# Electrocoalescence of a pair of conducting drops in an insulating oil

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## Supplementary Information

Two uncharged drops of infinitely conducting fluid of radius $a$ are suspended in a non conducting medium which is incompressible and Newtonian. The center to center distance between the drops is given by $s = 2a + h$. The drops are aligned along the direction of applied electric field ($E_0$) as shown in figure 1.

The system is non-inertial and is considered to be in Stokes' flow regime. The parameters used to characterize the system are viscosity ratio ($\lambda = \mu_i/\mu_o$), permittivity ratio ($Q = \epsilon_i/\epsilon_o$), conductivity ratio ($R = \sigma_o/\sigma_i$) and the electrocapillary number ($Ca = \frac{a\epsilon\epsilon_o E_0^2}{\gamma}$). Taylor's leaky dielectric theory is used to model the system (Saville, 1997). The electric current in the system can be neglected and therefore the magnetic field is also considered to be zero. Thus the electric field is irrotational ($E = -\nabla\phi$) modeling using electrostatics and get,

$$\nabla \cdot \mathbf{E_{i,o}} = 0 \tag{1}$$

or

$$\nabla^2 \phi_{i,o} = 0 \tag{2}$$

where $\phi$ is the electric potential, and $i$ and $o$ are the drop and medium phases respectively.

The boundary conditions at the fluid interfaces are

$$\mathbf{n} \cdot \mathbf{E_o} = R\mathbf{n} \cdot \mathbf{E_i} \;\; \text{(current continuity, } n \text{ is the outward unit normal)} \tag{3}$$

and

$$\mathbf{t} \cdot \mathbf{E_d} = \mathbf{t} \cdot \mathbf{E_m} \;\; \text{(Continuity of tangential electric field, } t \text{ is the unit tangent)} \tag{4}$$

Maxwell stresses at the interface is given by

$$\tau^E_{i,o} = \epsilon_{i,o}(\mathbf{EE} - \frac{1}{2}E^2\mathbf{I}) \tag{5}$$

Therefore fluid velocity $\mathbf{u}$ is given by Stokes and continuity equation as

$$\mu\nabla^2\mathbf{u} - \nabla p = \nabla \cdot \tau^H \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0 \tag{6}$$

where $p$ is the pressure and $\tau^{\mathbf{H}}$ is the hydrodynamic stress. At the interface continuity of velocity is applicable and the interfacial tension balances the stress jump and is written as

$$[[\mathbf{n} \cdot \tau^{\mathbf{H}}]] = \gamma\kappa\mathbf{n} - [[\mathbf{n} \cdot \tau^{\mathbf{E}}]] \tag{7}$$

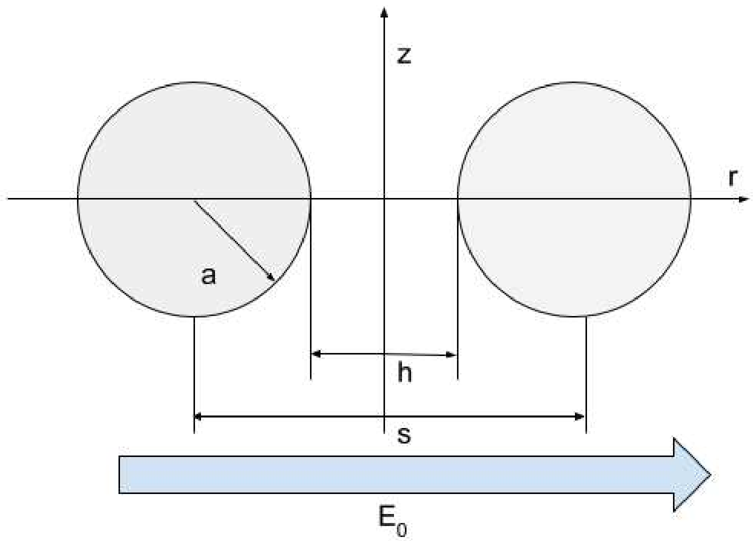


Figure 1: Schematic representation of similar sized drops under electric field $E_0$.

where $\kappa = ((\mathbf{I} - \mathbf{nn}) \cdot \nabla) \cdot \mathbf{n}$ is the mean curvature of the interface and $[[.]]$ denotes jump in stresses across the interface.

