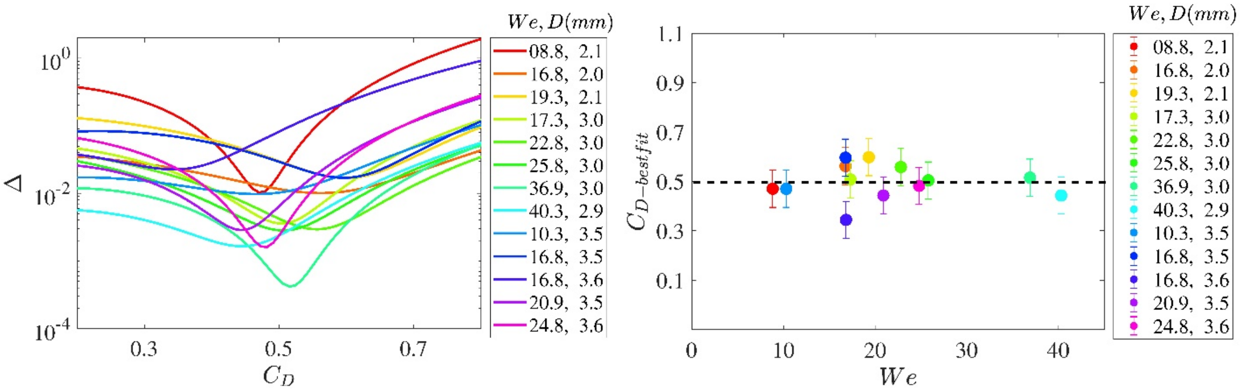
1. Evaluation of C_D from experimental data

Here, we show how we used the experimental data to estimate "best-fit" C_D value used in stage-III for each and individual experimental conditions. First, we calculated Δ, the sum of the square of the difference between the experimentally evaluated LHS and theoretically evaluated RHS of Eq. 3.2 for a range of possible C_D (Fig. S1a). Thus, $\Delta = \sum_{j=i}^{N}\left[\left(\frac{h(t)-h_0}{h_v}\right)_{exp} - \left(\ln\left[\frac{t-t_0}{t_v}+1\right]\right)_{theo}\right]^2$, N= number of data points in stage III. Subsequently, we find $(C_D)_{bestfit}$ as the C_D for which Δ is smallest (Fig. S1b). Although there is a small scatter in $(C_D)_{best-fit}$ for various experiments, most of them are within the close range of 0.5, which is the value predicted by the solid sphere model.



(a) (b)

Figure S1: (a) Error plot: Difference between theoretical and experiments values of LHS of Eq. 3.2 for different choice of C_D. (b) Best-fit C_D from the error plot.

2. Comparison of viscous and capillary terms in Stage II

Although the effect of capillarity can only be seen in Stage II, it is not necessarily stronger than the viscous effect. The relative importance of these two effects can be seen by comparing different terms of Eq. 3.3, which can be expressed in the non-dimensional form as $\frac{h(t)}{D}-\frac{1}{2}=\frac{\tau_c}{\tau_i}\frac{1}{4\pi}\sin\left[4\pi\frac{t-\tau_i}{\tau_c}\right]+\frac{h_v}{D}\ln\left[\frac{t-t_0}{\tau_v}+1\right]$. In the figure below, for various experimental conditions we compare the terms $(h_2)_I=\frac{\tau_c}{\tau_i}\frac{1}{4\pi}\sin\left[4\pi\frac{t-\tau_i}{\tau_c}\right]$ and $(h_2)_{II}=\frac{h_v}{D}\ln\left[\frac{t-t_0}{\tau_v}+1\right]$, which represent the capillary and viscous effects respectively. Clearly, the relative significance of $(h_2)_I$ and $(h_2)_{II}$ changes with We and D, and one cannot make the statement that capillary effect is stronger than viscous effect in stage II.

