Supplementary Material

1. Statistical analysis of power scaling laws

In log-space the scaling relationships for the eddy azimuthal velocity and eddy length scale establish a general linear system for which the scaling parameters can be estimated using ordinary linear regression or Bayesian statistical methods. The general linear system for the eddy velocity is

$$\log(\mu_U) = \beta_{0,f_U} + \beta_{h/D,f_U}\log(h/D) + \beta_{KC,f_U}\log(KC) + \beta_{\delta^+,f_U}\log(\delta^+) + \epsilon_U \tag{1.1}$$

and for the eddy length scale is

$$\log(\mu_L) = \beta_{0,L} + \beta_{h/D,f_L}\log(h/D) + \beta_{KC,f_L}\log(KC) + \beta_{\delta^+,f_L}\log(\delta^+) + \epsilon_L \tag{1.2}$$

where μ_U and μ_L are the expected value of the eddy velocity and eddy length scale and β_p, r are the scaling coefficients for p variables and r indicating the eddy length (f_L) or eddy velocity (f_U) scaling relationship. Assuming that the eddy velocity scales $U_e \sim \langle\overline{v}\rangle_{rms}$, we can use $\langle\overline{v}\rangle_{rms}$ and $\langle\overline{w}\rangle_{rms}$ as two readily measurable quantities from which to estimate the scaling coefficients of f_U and f_L.

By assuming flat (uninformed) prior distributions and normally distributed errors in the MCMC method, the MCMC and ordinary least squares methods are functionally equivalent and yield similar results (Table 1). An informative graphical representation of the MCMC coefficient estimates and relative size of confidence intervals is presented in a Forest plot (figure 1) which shows the mean, inter-quartile range and 95% confidence intervals of the posterior probability distributions. This highlights small effect sizes for h/D and KC in f_U and h/D in f_L. This model has a coefficient of determination of $r^2 = 0.88$ for both $\langle\overline{v}\rangle_{rms}/U_0$ and $\langle\overline{w}\rangle_{rms}/U_0$, however with large confidence intervals for h/D, δ^+ and the intercepts.

An advantage of the Bayesian statistical approach implemented with the MCMC method is the ability to examine the joint probability of the posterior distributions of the scaling coefficients. The joint-marginal distributions provide insight into the structure and potential correlation between scaling coefficient estimates. In addition to the small effect size of h/D (figure 1), there is significant correlation between the scaling coefficient for h/D and δ^+ in f_U and f_L (figure 2). This is not surprising given that the flow depth h is present in both δ^+ and h/D. Due to the low effect size of h/D and correlation with δ^+, it is justified to remove dependence on h/D from the models of f_U and f_L. Removing dependence on h/D significantly narrows the confidence intervals for the remaining coefficients (figure 3) without reducing the predictive capacity of the model, $r^2 = 0.88$ (figure 4).

The influence of KC on $\langle v\rangle_{rms}/U_0$ and of δ^+ and KC on $\langle w\rangle_{rms}/U_0$ remains weak and to first-order $\langle v\rangle_{rms}/U_0 \sim \delta^{+-2/3}$ and $\langle w\rangle_{rms}/U_0 \sim h/D$ (as would be predicted based on conservation of mass arguments). The tidally-forced, shallow island wake has weak non-linearity (with respect to upwelling and lateral velocity fluctuations). However, there is sufficient non-linearity to establish a range of wake forms (symmetric through vortex shedding) with a variety of complex flow structures that influences the spatial distribution of upwelling significantly.

