[du_{w,x}u_{a,x} + dv_{w,y}v_{a,y} + v_{w,x}v_{a,x} \right] -$$
$$\frac{\Gamma\bar{\mu}}{R} \left[2u_{w,y}u_{a,y} - d\left(u_x v_y \right)_{w,a} + 2\left(v_x u_y \right)_{w,a} - i\beta d\left(w_w v_{a,y} + w_w u_{a,x} \right) \right] -$$
$$\frac{\bar{\mu}_T}{R} \left[2\theta_{a,x}\theta_{w,x} + 2\theta_{a,y}\theta_{w,y} + \left(\theta\theta_{xx} \right)_{w,a} + \left(\theta\theta_{yy} \right)_{w,a} + d\Gamma\mathcal{A}_8 \left(\theta u_x \right)_{w,a} \right] +$$
$$\frac{\bar{\mu}_T}{R} \left[d\Gamma\mathcal{A}_9 \left(\theta v_y \right)_{w,a} + d\Gamma i\beta\mathcal{A}_1\theta_a w_w - 2\Gamma\mathcal{A}_3 \left(\left(\theta v_x \right)_{w,a} + \left(\theta u_y \right)_{w,a} \right) \right] .$$

We have introduced the constants

$$d = 4/3 , \quad s = 1/3 , \quad m = -2/3 ,$$

and the following notation

$$\mathcal{A}_1 = \frac{\partial\bar{U}}{\partial x} + \frac{\partial\bar{V}}{\partial y} , \quad \mathcal{A}_2 = \bar{U}\frac{\partial\bar{U}}{\partial x} + \bar{V}\frac{\partial\bar{U}}{\partial y} , \quad \mathcal{A}_3 = \frac{\partial\bar{U}}{\partial y} + \frac{\partial\bar{V}}{\partial x} , \quad \mathcal{A}_4 = d\frac{\partial\bar{U}}{\partial x} + m\frac{\partial\bar{V}}{\partial y} ,$$
$$\mathcal{A}_5 = \bar{U}\frac{\partial\bar{V}}{\partial x} + \bar{V}\frac{\partial\bar{V}}{\partial y} , \quad \mathcal{A}_6 = d\frac{\partial\bar{V}}{\partial y} + m\frac{\partial\bar{U}}{\partial x} , \quad \mathcal{A}_7 = \bar{U}\frac{\partial\bar{T}}{\partial x} + \bar{V}\frac{\partial\bar{T}}{\partial y} , \quad \mathcal{A}_8 = 2\frac{\partial\bar{U}}{\partial x} - \frac{\partial\bar{V}}{\partial y} ,$$
$$\mathcal{A}_9 = \frac{\partial\bar{U}}{\partial x} - 2\frac{\partial\bar{V}}{\partial y} , \quad \mathcal{B}_1 = \frac{\bar{\rho}}{\bar{P}\bar{T}} , \quad \mathcal{B}_2 = \frac{\bar{\rho}}{\bar{T}^2} , \quad \mathcal{B}_3 = \frac{\bar{\rho}}{\bar{P}} , \quad \mathcal{B}_4 = \frac{\bar{\rho}}{\bar{T}} , \quad \mathcal{B}_5 = \frac{1}{\bar{P}\bar{T}} ,$$
$$\mathcal{C}_1 = \frac{\bar{U}}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial x} + \frac{\bar{V}}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial y} , \mathcal{C}_2 = d\bar{U}_{xx} + \bar{U}_{yy} + s\bar{V}_{xy} , \quad \mathcal{C}_3 = d\bar{V}_{yy} + \bar{V}_{xx} + s\bar{U}_{xy} .$$

The subscripts accommodate both the designation of the disturbance (w - wall-induced steady disturbance, a - unsteady acoustic disturbance, b - boundary layer instability) and the partial derivatives, separated by a comma in this order. Moreover, the bilinear forcing terms have been contracted for brevity. For example $\left(up_x \right)_{w,a} = u_w p_{a,x} + u_a p_{w,x}$.

2. Adjoint linearised Navier-Stokes equations

The explicit form of equation 2.30 is

Continuity ($\tilde{\mathcal{L}}_c\tilde{\mathbf{q}}_{s,b}=0$):

$$(i\omega-\mathcal{C}_1)\frac{\tilde{p}_b}{\bar{P}}+\frac{\bar{U}}{\bar{P}}\tilde{p}_{b,x}+\frac{\bar{V}}{\bar{P}}\tilde{p}_{b,y}+\mathcal{B}_3\mathcal{A}_2\tilde{u}_b-\tilde{u}_{b,x}+\frac{\mathcal{B}_3\mathcal{A}_5}{d}\tilde{v}_b-\frac{1}{d}\tilde{v}_{b,y}+i\beta\tilde{w}_b+$$
$$[\sigma\mathcal{B}_3\mathcal{A}_7+\Gamma(i\omega+\mathcal{A}_1)]\tilde{\theta}_b+\Gamma\left(\bar{U}\tilde{\theta}_{b,x}+\bar{V}\tilde{\theta}_{b,y}\right)=0\,,$$

