Supplementary Material: An information-theoretic approach to study fluid-structure interactions

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The Supplementary Material is organized as follows. First, we present detailed statistics of the robustness analyses performed in Sec. 5.3 of the main document. We then present results on an additional experimental condition in which neither airfoil is actuated. Finally, we illustrate further experimental results to exclude the possibility of spurious mechanical coupling between the airfoils due to mechanical vibration of the facility. Throughout the statistical analysis, we adjust all the p-values by Bonferroni correction to correct for multiple comparisons; different from the main document, we omit the word "adjusted."

Detailed statistics on the robustness analysis

Transfer entropy computations at different downsampling resolutions for binary symbols

Here, we present details of the statistical analysis for $\mathrm{TE}_{U\to D}$ and $\mathrm{TE}_{D\to U}$ for both experimental conditions with a time step between actuation events $\Delta t_{\mathrm{act}} = 0.5$ s, using binary symbols ($m = 2$) and varying the resolution as $f = 9$, 11, and 12.

Through paired t-tests, we find that $\mathrm{TE}_{U\to D} > \mathrm{TE}_{D\to U}$ in Upstream Active (adjusted $p < 0.05$ in paired t-tests, except for $f = 9$, $L/c = 5$, when $p > 0.99$; and $f = 11$, $L/c = 5$, when $p = 0.10$), and $\mathrm{TE}_{U\to D} < \mathrm{TE}_{D\to U}$ in Downstream Active ($p < 0.05$ in paired t-tests, except for $f = 9$, $L/c = 0.5$, 1, 5; $f = 11$, $L/c = 0.2$, 0.5 and 5; $f = 12$, $L/c = 0.2$, 0.5, 1, and 5 with $p > 0.05$ in paired t-tests, and $f = 9$, $L/c = 0.2$, when $\mathrm{TE}_{U\to D} > \mathrm{TE}_{D\to U}$ with $p < 0.01$ in paired t-tests).

Linear regression tests indicate that $\mathrm{TE}_{U\to D}$ for Upstream Active and $\mathrm{TE}_{D\to U}$ for Downstream Active are negatively dependent on L/c (Upstream Active: $f = 9$: $\mathrm{TE}_{U\to D} = -0.0275L/c + 0.1232$; $f = 11$: $\mathrm{TE}_{U\to D} = -0.0318L/c + 0.1436$; and $f = 12$: $\mathrm{TE}_{U\to D} = -0.0299L/c + 0.1437$; $p < 0.01$ for t-statistics on slope. Downstream Active: $f = 9$: $\mathrm{TE}_{D\to U} = -0.00300L/c + 0.0192$; and $f = 11$: $\mathrm{TE}_{D\to U} = -0.00463L/c + 0.0344$; $p < 0.01$ for t-statistics on slope). The only exception is Downstream Active at $f = 12$, when no dependence of $\mathrm{TE}_{D\to U}$ on L/c is detected, with $p = 0.70$ for t-statistics on slope.

Through two-sample t-tests, we compare the values of $\mathrm{TE}_{U\to D}$ in Upstream Active and $\mathrm{TE}_{D\to U}$ in Downstream Active for all values of L/c and $f = 9$, 11, and 12. We find that $\mathrm{TE}_{U\to D}$ in Upstream Active is significantly higher than $\mathrm{TE}_{D\to U}$ in Downstream Active for $f = 9$, $L/c \leq 2$; $f = 11$, $L/c \leq 1$; and $f = 12$, $L/c \leq 1$ ($p < 0.01$ in two-sample t-tests). No difference is found between $\mathrm{TE}_{U\to D}$ in Upstream Active and $\mathrm{TE}_{D\to U}$ in Downstream Active for $f = 9$, $L/c = 5$ ($p > 0.99$ in two-sample t-tests); $f = 11$, $L/c > 1$ ($p > 0.99$ at $L/c = 2$ and 5 in two-sample t-tests); and $f = 12$, $L/c > 1$ ($p = 0.22$ at $L/c = 2$ and $p > 0.99$ at $L/c = 5$ in two-sample t-tests).