	OLS			HMC		
	95% LCL	MLE	95% UCL	95% LCL	Mean	95% UCL
β_{0,f_U}	-0.83	-0.75	-0.66	-0.85	-0.75	-0.65
$\beta_{h/D,f_U}$	-0.16	0.01	0.18	-0.18	0.01	0.20
β_{KC,f_U}	0.02	0.07	0.12	0.02	0.07	0.13
$\beta_{\delta+,f_U}$	-0.87	-0.67	-0.47	-0.90	-0.67	-0.45
β_{0,f_L}	0.93	1.22	1.50	0.90	1.21	1.54
$\beta_{h/D,f_L}$	-0.42	0.16	0.74	-0.50	0.16	0.79
β_{KC,f_L}	-1.30	-1.14	-0.97	-1.33	-1.14	-0.96
$\beta_{\delta+,f_L}$	0.47	1.14	1.81	0.37	1.14	1.88

TABLE 1. Estimates of scaling coefficients of f_U and f_L. Maximum likelihood estimate (MLE) (from ordinary least squares - OLS) and the zeroth moment of the posterior probability distribution (from Hamiltonian Monte Carlo - HMC). The 95% lower confidence level (LCL) and 95% upper confidence level of the coefficient estimates. This model has a coefficient of determination of $r^2 = 0.88$ for both $\langle \overline{v} \rangle_{rms}/U_0$ and $\langle \overline{w} \rangle_{rms}/U_0$.

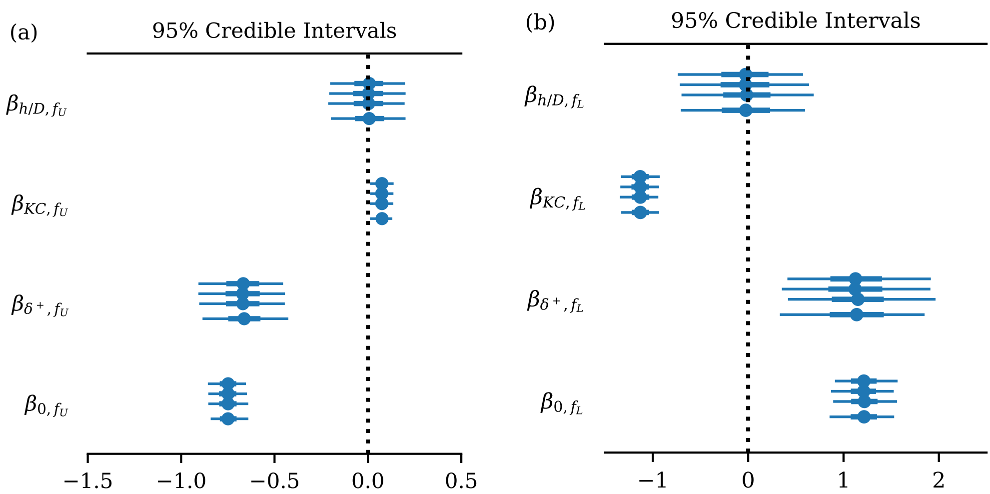

FIGURE 1. Forest plot showing mean value (dot), inter-quartile range (thick line) and 95% confidence interval of the posterior probability distributions for the scaling parameters of (a) f_U and (b) f_L

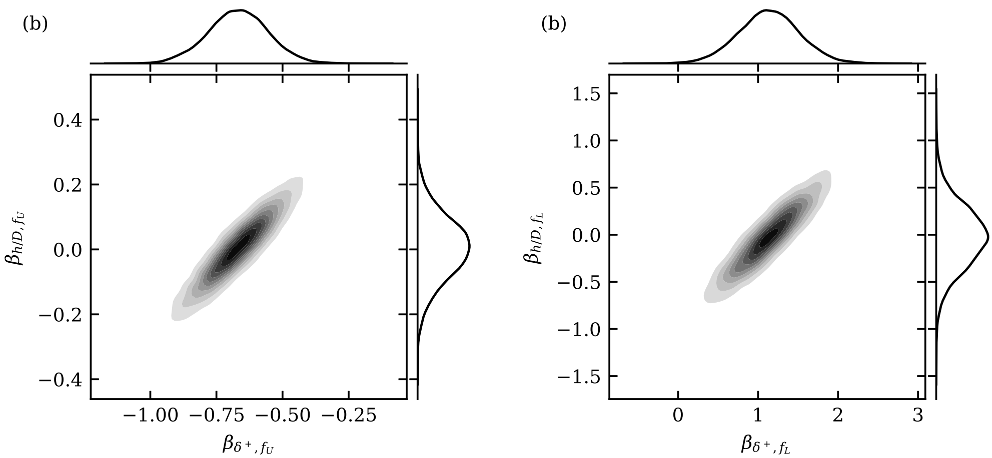

FIGURE 2. Joint posterior probability distributions between scaling parameter estimates for h/D and δ^+ in (a) f_U and (b) f_L