X-momentum ($\tilde{\mathcal{L}}_x\tilde{\mathbf{q}}_{s,b}=0$):

$$-\frac{\bar{\rho}_x}{\bar{\rho}}\tilde{p}_b+\tilde{p}_{b,x}-\frac{d\bar{\mu}}{R}\tilde{u}_{b,xx}-\left(\frac{d\bar{\mu}_x}{R}+\bar{\rho}\bar{U}\right)\tilde{u}_{b,x}-\frac{\bar{\mu}}{R}\tilde{u}_{b,yy}-\left(\frac{\bar{\mu}_y}{R}+\bar{\rho}\bar{V}\right)\tilde{u}_{b,y}$$
$$+\left(\bar{\rho}\bar{U}_x-\bar{\rho}i\omega+\frac{\beta^2\bar{\mu}}{R}\right)\tilde{u}_b-\frac{s\bar{\mu}}{dR}\tilde{v}_{b,xy}+\frac{m-s}{dR}\bar{\mu}_y\tilde{v}_{b,x}+\frac{1-s}{dR}\bar{\mu}_x\tilde{v}_{b,y}$$
$$+\frac{\bar{\rho}\bar{V}_x}{d}\tilde{v}_b+\frac{si\beta\bar{\mu}}{R}\tilde{w}_{b,x}+\frac{(s-1)i\beta\bar{\mu}_x}{R}\tilde{w}_b+\frac{d\Gamma\bar{\mu}\mathcal{A}_8}{R}\tilde{\theta}_{b,x}+\frac{2\Gamma\bar{\mu}\mathcal{A}_3}{R}\tilde{\theta}_{b,y}+$$
$$\left[\left(\sigma\bar{\rho}\bar{T}_x-\Gamma\bar{P}_x\right)+\frac{\bar{\mu}d\Gamma\mathcal{A}_{8,x}}{R}+\frac{d\Gamma\bar{\mu}_x\mathcal{A}_8}{R}+\frac{2\Gamma\bar{\mu}\mathcal{A}_{3,y}}{R}+\frac{2\Gamma\bar{\mu}_y\mathcal{A}_3}{R}\right]\tilde{\theta}_b=0\,,$$

Y-momentum ($\tilde{\mathcal{L}}_y\tilde{\mathbf{q}}_{s,b}=0$):

$$-\frac{\bar{\rho}_y}{\bar{\rho}}\tilde{p}_b+\tilde{p}_{b,y}-\frac{s\bar{\mu}}{R}\tilde{u}_{b,xy}+\bar{\rho}\bar{U}_y\tilde{u}_b+\frac{1-s}{R}\bar{\mu}_y\tilde{u}_{b,x}+\frac{m-s}{R}\bar{\mu}_x\tilde{u}_{b,y}-\frac{\bar{\mu}}{R}\tilde{v}_{b,yy}+$$
$$\left(\frac{\bar{\rho}\bar{V}_y}{d}-\frac{\bar{\rho}i\omega}{d}+\frac{\bar{\mu}\beta^2}{dR}\right)\tilde{v}_b-\left(\frac{\bar{\mu}_y}{R}+\frac{\bar{\rho}\bar{V}}{d}\right)\tilde{v}_{b,y}-\frac{\bar{\mu}}{dR}\tilde{v}_{b,xx}-\left(\frac{\bar{\mu}_x}{dR}+\frac{\bar{\rho}\bar{U}}{d}\right)\tilde{v}_{b,x}-$$
$$\frac{\bar{\mu}d\Gamma\mathcal{A}_9}{R}\tilde{\theta}_{b,y}+\frac{2\bar{\mu}\Gamma\mathcal{A}_3}{R}\tilde{\theta}_{b,x}+\frac{s\bar{\mu}i\beta}{R}\tilde{w}_{b,y}+\frac{(s-1)i\beta\bar{\mu}_y}{R}\tilde{w}_b+$$
$$\left[\left(\sigma\bar{\rho}\bar{T}_y-\Gamma\bar{P}_y\right)-\frac{d\bar{\mu}\Gamma\mathcal{A}_{9,y}}{R}-\frac{d\Gamma\bar{\mu}_y\mathcal{A}_9}{R}+\frac{2\bar{\mu}\Gamma\mathcal{A}_{3,x}}{R}+\frac{2\Gamma\bar{\mu}_x\mathcal{A}_3}{R}\right]\tilde{\theta}_g=0\,,$$