The dependence of the delay $T(\delta_c)$ on L/c for $f = 9$, 11, and 12 is examined through linear regression fits. In Upstream Active, we find a marked positive dependence of the delay on the separation distance for all the tested values of f ($f = 9$: $T(\delta_c)/T_0 = 0.148L/c + 0.426$; $f = 11$: $T(\delta_c)/T_0 = 0.312L/c + 0.261$; $f = 12$: $T(\delta_c)/T_0 = 0.253L/c + 0.379$; $p < 0.01$ for t-statistic on the slope). In Downstream Active, no significant dependence of $T(\delta_c)$ on L/c can be established (f = 11: $T(\delta_c)/T_0 = 0.0206L/c + 0.432$, p = 0.72 for t-statistic on the slope; and f = 12: $T(\delta_c)/T_0 = 0.00804L/c + 0.443$, p = 0.91 for t-statistic on the slope) except for f = 9 ($T(\delta_c)/T_0 = 0.293L/c + 0.274$, $p < 0.01$ for t-statistic on the slope).

Transfer entropy computations at different downsampling resolutions for six symbols

Here, we present details of the statistical analysis for $\mathrm{TE}_{U\to D}$ and $\mathrm{TE}_{D\to U}$ for both experimental conditions with a time step between actuation events $\Delta t_{\mathrm{act}} = 0.5$ s, using six symbols (m = 3) and varying the resolution as $5 \leq f \leq 8$.

Through paired t-tests, we find that $\mathrm{TE}_{U\to D} > \mathrm{TE}_{D\to U}$ in Upstream Active ($p < 0.01$ in paired t-tests, except for f = 6, L/c = 5, when p = 0.14, f = 8, L/c = 5, when p = 0.31) and $\mathrm{TE}_{U\to D} < \mathrm{TE}_{D\to U}$ in Downstream Active ($p < 0.05$ in paired t-tests, except for f = 5, L/c = 0.5, when p = 0.24; f = 6, L/c = 5, when $p > 0.99$; and f = 8, L/c = 0.2, when p = 0.20, L/c = 0.5, when p = 0.12, and L/c = 5, when $p > 0.99$).

Linear regression tests confirm that $\mathrm{TE}_{U\to D}$ in Upstream Active and $\mathrm{TE}_{D\to U}$ in Downstream Active conditions are negatively correlated to L/c (Upstream Active: f = 5: $\mathrm{TE}_{U\to D} = -0.0242L/c + 0.142$; f = 6: $\mathrm{TE}_{U\to D} = -0.0337L/c + 0.187$; f = 7: $\mathrm{TE}_{U\to D} = -0.0238L/c + 0.171$; and f = 8: $\mathrm{TE}_{U\to D} = -0.0339L/c + 0.207$; $p < 0.01$ for t-statistics on slope and intercept. Downstream Active: f = 5: $\mathrm{TE}_{D\to U} = -0.0210L/c + 0.151$; f = 6: $\mathrm{TE}_{D\to U} = -0.0239L/c + 0.160$; f = 7: $\mathrm{TE}_{D\to U} = -0.0197L/c + 0.156$; and f = 8: $\mathrm{TE}_{D\to U} = -0.0131L/c + 0.123$; $p < 0.01$ for t-statistics on slope and intercept).

Through two-sample t-tests, we compare the values of $\mathrm{TE}_{U\to D}$ in Upstream Active and $\mathrm{TE}_{D\to U}$ in Downstream Active for all values of L/c within the range $5 \leq f \leq 8$. We find that values of $\mathrm{TE}_{U\to D}$ in Upstream Active are significantly higher than $\mathrm{TE}_{D\to U}$ in Downstream Active for f = 6, $L/c \leq 0.5$; f = 7, L/c = 0.5; and f = 8, $L/c \leq 0.5$ ($p < 0.05$ in two-sample t-tests). No difference is detected between $\mathrm{TE}_{U\to D}$ in Upstream Active and $\mathrm{TE}_{D\to U}$ in Downstream Active for f = 5, L/c = 0.2 (p = 0.23 in two-sample t-tests), L/c = 0.5 (p = 0.058 in two-sample t-tests), and L/c = 5 ($p > 0.99$ in two-sample t-tests); f = 6, L/c = 1 (p = 0.12 in two-sample t-tests) and L/c = 5 (p = 0.78 in two-sample t-tests); f = 7, L/c = 0.2 (p = 0.52 in two-sample t-tests), L/c = 2 (p = 0.44 in two-sample t-tests), and L/c = 5 ($p > 0.99$ in two-sample t-tests); and f = 8, $L/c > 0.5$ ($p > 0.99$ at L/c = 1, p = 0.61 at L/c = 2, and $p > 0.99$ at L/c = 5 in two-sample t-tests). We also find that values of $\mathrm{TE}_{D\to U}$ in Downstream Active are significantly higher than $\mathrm{TE}_{U\to D}$ in Upstream Active for f=5, L/c =1 and 2; f=6, L/c = 2; and f=7, L/c =1 ($p < 0.05$ in two-sample t-tests).