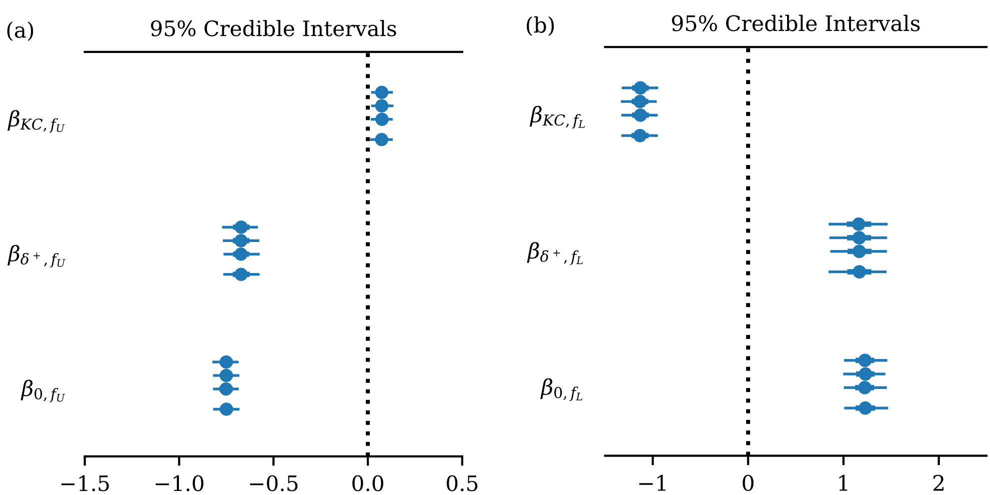

FIGURE 3. Forest plot showing mean value (dot), inter-quartile range (thick line) and 95% confidence interval of the posterior probability distributions for the scaling parameters of (a) $f_U(KC, \delta^+)$ and (b) $f_L(KC, \delta^+)$

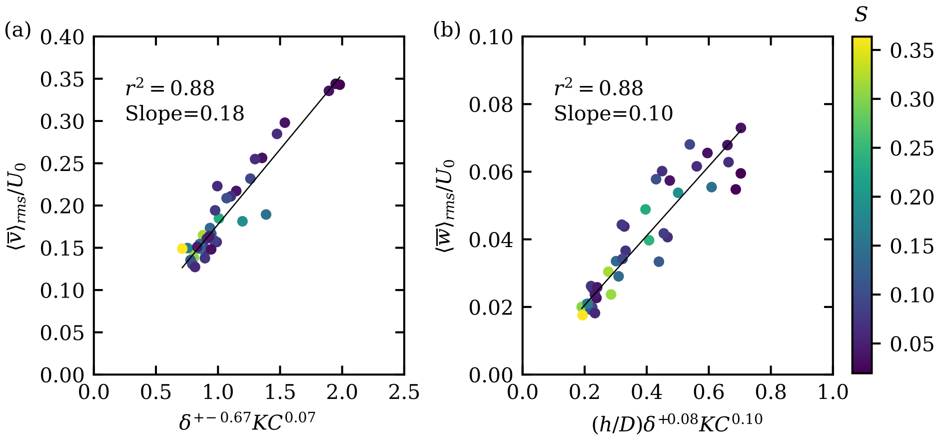

FIGURE 4. Scaling relationships for (a) $\langle \overline{v} \rangle_{rms}/U_0$ and (b) $\langle \overline{w} \rangle_{rms}/U_0$ based on the reduced parametrisation of $U_e/U_0 \sim f_U(KC, \delta^+)$ and $L_e/D \sim f_L(KC, \delta^+)$. The black line indicates a 1:1 slope.