Z-momentum ($\tilde{\mathcal{L}}_z\tilde{\mathbf{q}}_{s,b}=0$):

$$-i\beta\tilde{p}_b+\frac{i\bar{\mu}\beta s}{R}\tilde{u}_{b,x}+\frac{i\beta(s-m)}{R}\left(\bar{\mu}_x\tilde{u}_b+\frac{\bar{\mu}_y}{d}\tilde{v}_b\right)+\frac{i\beta s\bar{\mu}}{dR}\tilde{v}_{b,y}-\frac{\bar{\mu}}{R}\left(\tilde{w}_{b,xx}+\tilde{w}_{b,yy}\right)+$$
$$\left(\frac{d\beta^2\bar{\mu}}{R}-\bar{\rho}i\omega\right)\tilde{w}_b-\left(\frac{\bar{\mu}_x}{R}+\bar{\rho}\bar{U}\right)\tilde{w}_{b,x}-\left(\frac{\bar{\mu}_y}{R}+\bar{\rho}\bar{V}\right)\tilde{w}_{b,y}+\frac{i\beta d\Gamma\bar{\mu}\mathcal{A}_1}{R}\tilde{\theta}_b=0\,,$$

Energy ($\tilde{\mathcal{L}}_e\tilde{\mathbf{q}}_{s,b}=0$):

$$(-i\omega+\mathcal{C}_1)\frac{\tilde{p}_b}{\bar{T}}-\frac{\bar{U}}{\bar{T}}\tilde{p}_{b,x}-\frac{\bar{V}}{\bar{T}}\tilde{p}_{b,y}-\mathcal{B}_4\mathcal{A}_2\tilde{u}_b+\frac{\bar{\mu}_T}{R}\left(\mathcal{A}_3\tilde{u}_{b,y}+\mathcal{A}_4\tilde{u}_{b,x}\right)-\frac{\mathcal{B}_4\mathcal{A}_5}{d}\tilde{v}_b$$
$$+\frac{\bar{\mu}_T}{dR}\left(\mathcal{A}_3\tilde{v}_{b,x}+\mathcal{A}_6\tilde{v}_{b,y}\right)-\frac{i\beta m\bar{\mu}_T\mathcal{A}_1}{R}\tilde{w}_b-\frac{\bar{\mu}}{R}\left(\tilde{\theta}_{b,xx}+\tilde{\theta}_{b,yy}\right)-\sigma\bar{\rho}\left(\bar{U}\tilde{\theta}_{b,x}+\bar{V}\tilde{\theta}_{b,y}\right)+$$
$$\left[-i\omega\sigma\bar{\rho}+\frac{\bar{\mu}\beta^2}{R}-\sigma\mathcal{B}_4\mathcal{A}_7-\frac{\Gamma\bar{\mu}_T}{R}\left(d\bar{U}_x^2-d\bar{U}_x\bar{V}_y+d\bar{V}_y^2+\bar{V}_x^2+2\bar{V}_x\bar{U}_y+\bar{U}_y^2\right)\right]\tilde{\theta}_b=0\,.$$

The right-hand side of equation 2.34 is given by

$$\tilde{\mathcal{F}}_c = \frac{1}{\bar{P}}\left(\tilde{p}_{b,x}u_a + \tilde{p}_{b,y}v_a\right) + \left[\mathcal{B}_5\left(-i\omega + \mathcal{A}_1\right) + \frac{\left(\bar{U}\mathcal{B}_{1,x} + \bar{V}\mathcal{B}_{1,y}\right)}{\bar{\rho}}\right]\theta_a\tilde{p}_b - \left(\bar{U}\mathcal{B}_5\theta_a\tilde{p}_b\right)_x -$$
$$\left(\bar{V}\mathcal{B}_5\theta_a\tilde{p}_b\right)_y + \mathcal{B}_5\tilde{p}_b\left(\bar{U}\theta_{a,x} + \bar{V}\theta_{a,y}\right) - \left(\frac{\bar{\rho}_x u_a}{\bar{\rho}} + \frac{\bar{\rho}_y v_a}{\bar{\rho}}\right)\frac{\tilde{p}_b}{\bar{P}} + \mathcal{B}_3\left(\bar{U}_x - i\omega\right)u_a\tilde{u}_b +$$
$$\mathcal{B}_3\bar{U}u_{a,x}\tilde{u}_b - \mathcal{A}_2\mathcal{B}_1\theta_a\tilde{u}_b + \mathcal{B}_3\bar{U}_y v_a\tilde{u}_b + \mathcal{B}_3\bar{V}u_{a,y}\tilde{u}_b + \frac{\mathcal{B}_3}{d}\left(\bar{V}_y - i\omega\right)v_a\tilde{v}_b + \frac{\mathcal{B}_3\bar{V}}{d}v_{a,y}\tilde{v}_b -$$
$$\frac{\mathcal{A}_5\mathcal{B}_1}{d}\theta_a\tilde{v}_b + \frac{\mathcal{B}_3\bar{V}_x}{d}u_a\tilde{v}_b + \frac{\mathcal{B}_3\bar{U}}{d}v_{a,x}\tilde{v}_b - \sigma\left[\mathcal{B}_1\mathcal{A}_7 + i\omega\mathcal{B}_3\right]\theta_a\tilde{\theta}_b + \sigma\mathcal{B}_3\tilde{\theta}_b\left(\bar{U}\theta_{a,x} + \bar{V}\theta_{a,y}\right) +$$
$$\sigma\mathcal{B}_3\tilde{\theta}_b\left(\bar{T}_x u_a + \bar{T}_y v_a\right) + \Gamma\tilde{\theta}\left(u_{a,x} + v_{a,y}\right) + \Gamma\left(u_a\tilde{\theta}_{b,x} + v_a\tilde{\theta}_{b,y}\right),$$