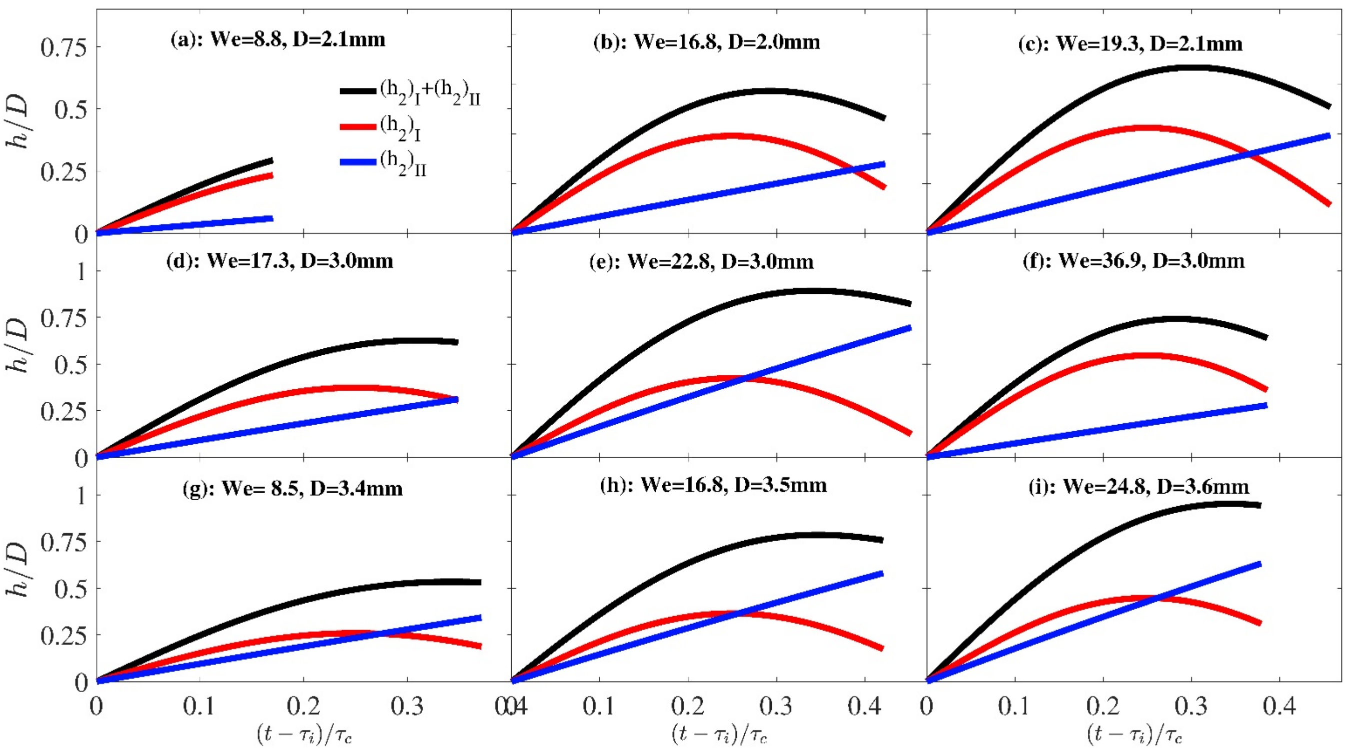


Figure S2: Relative importance of capillary and viscous terms in stage II for various experimental conditions.

3. Additional information related to Figure 6

Below are the *dh/*dt plot for set of nine experimental conditions for which *h* vs t data are plotted in Fig. 6 of the manuscript. All these experiments clearly show three stages. Stage I: constant dh/dt Stage II: oscillating dh/dt and Stage III: decaying dh/dt.

