

- Figure 1. " **ϕ steady**" : Vertical and horizontal distributions of the steady states of the geopotential $\phi(x, y, \hat{z})$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 2. " **ϕ unsteady**" : Vertical and horizontal distributions of the unsteady state (at time $t_0 = 0$) of the geopotential $\tilde{\phi}(x, y, \hat{z}, t_0)$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 3. " **ϖ steady**" : Vertical and horizontal distributions of the steady state of the potential vorticity anomaly $\varpi^q(x, y, \hat{z})$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 4. " **ϖ unsteady**" : Vertical and horizontal distributions of the unsteady state (at time $t_0 = 0$) of the potential vorticity anomaly $\tilde{\varpi}^q(x, y, \hat{z}, t_0)$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 5. " **$d \times c$ steady**" : Vertical and horizontal distributions of the steady state of density $d(x, y, \hat{z}) \times c$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 6. " **ζ^g steady**" : Vertical and horizontal distributions of the steady state of the geostrophic vertical vorticity $\zeta^g(x, y, \hat{z})$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 7. " **$\tilde{\zeta}^g$ unsteady**" : $\tilde{\zeta}^g(x, y, \hat{z}, t_0)$. Vertical and horizontal distributions of the unsteady state (at time $t_0 = 0$) of the geostrophic vertical vorticity $\tilde{\zeta}^g(x, y, \hat{z}, t_0)$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 8. " **\mathcal{S} steady**" : Vertical and horizontal distributions of the steady state of the vertical stratification $\mathcal{S}(x, y, \hat{z})$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 9. " **$\tilde{\mathcal{S}}$ unsteady**" : Vertical and horizontal distributions of the unsteady state (at time $t_0 = 0$) of the vertical stratification $\tilde{\mathcal{S}}(x, y, \hat{z}, t_0)$. Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.
- Figure 10. " **w^q steady and unsteady**" : Vertical and horizontal distributions of the steady state of the quasi-geostrophic vertical velocity (equal to the unsteady state at time $t_0 = 0$) $w^q(x, y, \hat{z}) = \tilde{w}^q(x, y, \hat{z}, t_0) (\times 10^5)$. We denote

$$\begin{aligned} w_{ij} &= \mathbf{u}_i^g \cdot \nabla_h \mathcal{D}_j, \\ w_{1221} &= w_{12} + w_{21}. \end{aligned}$$

Modal amplitudes $\{\hat{\phi}_0, \hat{\phi}_1, \hat{\phi}_2\} = \{1, 1, 1\}$ and horizontal to vertical aspect ratio $c = 100$.