$$\tilde{\mathcal{F}}_x = \left(\tilde{p}_{b,x} - \frac{\bar{\rho}_x}{\bar{\rho}}\tilde{p}_b\right)\left(\frac{p_a}{\bar{P}} - \frac{\theta_a}{\bar{T}}\right) - \left(\bar{\rho}\tilde{u}_b\right)_x u_a + \left(\frac{\bar{\mu}_T\beta^2}{R} - \mathcal{B}_4\bar{U}_x\right)\theta_a\tilde{u}_b - \bar{U}\left(\mathcal{B}_3 p_a\tilde{u}_b\right)_x +$$
$$\left[\left(\bar{U}\mathcal{B}_4 + \frac{d\bar{\mu}_{TT}\bar{T}_x}{R}\right)\theta_a\tilde{u}_b\right]_x - \left(\bar{\rho}v_a\tilde{u}_b\right)_y - \left(\bar{V}\mathcal{B}_3 p_a\tilde{u}_b\right)_y + \left[\left(\bar{V}\mathcal{B}_4 + \frac{\bar{\mu}_{TT}\bar{T}_y}{R}\right)\theta_a\tilde{u}_b\right]_y +$$
$$\left(\frac{d\bar{\mu}_T}{R}\theta_{a,x}\tilde{u}_b\right)_x + \left(\frac{\bar{\mu}_T}{R}\theta_{a,y}\tilde{u}_b\right)_y - \frac{d}{R}\left(\bar{\mu}_T\theta_a\tilde{u}_b\right)_{xx} - \frac{\left(\bar{\mu}_T\theta_a\tilde{u}_b\right)_{yy}}{R} + \frac{\bar{\rho}}{d}v_{a,x}\tilde{v}_b + \frac{\mathcal{B}_3\bar{V}_x}{d}p_a\tilde{v}_b -$$
$$\frac{\mathcal{B}_4\bar{V}_x}{d}\theta_a\tilde{v}_b + \frac{m}{dR}\left(\bar{\mu}_{TT}\bar{T}_y\theta_a\tilde{v}_b\right)_x + \frac{m}{dR}\left(\bar{\mu}_T\theta_{a,y}\tilde{v}_b\right)_x + \frac{\left(\bar{\mu}_{TT}\bar{T}_x\theta_a\tilde{v}_b\right)_y}{dR} + \frac{\left(\bar{\mu}_T\theta_{a,x}\tilde{v}_b\right)_y}{dR} -$$
$$\frac{s}{dR}\left(\bar{\mu}_T\theta_a\tilde{v}_b\right)_{xy} - \frac{i\beta\bar{\mu}_{TT}\bar{T}_x}{R}\theta_a\tilde{w}_b - \frac{i\beta\bar{\mu}_T}{R}\theta_{a,x}\tilde{w}_b + \frac{i\beta s}{R}\left(\bar{\mu}_T\theta_a\tilde{w}_b\right)_x + \sigma\bar{\rho}\theta_{a,x}\tilde{\theta}_b + \sigma\mathcal{B}_3\bar{T}_x p_a\tilde{\theta}_b -$$
$$\sigma\mathcal{B}_4\bar{T}_x\theta_a\tilde{\theta}_b - \Gamma p_{a,x}\tilde{\theta}_b + \frac{2\Gamma d}{R}\left(\bar{\mu}u_{a,x}\tilde{\theta}_b\right)_x + \frac{2\Gamma}{R}\left(\bar{\mu}u_{a,y}\tilde{\theta}_b\right)_y - \frac{\Gamma d}{R}\left(\bar{\mu}v_{a,y}\tilde{\theta}_b\right)_x +$$
$$\frac{2\Gamma}{R}\left(\bar{\mu}v_{a,x}\tilde{\theta}_b\right)_y + \frac{\Gamma d}{R}\left(\bar{\mu}_T\mathcal{A}_8\theta_a\tilde{\theta}_b\right)_x + \frac{2\Gamma}{R}\left(\bar{\mu}_T\mathcal{A}_3\theta_a\tilde{\theta}_b\right)_y,$$

