Effect of polymer-stress diffusion in the numerical simulation of elastic turbulence

SUPPLEMENTARY MATERIAL

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This supplementary material consists of additional simulations of the Oldroyd-B model on the periodic square $[0, 2\pi]^2$ with a cellular forcing (see (2.1) and (2.2) in the main text). The integration method is described in § 2 of the manuscript.

Figures 1 and 2 are the analogues of figures 2 (right panel), 6 and 8 (left panel) in the main text for $\tau = 40$, $\mu = 0.02$ and same K, f_0, ν as in § 2. In particular, the Deborah number is now $De = 8$ and the coupling coefficient μ is doubled.

Figures 3 and 4 are the analogues of figures 2 (right panel), 6 and 8 (left panel) in the main text for $K = 4$, $\tau = 10$, $f_0 = 0.16$ and same ν and μ as in § 2. The Deborah number is $De = 8$ and the spatial frequency of the forcing is doubled compared to the main text.

These additional simulations support the validity of the conclusions reported in the manuscript.

Finally, figure 5 shows the quantity $d^2(\mathsf{I}, \mathsf{C}) = \mathrm{tr}\log^2(\mathsf{C})$, which represents the geodesic distance between the conformation tensor and the identity tensor in the space of positive-definite tensors [see Hameduddin *et al.*, *J. Fluid Mech.* **842**, 395 (2018)] for the same parameters as in § 3 of the main text.

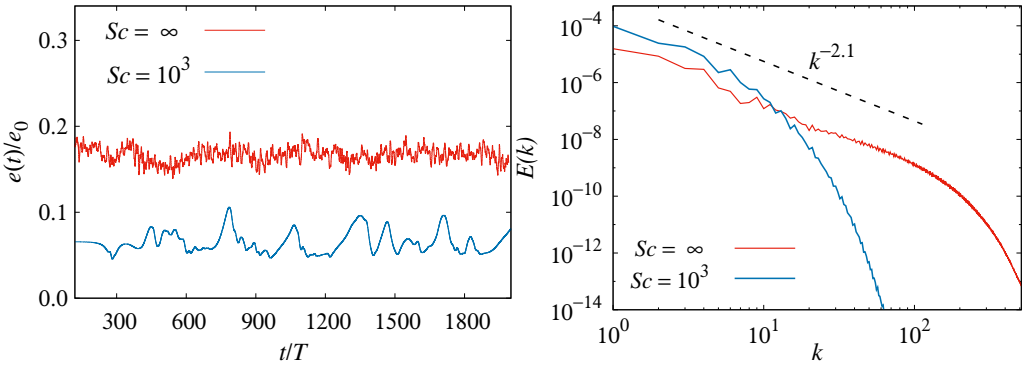

Fig. 1. Time series of the rescaled kinetic energy in the steady state (left) and kinetic-energy spectra (right) for the Oldroyd-B model with $\nu = 0.05$, $f_0 = 0.02$, $K = 2$, $De = 8$, $\mu = 0.02$.

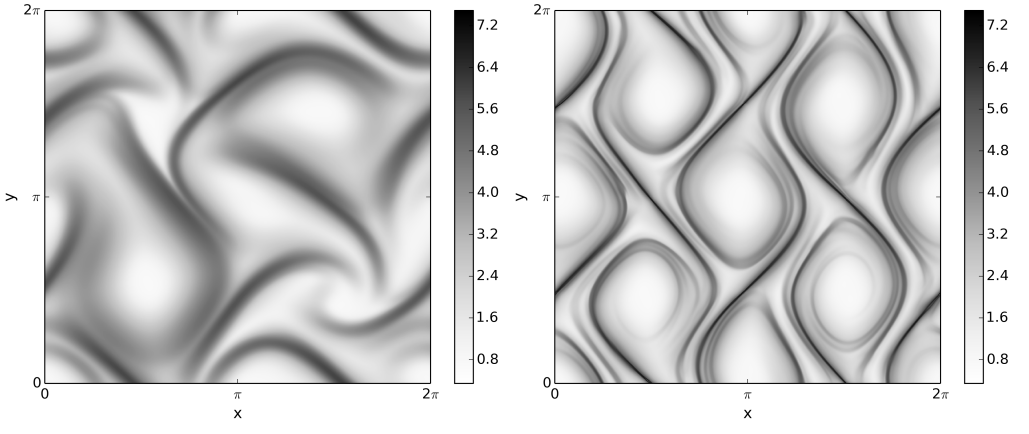

Fig. 2. Representative snapshots of $\ln(\mathrm{tr}\,\boldsymbol{C})$ in the steady state for the Oldroyd-B model with $\nu = 0.05$, $f_0 = 0.02$, $K = 2$, $De = 8$, $\mu = 0.02$ and $Sc = 10^3$ (left) and $Sc = \infty$ (right).