The dependence of the delay $T(\delta_c)$ on L/c for $5 \leq f \leq 8$ is examined through linear regression fits. For condition Upstream Active, we find a marked positive dependence of the delay on the separation distance for all f tested (f = 5: $T(\delta_c)/T_0 = 0.141L/c + 0.385$; f = 6: $T(\delta_c)/T_0 = 0.247L/c + 0.348$; f = 7: $T(\delta_c)/T_0 = 0.255L/c + 0.332$; f = 8: $T(\delta_c)/T_0 = 0.236L/c + 0.290$; $p < 0.01$ for t-statistic on the slope). For condition Downstream Active, no significant dependence of $T(\delta_c)$ on L/c can be established for f = 6 and 7 (f = 6: $T(\delta_c)/T_0 = 0.0225L/c + 0.267$, p = 0.64 for t-statistic on the slope; and f = 7: $T(\delta_c)/T_0 = -0.0851L/c + 0.352$, p = 0.07 for t-statistic on the slope). We find a significant negative dependence of $T(\delta_c)$ on L/c for f = 5 and 8 (f = 5: $T(\delta_c)/T_0 = -0.145L/c + 0.337$, $p < 0.05$ for t-statistic on the slope; and f = 8: $T(\delta_c)/T_0 = -0.273L/c + 0.745$, $p < 0.05$ for t-statistic on the slope).

Transfer entropy computations for independent experiments with different time intervals between airfoil actuation events

Here, we present details of the statistical analysis for $\mathrm{TE}_{U\to D}$ and $\mathrm{TE}_{D\to U}$ for both experimental conditions with larger time interval between actuation events, $\Delta t_{\mathrm{act}}/T_0 = 0.6$ and $\Delta t_{\mathrm{act}}/T_0 = 1.2$, using binary symbols ($m$ = 2) and f = 10.

For $\Delta t_{\mathrm{act}}/T_0 = 0.6$ and $\Delta t_{\mathrm{act}}/T_0 = 1.2$, we find that $\mathrm{TE}_{U\to D} > \mathrm{TE}_{D\to U}$ for Upstream Active ($p < 0.05$ in paired t-tests for all separation distances except for $\Delta t_{\mathrm{act}}/T_0 = 0.6$, L/c = 5, when $p > 0.99$; $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 0.2, when p = 0.47; $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 0.5, when p = 0.051; $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 1, when p = 0.29; $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 2, when p = 0.36; and $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 5, when p = 0.16), and $\mathrm{TE}_{U\to D} < \mathrm{TE}_{D\to U}$ for Downstream Active for $\Delta t_{\mathrm{act}}/T_0 = 0.6$, L/c = 0.2 ($p < 0.01$ in paired t-tests; for $\Delta t_{\mathrm{act}}/T_0 = 0.6$, L/c = 0.5, p = 0.61; $\Delta t_{\mathrm{act}}/T_0 = 0.6$, L/c = 1, p = 0.12; $\Delta t_{\mathrm{act}}/T_0 = 0.6$, L/c = 2 and 5, $p > 0.99$; and $\Delta t_{\mathrm{act}}/T_0 = 1.2$ at all separation distances, when $p > 0.99$).