$$\tilde{\mathcal{F}}_y = \left(\tilde{p}_{b,y} - \frac{\bar{\rho}_y}{\bar{\rho}}\tilde{p}_b\right)\left(\frac{p_a}{\bar{P}} - \frac{\theta_a}{\bar{T}}\right) + \bar{\rho}u_{a,y}\tilde{u}_b + \mathcal{B}_3\bar{U}_y p_a\tilde{u}_b - \mathcal{B}_4\bar{U}_y\theta_a\tilde{u}_b + \frac{m}{R}\left(\bar{\mu}_{TT}\bar{T}_x\theta_a\tilde{u}_b\right)_y +$$
$$\frac{m}{R}\left(\bar{\mu}_T\theta_{a,x}\tilde{u}_b\right)_y + \frac{\left(\bar{\mu}_{TT}\bar{T}_y\theta_a\tilde{u}_b\right)_x}{R} + \frac{\left(\bar{\mu}_T\theta_{a,y}\tilde{u}_b\right)_x}{R} - \frac{s}{R}\left(\bar{\mu}_T\theta_a\tilde{u}_b\right)_{xy} - \frac{\left(\bar{\rho}v_a\tilde{v}_b\right)_y}{d} + \frac{\bar{\rho}}{d}v_{a,y}\tilde{v}_b +$$
$$\left(\frac{\bar{\mu}_T\beta^2}{Rd} - \frac{\mathcal{B}_4\bar{V}_y}{d}\right)\theta_a\tilde{v}_b + \frac{\mathcal{B}_3\bar{V}_y}{d}p_a\tilde{v}_b - \frac{\left(\mathcal{B}_3\bar{V}p_a\tilde{v}\right)_y}{d} + \left[\left(\frac{\mathcal{B}_4\bar{V}}{d} + \frac{\bar{\mu}_{TT}\bar{T}_y}{R}\right)\theta_a\tilde{v}_b\right]_y -$$
$$\frac{\left(\bar{\rho}u_a\tilde{v}_b\right)_x}{d} - \frac{\left(\mathcal{B}_3\bar{U}p_a\tilde{v}\right)_x}{d} + \left[\left(\frac{\mathcal{B}_4\bar{U}}{d} + \frac{\bar{\mu}_{TT}\bar{T}_x}{Rd}\right)\theta_a\tilde{v}_b\right]_x + \frac{\left(\bar{\mu}_T\theta_{a,y}\tilde{v}_b\right)_y}{R} + \frac{\left(\bar{\mu}_T\theta_{a,x}\tilde{v}_b\right)_x}{Rd} -$$
$$\frac{\left(\bar{\mu}_T\theta_a\tilde{v}_b\right)_{yy}}{R} - \frac{\left(\bar{\mu}_T\theta_a\tilde{v}_b\right)_{xx}}{Rd} - \frac{i\beta\bar{\mu}_{TT}\bar{T}_y}{R}\theta_a\tilde{w}_b - \frac{i\beta\bar{\mu}_T}{R}\theta_{a,y}\tilde{w}_b + \frac{i\beta s}{R}\left(\bar{\mu}_T\theta_a\tilde{w}_b\right)_y + \sigma\bar{\rho}\theta_{a,y}\tilde{\theta}_b +$$
$$\sigma\mathcal{B}_3\bar{T}_y p_a\tilde{\theta}_b - \sigma\mathcal{B}_4\bar{T}_y\theta_a\tilde{\theta}_b - \Gamma p_{a,y}\tilde{\theta}_b + \frac{2\Gamma d}{R}\left(\bar{\mu}v_{a,y}\tilde{\theta}_b\right)_y + \frac{2\Gamma}{R}\left(\bar{\mu}v_{a,x}\tilde{\theta}_b\right)_x - \frac{\Gamma d}{R}\left(\bar{\mu}u_{a,x}\tilde{\theta}_b\right)_y +$$
$$\frac{2\Gamma}{R}\left(\bar{\mu}u_{a,y}\tilde{\theta}_b\right)_x - \frac{\Gamma d}{R}\left(\bar{\mu}_T\mathcal{A}_9\theta_a\tilde{\theta}_b\right)_y + \frac{2\Gamma}{R}\left(\bar{\mu}_T\mathcal{A}_3\theta_a\tilde{\theta}_b\right)_x,$$