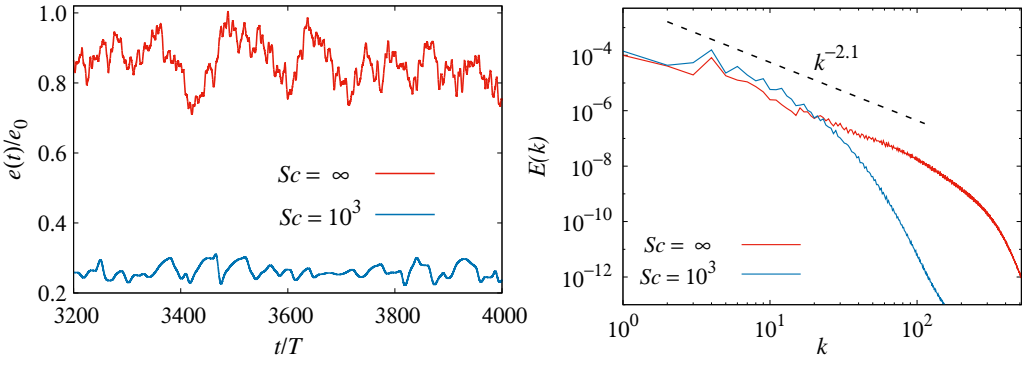

Fig. 3. Time series of the rescaled kinetic energy in the steady state (left) and kinetic-energy spectra (right) for the Oldroyd-B model with $\nu = 0.05$, $f_0 = 0.16$, $K = 4$, $De = 8$, $\mu = 0.01$.

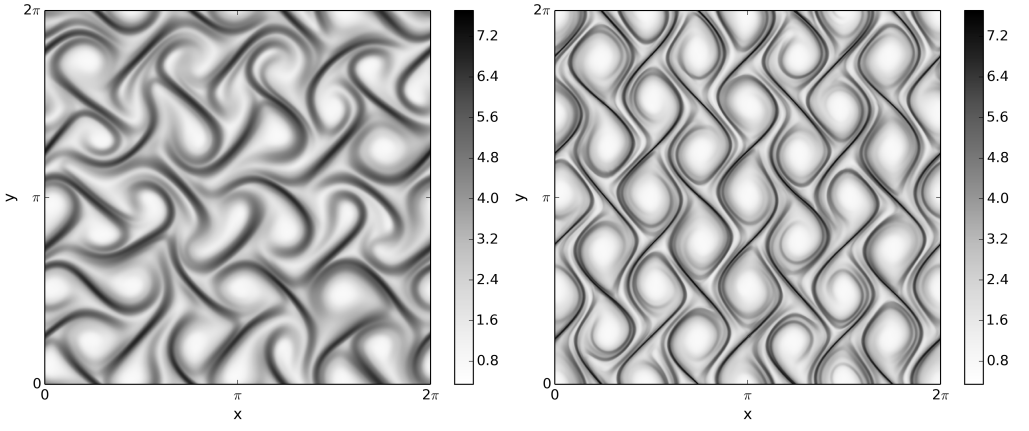

Fig. 4. Representative snapshots of $\ln(\mathrm{tr}\,\boldsymbol{C})$ in the steady state for the Oldroyd-B model with $\nu = 0.05$, $f_0 = 0.16$, $K = 4$, $De = 8$, $\mu = 0.01$ and $Sc = 10^3$ (left) and $Sc = \infty$ (right).

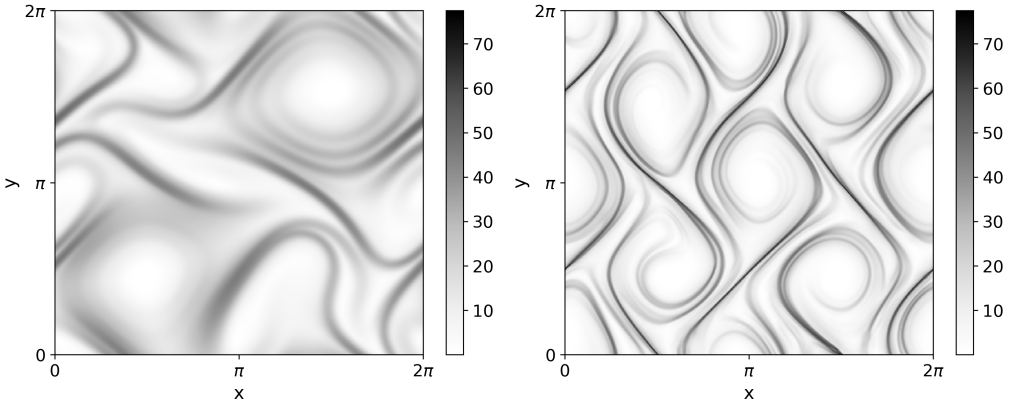

Fig. 5. Snapshots of $d^2(\mathbf{I}, \mathbf{C})$ for $Sc = 10^3$ (left) and $Sc = \infty$ (right) and the same parameters and time instants as in figure 6 of the main text.