For $\Delta t_{\mathrm{act}}/T_0 = 0.6$ and $\Delta t_{\mathrm{act}}/T_0 = 1.2$, linear regression fits confirm that $\mathrm{TE}_{U\to D}$ for Upstream Active and $\mathrm{TE}_{D\to U}$ for Downstream Active are negatively dependent on L/c (Upstream Active: $\Delta t_{\mathrm{act}}/T_0 = 0.6$: $\mathrm{TE}_{U\to D} = -0.0131L/c + 0.0668$; $\Delta t_{\mathrm{act}}/T_0 = 1.2$: $\mathrm{TE}_{U\to D} = -0.00225L/c + 0.0179$; $p < 0.05$ for t-statistics on slope. Downstream Active: $\Delta t_{\mathrm{act}}/T_0 = 0.6$: $\mathrm{TE}_{D\to U} = -0.00281L/c + 0.0184$; $p < 0.01$ for t-statistics on slope), although we lose significance in the dependence of Downstream Active on $\Delta t_{\mathrm{act}}/T_0 = 1.2$ ($\mathrm{TE}_{D\to U} = 0.000230L/c + 0.00370$, p = 0.38 for t-statistics on slope).

Through two-sample t-tests, we compare the values of $\mathrm{TE}_{U\to D}$ in Upstream Active and $\mathrm{TE}_{D\to U}$ in Downstream Active for $\Delta t_{\mathrm{act}}/T_0 = 0.6$ and $\Delta t_{\mathrm{act}}/T_0 = 1.2$ at all L/c. Statistical analysis reveals that values of $\mathrm{TE}_{U\to D}$ in Upstream Active are significantly higher than $\mathrm{TE}_{D\to U}$ in Downstream Active for $\Delta t_{\mathrm{act}}/T_0 = 0.6$ and $\Delta t_{\mathrm{act}}/T_0 = 1.2$ ($p < 0.01$ in two-sample t-tests at all separation distances, except for $\Delta t_{\mathrm{act}}/T_0 = 0.6$, L/c = 5, when $p > 0.99$; $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 1, when p = 0.090; $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 2, when p = 0.48; and $\Delta t_{\mathrm{act}}/T_0 = 1.2$, L/c = 5, when p = 0.33).

The dependence of the delay $T(\delta_c)$ for $\Delta t_{\mathrm{act}}/T_0 = 0.6$ and $\Delta t_{\mathrm{act}}/T_0 = 1.2$ on L/c is again examined through linear regression analysis. In Upstream Active, we find a marked positive dependence of the delay on the separation distance ($\Delta t_{\mathrm{act}}/T_0 = 0.6$: $T(\delta_c)/T_0 = 0.301L/c + 0.362$; $\Delta t_{\mathrm{act}}/T_0 = 1.2$: $T(\delta_c)/T_0 = 0.217L/c + 0.390$; $p < 0.05$ for t-statistic on the slope). In Downstream Active, a dependence of $T(\delta_c)$ on L/c cannot be established ($\Delta t_{\mathrm{act}}/T_0 = 0.6$: $T(\delta_c)/T_0 = 0.0295L/c + 0.844$, p = 0.85 for t-statistic on the slope; and $\Delta t_{\mathrm{act}}/T_0 = 1.2$: $T(\delta_c)/T_0 = 0.151L/c + 1.07$, p = 0.36 for t-statistic on the slope).

Experiments when neither airfoil is actuated

In addition to the control and experimental conditions presented in the main document, we also experiment on the case in which neither airfoil is actuated. More specifically, we conduct ten trials for each of the five separation distances considered above, allowing the airfoils to freely rotate. We present below statistical details of comparisons between $\mathrm{TE}_{U\to D}$ and $\mathrm{TE}_{D\to U}$ under this condition through paired t-test. Transfer entropy is computed with m = 2, $9 \le f \le 12$ and m = 3, $5 \le f \le 8$.

For transfer entropy analysis using binary symbols (m = 2) within the range of downsampling resolution $9 \le f \le 12$ (Fig. S1), we detect higher $\mathrm{TE}_{U\to D}$ than $\mathrm{TE}_{D\to U}$ only at f = 9, L/c = 2 ($p < 0.01$ in paired t-tests). For transfer entropy analysis using m = 3 within the range of

$5 \leq f \leq 8$ (Fig. S2), we detect higher $TE_{U \to D}$ than $TE_{D \to U}$ at $f = 5$, $L/c = 2$ and $f = 7$, $L/c = 0.2$ and 5 ($p < 0.05$ in paired t-tests). From Figs. S1 and S2, we conclude that when neither airfoil is actuated, interactions between the two airfoils are negligible.