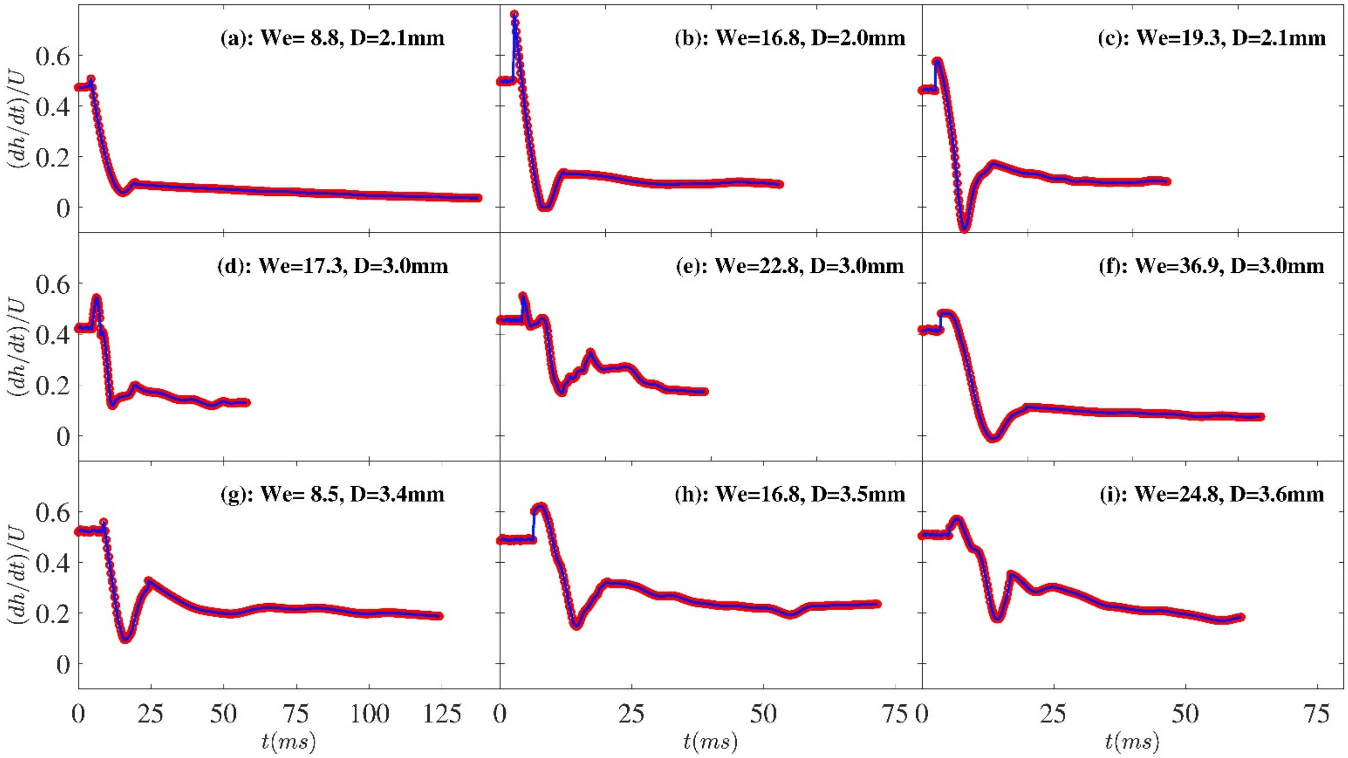
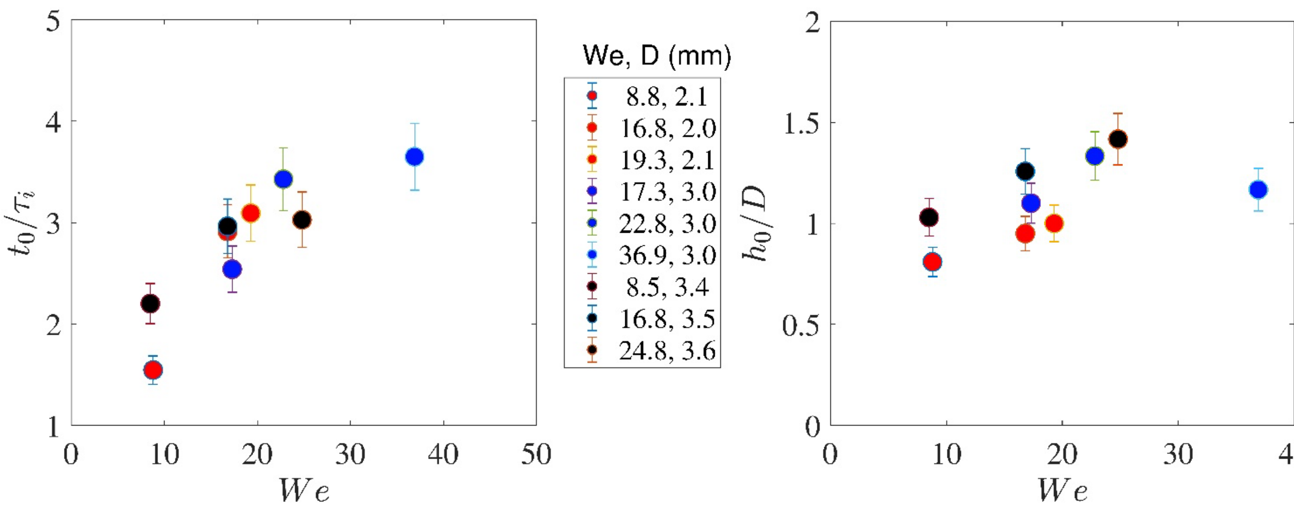


Figure S3: Normalized penetration velocity for various experimental conditions.

Below we also report the corresponding values of t_0 and h_0.



(a) (b)

Figure S4: Normalized values of (a) t_0 and (b)h_0 for the nine experimental conditions reported in Fig. 6.

<u>Possibility of unified expression of viscous drag in stage -II and III:</u>

Although the drag acting onto the vortex ring is relevant during the stage II and III, they are expressed separately with different initial conditions (Eqn. 3.2 and 3.3). The unification of viscous drag terms in two stages were not possible due to the “randomness” of t_0 (time at which stage II end and stage III begins), which changes from with We and D as shown in Fig. S4. The behavior and scaling of t_0 will be a part of our future study in which we will also seek the possibility of unification of expression for drag in stages II and III.