$$\tilde{\mathcal{F}}_z = i\beta\left(\frac{\theta_a}{\bar{T}} - \frac{p_a}{\bar{P}}\right)\tilde{p}_b - \frac{i\beta m\bar{\mu}_{TT}\theta_a}{R}\left(\bar{T}_x\tilde{u}_b + \frac{\bar{T}_y}{d}\tilde{v}_b\right) - \frac{i\beta m\bar{\mu}_T}{R}\left(\theta_{a,x}\tilde{u}_b + \frac{\theta_{a,y}\tilde{v}_b}{d}\right) +$$
$$\frac{i\beta s}{R}\left(\bar{\mu}_T\theta_a\tilde{u}_b\right)_x + \frac{i\beta s}{Rd}\left(\bar{\mu}_T\theta_a\tilde{v}_b\right)_y - \left(\bar{\rho}v_a\tilde{w}_b\right)_y - \left(\mathcal{B}_3\bar{V}p_a\tilde{w}_b\right)_y + \left[\left(\mathcal{B}_4\bar{V} + \frac{\bar{\mu}_{TT}\bar{T}_y}{R}\right)\theta_a\tilde{w}_b\right]_y +$$
$$\left(\frac{\bar{\mu}_T}{R}\theta_{a,y}\tilde{w}_b\right)_y + \left[\left(\mathcal{B}_4\bar{U} + \frac{\bar{\mu}_{TT}\bar{T}_x}{R}\right)\theta_a\tilde{w}_b\right]_x + \left(\frac{\bar{\mu}_T}{R}\theta_{a,x}\tilde{w}_b\right)_x - \left(\bar{\rho}u_a\tilde{w}_b\right)_x - \left(\mathcal{B}_3\bar{U}p_a\tilde{w}_b\right)_x +$$
$$\frac{d\beta^2\bar{\mu}_T}{R}\theta_a\tilde{w}_b - \left(\frac{\bar{\mu}_T\theta_a\tilde{w}_b}{R}\right)_{yy} - \left(\frac{\bar{\mu}_T\theta_a\tilde{w}_b}{R}\right)_{xx} + \frac{i\beta\Gamma d\bar{\mu}}{R}\left(v_{a,y}\tilde{\theta}_b + u_{a,x}\tilde{\theta}_b\right) + \frac{i\beta d\Gamma\mathcal{A}_1\bar{\mu}_T}{R}\theta_a\tilde{\theta}_b\ ,$$