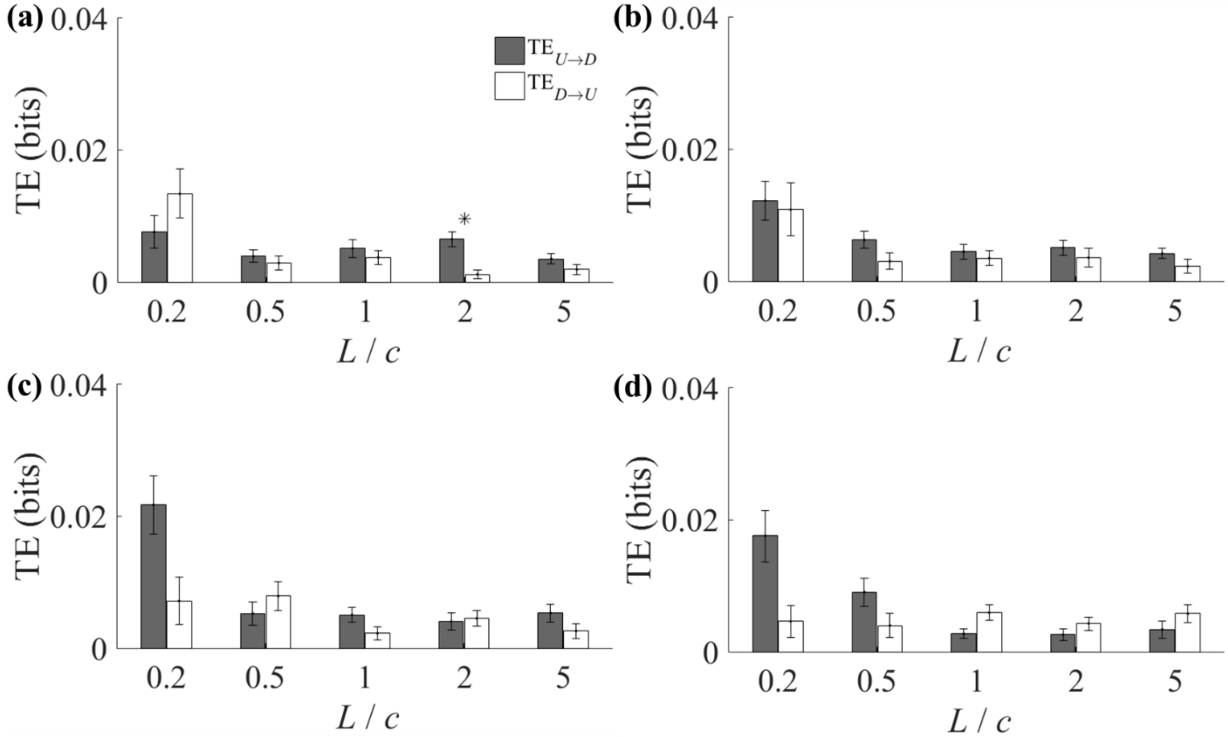

Figure S1. Transfer entropies $TE_{U \to D}$ and $TE_{D \to U}$ for the five separation distances when neither airfoil is actuated. Computations are performed with $m = 2$ and (a) f=9, (b) f=10, (c) f=11, and (d) f=12. Error bars represent standard errors, and an asterisk identifies significant difference between $TE_{U \to D}$ and $TE_{D \to U}$ from paired t-tests (adjusted $p <$ 0.05).