Gauss divergence theorem can be used to convert all the volume integrals to surface integrals and reduce the model from three dimensions to two dimensions and since we assume the system to be axisymmetric we can carry out our calculations on one half of the two dimensional arc since analytical integration in azimuthal direction is enough to calculate values on the whole two dimensional arc. Now the one half arc is discretized into N+1 nodes and using the arc length ($c$) as a parameter cubic spline interpolation is used to describe the node points in cylindrical coordinate system.

We can calculate the normal electric field using the following integral equation (Sherwood, 1988; Baygent et al., 1998),

$$E_{nm}(x_0) = \frac{2}{1+R}\mathbf{n}(\mathbf{x_0}) \cdot \mathbf{E_0}(\mathbf{x_0}) + \frac{1-R}{2\pi(1+R)}\mathbf{n}(\mathbf{x_0}) \cdot \sum_{i=1}^{2} \int_{\Lambda_i} \frac{\mathbf{x_0} - \mathbf{x}}{(|\mathbf{x_0} - \mathbf{x}|)^{\mathbf{3}}} E_{ni}(\mathbf{x}) dc(\mathbf{x}) \tag{8}$$

where $\mathbf{x_0}$ is the position vector of the point on the interface and m is the counter for two drops. If we know the electric field, then the potential is given by

$$\phi_m(\mathbf{x_0}) = \phi_m(\mathbf{x_0}) - (\frac{1-R}{4\pi}) \sum_{i=1}^{2} \int_{\Lambda_i} \frac{E_{ni}(\mathbf{x})}{|\mathbf{x_0} - \mathbf{x}|} dc(\mathbf{x}) \tag{9}$$

where $\phi_0$ is the applied potential. The instantaneous distribution of non dimensional velocity over the interface at any point $x_0$ is given by Fredholm integral equation of second kind, (Pozrikidis, 1992; Pozrikidis, 2001)

$$\mathbf{u_m}(\mathbf{x_0}) = -\frac{1}{4\pi(1+\lambda)} \sum_{i=1}^{2} \int_{\Lambda_i} \mathbf{\Delta f_i}(\mathbf{x}) \cdot \mathbf{G_i}(\mathbf{x}, \mathbf{x_0}) dc(\mathbf{x})$$
$$+ \frac{1-\lambda}{4\pi(1+\lambda)} \sum_{i=1}^{2} \int_{\Lambda_i}^{PV} \mathbf{n_i}(\mathbf{x}) \cdot \mathbf{T_i}(\mathbf{x}, \mathbf{x_0}) \cdot \mathbf{u_i}(\mathbf{x}) dc(\mathbf{x}) \tag{10}$$

where $PV$ is the principal value integral pertaining to the drop over which we are integrating and G and T are known kernels of velocity and stress field respectively. The force can be calculated from equation (7) and $[[\mathbf{n} \cdot \tau^{\mathbf{E}}]]$ is given by

$$[[\mathbf{n} \cdot \tau^{\mathbf{E}}]] = \frac{\epsilon_m}{2}[E_{nm}^2(1 - RQ^2) + E_{tm}^2(1 - \frac{1}{Q})]\mathbf{n} + \epsilon_m E_{nm} E_{tm}(1 - RQ)t \tag{11}$$

To simulate perfectly conducting drops in perfectly dielectric system conductivity ratio $R$ is taken as 0.001 meaning drop conductivity is 1000 times that of oil medium. This would effectively mean that the leaky dielectric model behaves as perfect conductor in perfect dielectric system. Once the instantaneous velocity for every node point is known to us we can advance the interface using Euler method for each time step $\delta t$

$$\mathbf{x_0^{t+\delta t}} = \mathbf{x_0^t} + \mathbf{u}(\mathbf{x_c})\delta t \tag{12}$$

Adaptive time step was used by introducing Courant–Friedrichs–Lewy (CFL) condition as

$$C = \frac{u\delta t}{\delta x} \tag{13}$$

where $C$ is the Courant number, $u$ is the largest magnitude of the velocity of the nodes (typically the polar nodes of the drops approaching each other), $\delta t$ is the time step, and $\delta x$ is the distance between the polar node and the node adjacent to it.

So as the velocity of the nodes of drops approaching each other, increases, the $\delta t$ would decrease thus letting us capture the ultra fast dynamics at play. It was also made sure that the minimum separation between the two drops is always greater than the nodal distances.

# References


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