$$\tilde{\mathcal{F}}_e = \left(\frac{\bar{\rho}_y}{\bar{\rho}}v_a + \frac{\bar{\rho}_x}{\bar{\rho}}u_a\right)\frac{\tilde{p}_b}{\bar{T}} - \frac{1}{\bar{T}}\left(\tilde{p}_{b,y}v_a + \tilde{p}_{b,x}u_a\right) + \left[\mathcal{B}_5\left(-i\omega + \mathcal{A}_1\right) + \frac{\bar{U}\mathcal{B}_{1,x}}{\bar{\rho}} + \frac{\bar{V}\mathcal{B}_{1,y}}{\bar{\rho}}\right]p_a\tilde{p}_b -$$
$$2\left[\frac{\left(-i\omega + \mathcal{A}_1\right)}{\bar{T}^2} + \frac{\bar{U}\mathcal{B}_{2,x}}{\bar{\rho}} + \frac{\bar{V}\mathcal{B}_{2,y}}{\bar{\rho}}\right]\theta_a\tilde{p}_b - \left(\bar{U}\mathcal{B}_5\tilde{p}_b\right)_x p_a + 2\left(\frac{\bar{U}\tilde{p}_b}{\bar{T}^2}\right)_x\theta_a - \left(\bar{V}\mathcal{B}_5\tilde{p}_b\right)_y p_a +$$
$$2\left(\frac{\bar{V}\tilde{p}_b}{\bar{T}^2}\right)_y\theta_a + \mathcal{B}_4\left(i\omega - \bar{U}_x\right)u_a\tilde{u}_b - \left(\bar{U}\mathcal{B}_4 + \frac{d\bar{\mu}_{TT}\bar{T}_x}{R}\right)u_{a,x}\tilde{u}_b - \mathcal{B}_1\mathcal{A}_2p_a\tilde{u}_b +$$
$$\left[2\mathcal{B}_2\mathcal{A}_2 - \frac{\bar{\mu}_{TTT}}{R}\left(\bar{T}_y\mathcal{A}_3 + \bar{T}_x\mathcal{A}_4\right) - \frac{\bar{\mu}_{TT}}{R}\left(d\bar{U}_{xx} + \bar{U}_{yy} + s\bar{V}_{xy}\right)\right]\theta_a\tilde{u}_b - \mathcal{B}_4\bar{U}_y v_a\tilde{u}_b -$$
$$\left(\bar{V}\mathcal{B}_4 + \frac{\bar{\mu}_{TT}\bar{T}_y}{R}\right)u_{a,y}\tilde{u}_b + \frac{d}{R}\left(\bar{\mu}_T u_{a,x}\tilde{u}_b\right)_x + \theta_a\left(\frac{\bar{\mu}_{TT}\mathcal{A}_4}{R}\tilde{u}_b\right)_x + \frac{m\bar{\mu}_T}{R}\left(v_{a,y}\tilde{u}_b\right)_x +$$
$$\frac{\left(\bar{\mu}_T u_{a,y}\tilde{u}_b\right)_y}{R} + \theta_a\frac{\left(\bar{\mu}_{TT}\mathcal{A}_3\tilde{u}_b\right)_y}{R} + \frac{\bar{\mu}_T}{R}\left(v_{a,x}\tilde{u}_b\right)_y - \frac{d\bar{\mu}_T}{R}u_{a,xx}\tilde{u}_b - \frac{\bar{\mu}_T}{R}u_{a,yy}\tilde{u}_b - \frac{s\bar{\mu}_T}{R}v_{a,xy}\tilde{u}_b +$$
$$\frac{\mathcal{B}_4}{d}\left(i\omega - \bar{V}_y\right)v_a\tilde{v}_b - \left(\frac{\bar{V}\mathcal{B}_4}{d} + \frac{\bar{\mu}_{TT}\bar{T}_y}{R}\right)v_{a,y}\tilde{v}_b - \frac{\mathcal{A}_5\mathcal{B}_1}{d}p_a\tilde{v}_b - \frac{\mathcal{B}_4\bar{V}_x}{d}u_a\tilde{v}_b - \sigma\left(\bar{U}\mathcal{B}_3p_a\tilde{\theta}_b\right)_x +$$
$$\left[\frac{2\mathcal{B}_2\mathcal{A}_5}{d} - \frac{\bar{\mu}_{TTT}}{Rd}\left(\bar{T}_x\mathcal{A}_3 + \bar{T}_y\mathcal{A}_6\right) - \frac{\bar{\mu}_{TT}}{Rd}\left(d\bar{V}_{yy} + \bar{V}_{xx} + s\bar{U}_{xy}\right)\right]\theta_a\tilde{v}_b + \left(\frac{\bar{\mu}_T v_{a,y}\tilde{v}_b}{R}\right)_y -$$
$$\left(\frac{\bar{U}\mathcal{B}_4}{d} + \frac{\bar{\mu}_{TT}\bar{T}_x}{Rd}\right)v_{a,x}\tilde{v}_b + \theta_a\left(\frac{\bar{\mu}_{TT}\mathcal{A}_6}{Rd}\tilde{v}_b\right)_y + \frac{m\bar{\mu}_T}{Rd}\left(u_{a,x}\tilde{v}_b\right)_y + \left(\frac{\bar{\mu}_T v_{a,x}\tilde{v}_b}{Rd}\right)_x +$$
$$\theta_a\left(\frac{\bar{\mu}_{TT}\mathcal{A}_3\tilde{v}_b}{Rd}\right)_x + \frac{\bar{\mu}_T}{Rd}\left(u_{a,y}\tilde{v}_b\right)_x - \frac{\bar{\mu}_T}{R}v_{a,yy}\tilde{v}_b - \frac{\bar{\mu}_T}{Rd}v_{a,xx}\tilde{v}_b - \frac{s\bar{\mu}_T}{Rd}u_{a,xy}\tilde{v}_b -$$
$$\frac{i\beta m\bar{\mu}_T\tilde{w}_b}{R}\left(v_{a,y} + u_{a,x}\right) - \frac{i\beta m\bar{\mu}_T\mathcal{A}_1}{R}\theta_a\tilde{w}_b - \sigma\mathcal{B}_1\mathcal{A}_7p_a\tilde{\theta}_b - \sigma\left(\bar{\rho}u_a\tilde{\theta}_b\right)_x - \sigma\left(\bar{\rho}v_a\tilde{\theta}_b\right)_y -$$
$$\sigma\left(\bar{V}\mathcal{B}_3p_a\tilde{\theta}_b\right)_y + \theta_a\left[\left(\sigma\bar{U}\mathcal{B}_4 + \frac{2\bar{\mu}_{TT}\bar{T}_x}{R}\right)\tilde{\theta}_b\right]_x + \theta_a\left[\left(\sigma\bar{V}\mathcal{B}_4 + \frac{2\bar{\mu}_{TT}\bar{T}_y}{R}\right)\tilde{\theta}_b\right]_y -$$
$$\sigma\mathcal{B}_4\bar{T}_x\tilde{\theta}_b\left(u_a + v_a\right) + \frac{2}{R}\left(\bar{\mu}_T\theta_{a,x}\tilde{\theta}_b\right)_x + \frac{2}{R}\left(\bar{\mu}_T\theta_{a,y}\tilde{\theta}_b\right)_y - \left(\frac{\bar{\mu}_T\theta_a\tilde{\theta}_b}{R}\right)_{xx} - \left(\frac{\bar{\mu}_T\theta_a\tilde{\theta}_b}{R}\right)_{yy} -$$
$$\frac{\bar{\mu}_T\theta_{a,xx}\tilde{\theta}_b}{R} - \frac{\bar{\mu}_T\theta_{a,yy}\tilde{\theta}_b}{R} - \frac{d\Gamma\bar{\mu}_T\mathcal{A}_8}{R}u_{a,x}\tilde{\theta}_b + \frac{d\Gamma\bar{\mu}_T\mathcal{A}_9}{R}v_{a,y}\tilde{\theta}_b - \frac{2\Gamma\bar{\mu}_T\mathcal{A}_3}{R}\tilde{\theta}_b\left(v_{a,x} + u_{a,y}\right)\ .$$