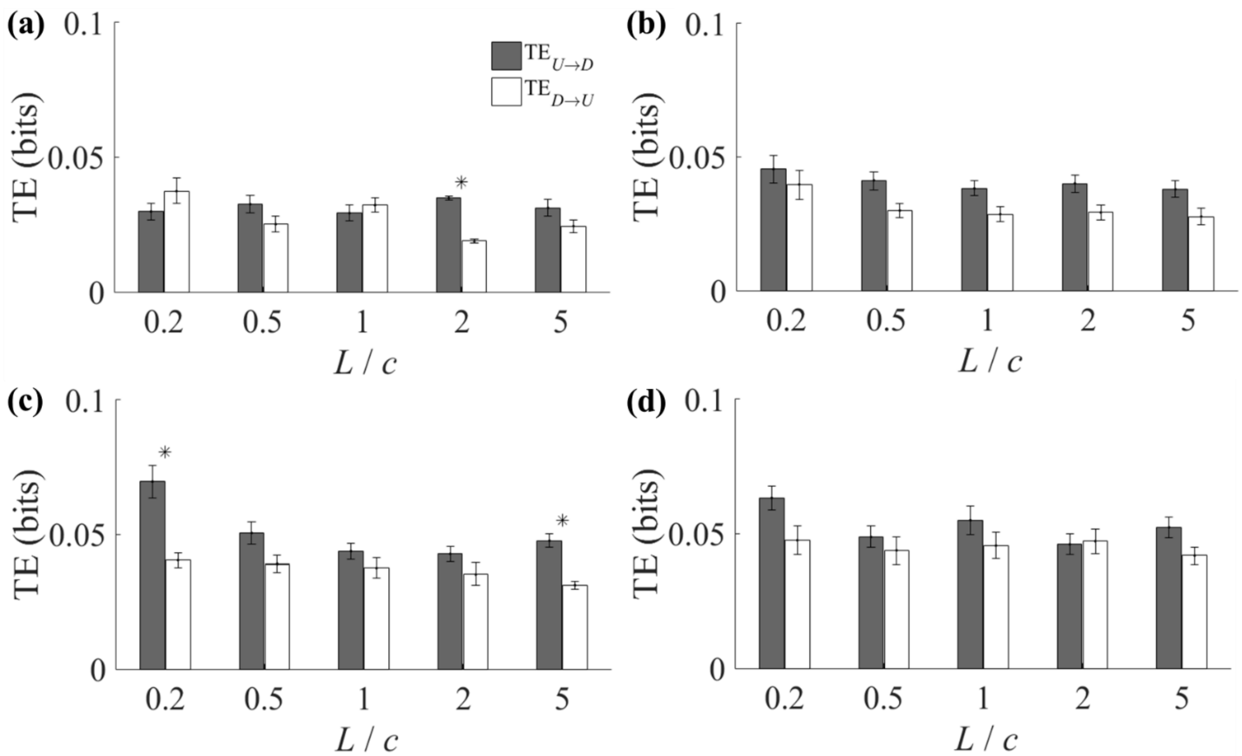

Figure S2. Transfer entropies $TE_{U\to D}$ and $TE_{D\to U}$ for the five separation distances when neither airfoil is actuated. Computations are performed with m = 3 and (a) f=5, (b) f=6, (c) f=7, and (d) f=8. Error bars represent standard errors, and an asterisk identifies significant difference between $TE_{U\to D}$ and $TE_{D\to U}$ from paired t-tests (adjusted $p <$ 0.05).

Interaction between airfoils in the absence of water

To exclude the possibility of structural coupling between the airfoils due to mechanical vibration of the experimental facility, we have performed additional trails in which there was no water in the channel. As evidenced in Fig. S3, no detectable change in the pitch angles is observed in the passive airfoil regardless of the actuation condition.

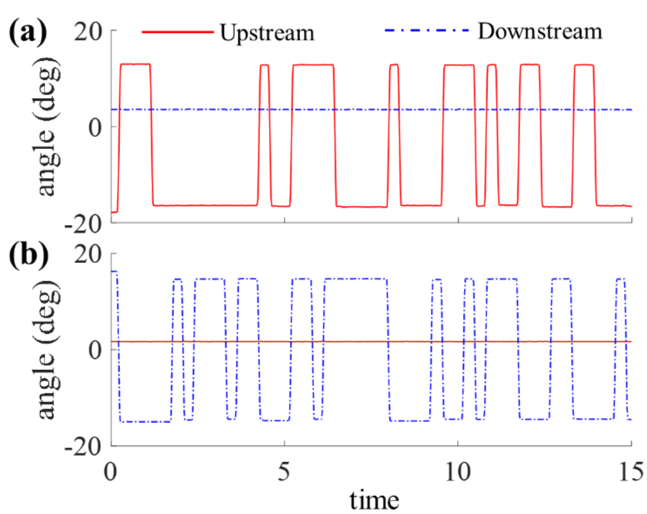

Figure S3. Segments of the time series of the pitch angles of the two airfoils for (a) Upstream Active and (b) Downstream Active at separation distance L/c = 1. Time is nondimensionalized by T_0.