The bilinear concomitant for the direct and adjoint fields $\mathbf{q}_{s,b}, \tilde{\mathbf{q}}_{s,b}$ is given by

$$K_1 = u_b\tilde{p}_b + \frac{\bar{U}}{\bar{P}}p_b\tilde{p}_b - \frac{\bar{U}}{\bar{T}}\theta_b\tilde{p}_b - \bar{\rho}\bar{U}\mathbf{v}_{b,2}\cdot\tilde{\mathbf{v}}_b + \frac{\bar{\mu}}{R}\left(\mathbf{v}_{b,x,3}\cdot\tilde{\mathbf{v}}_{b,2} - \mathbf{v}_{b,2}\cdot\tilde{\mathbf{v}}_{b,x,3}\right) - \frac{s\bar{\mu}}{R}v_b\tilde{u}_{b,y} - \frac{m\bar{\mu}_y}{R}\left(v_b\tilde{u}_b - \frac{u_b\tilde{v}_b}{d}\right) + \frac{\bar{\mu}_T\mathcal{A}_4}{R}\theta_b\tilde{u}_b - p_b\tilde{u}_b + \frac{i\beta s\bar{\mu}}{R}\left(w_b\tilde{u}_b + u_b\tilde{w}_b\right) - \frac{s\bar{\mu}}{Rd}u_b\tilde{v}_{b,y} - \frac{s\bar{\mu}_y}{dR}u_b\tilde{v}_b + \frac{\bar{\mu}_T\mathcal{A}_3}{dR}\theta_b\tilde{v}_b + \Gamma\bar{U}p_b\tilde{\theta}_b + \frac{d\Gamma\bar{\mu}\mathcal{A}_8}{R}u_b\tilde{\theta}_b + \frac{2\Gamma\bar{\mu}\mathcal{A}_3}{R}v_b\tilde{\theta}_b + \frac{\bar{\mu}}{R}\left(\theta_{b,x}\tilde{\theta}_b - \theta_b\tilde{\theta}_{b,x}\right) + \frac{\bar{\mu}_x}{R}\theta_b\tilde{\theta}_b - \sigma\bar{\rho}\bar{U}\theta_b\tilde{\theta}_b\ ,$$

$$K_2 = v_b\tilde{p}_b + \frac{\bar{V}}{\bar{P}}p_b\tilde{p}_b - \frac{\bar{V}}{\bar{T}}\theta_b\tilde{p}_b + \frac{s\bar{\mu}}{R}v_{b,x}\tilde{u}_b - \bar{\rho}\bar{V}\mathbf{v}_{b,2}\cdot\tilde{\mathbf{v}}_b + \frac{\bar{\mu}}{R}\left(\mathbf{v}_{b,y}\cdot\tilde{\mathbf{v}}_b - \mathbf{v}_b\cdot\tilde{\mathbf{v}}_{b,y}\right) + \frac{\bar{\mu}_T\mathcal{A}_3}{R}\theta_b\tilde{u}_b + \frac{m\bar{\mu}_x}{R}v_b\tilde{u}_b + \frac{s\bar{\mu}}{dR}u_{b,x}\tilde{v}_b + \frac{\bar{\mu}_x}{dR}u_b\tilde{v}_b + \frac{\bar{\mu}_T\mathcal{A}_6}{dR}\theta_b\tilde{v}_b - \frac{1}{d}p_b\tilde{v}_b + \frac{i\beta s\bar{\mu}}{dR}w_b\tilde{v}_b + \frac{i\beta s\bar{\mu}}{R}v_b\tilde{w}_b + \Gamma\bar{V}p_b\tilde{\theta}_b - \frac{d\Gamma\bar{\mu}\mathcal{A}_9}{R}v_b\tilde{\theta}_b + \frac{2\Gamma\bar{\mu}\mathcal{A}_3}{R}u_b\tilde{\theta}_b + \frac{\bar{\mu}}{R}\left(\theta_{b,y}\tilde{\theta}_b - \theta_b\tilde{\theta}_{b,y}\right) + \frac{\bar{\mu}_y}{R}\theta_b\tilde{\theta}_b - \sigma\bar{\rho}\bar{V}\theta_b\tilde{\theta}_b\ ,$$

where

$$\mathbf{v}_b = [u_b, v_b, w_b]^\top\ ,\quad \mathbf{v}_{b,1} = [u_b, v_b, dw_b]^\top\ ,\quad \mathbf{v}_{b,2} = [u_b, v_b/d, w_b]^\top\ ,$$
$$\mathbf{v}_{b,x,3} = [du_{b,x}, v_{b,x}, w_{b,x}]^\top\ ,\quad \mathbf{v}_{b,y} = [u_{b,y}, v_{b,y}, w_{b,y}]^\top\ .$$