Fundamentals of laminar free convection in internally-heated fluids at values of the Rayleigh-Roberts number up to 10^9

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Supplementary Material. Numerical Convergence and Precision Tests

Numerical simulations were carried out with a well-tested and benchmarked code (Tackley 1998, 2008). They are accurate, as shown by a comparison with laboratory experiments (Limare *et al.* 2015), but they also must be precise enough for scaling laws to be reliable and comparisons with other studies to be meaningful. Furthermore, many authors have investigated fluid layers with smaller aspect ratios than those of this study, so we must assess how results depend on the width of the domain.

Precision

For precise results, high numerical resolution must be achieved in both the horizontal and vertical directions. Table 1 lists results for a fixed aspect ratio and different grid spacings for both steady-state and time-dependent regimes.

For the steady-state regimes, variables reach constant values and comparisons are

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therefore straightforward. In order to properly account for the intricate geometrical patterns that emerge, which include spiky structures striking at 60° from one another, it was necessary to work in very high aspect ratio domains with a large number of grid points in the horizontal plane. It may be seen from table 1 that values for velocity and temperature vary, when changing the resolution, by less than 1% and that those for the number of axial downwellings and their horizontal cross-section differ by 5% at most. Variations of the latter two variables are anti-correlated, which is expected due to the energy constraint. One could attribute these differences to improvements of the numerical resolution, but they only affect the geometrical characteristics of the flow and we argue that they may well be an intrinsic feature of this type of convection regime in large aspect ratio chambers. For these regimes, as stated in the main text, all our simulations are initiated with random temperature fluctuations in the fluid layer. In the same conditions, laboratory experiments reveal periodic structures with defects that behave like dislocations (Whitehead Jr. 1984). These defects persist and migrate horizontally even for very long times, leading to small variations of the dominant wavenumber (Whitehead Jr. 1984). Thus, one might expect subtle differences of spatial pattern depending on the exact capture time. We attribute the slight changes of N and A_i in table 1 to such behaviour.

Determining the characteristics of time-dependent regimes is more challenging because the flow and temperature fields change at all times. These changes, however, remain confined to finite ranges that evolve as Ra_H is increased. We have sought results in what can be called "statistical steady-state" conditions, such that values remain in the same ranges over successive time intervals. Results at any given time can be considered as a sample of a larger ensemble and scaling laws were derived from relationships between mean values and Ra_H. Table 1 lists the ranges of values for the variables of interest for

$Ra_H = 10^8$ and two different numerical grids. No statistically significant change can be found. It is worth emphasizing the very small range of temperature values ($\approx 2\%$). The other variables vary within larger ranges ($\approx 15\%$), but there is no overlap for Ra_H values that are separated by one order of magnitude.

For the purposes of establishing a scaling law as a function of the Rayleigh-Roberts number, what matters is the width of the range of values in comparison to the changes that are due to an increasing Ra_H. For the worst case at hand, that of the number of downwellings, the variable varies within a $\approx 15\%$ range and changes by almost an order of magnitude as Ra_H is multiplied by 10^3.

Aspect Ratio Dependence

Numerical simulations are carried out in finite-size domains and, much like laboratory experiments, are likely to be sensitive to local flows that are generated at the side walls and at the domain corners. Table 2 shows results for different domains and the two main regimes (steady-state and time-dependent). For a meaningful comparison, one must evaluate the influence of the aspect ratio for a fixed horizontal grid spacing, implying that the number of grid points must be increased in proportion to the lateral dimensions of the domain.

For steady-state simulations at $Ra_H = 10^4$, values for temperature and velocity cannot be distinguished in any meaningful way. Results for the number and horizontal cross-section of downwellings do differ again (by about 13%) and are anti-correlated. This difference is slightly larger than that was obtained for different grid spacings at a fixed aspect ratio and may reflect a sampling artefact. The downwelling population is made of a small number of individuals and the impact of those that are generated at the boundaries is likely to get reduced as the aspect ratio is increased. This may account for part of the decrease that is observed when the aspect ratio is doubled (table 2).

Grid size	X:Y	Nd^2	A_i/d^2	$\Delta T_i k/Hd^2$	$W_i d/\kappa$
			$Ra_H = 10^4$		
512×512×64	16:16	0.144	1.436	0.323	19.7
1024×1024×32	16:16	0.144	1.435	0.323	19.6
1024×1024×64	16:16	0.152	1.362	0.324	19.4
			$Ra_H = 10^8$		
384×384×128	6:6	3.44–4.00	0.0128–0.0156	0.0389–0.0395	1092–1260
384×384×256	6:6	3.17–3.97	0.0132–0.0162	0.0386–0.0396	1078–1264

TABLE 1. Precision tests for calculations with free slip boundaries for two different values of the Rayleigh-Roberts number Ra_H. Entries are as follows. Grid size: the number of grid elements used in the X:Y:Z directions. X:Y are the horizontal dimensions of the computation domain (scaled to the fluid layer thickness). All variables are dimensionless values for the main characteristics of downwellings at mid-depth in the fluid layer. Nd^2 is the dimensionless number per unit area, A_i/d^2 is the average horizontal cross-section, $\Delta T_i k/Hd^2$ is the average temperature contrast and $W_i d/\kappa$ is the average vertical velocity. In simulations for time-dependent regimes at $Ra_H = 10^8$, variables take values that change with time and that vary within the indicated range.

Grid size	X:Y	Nd^2	A_i/d^2	$\Delta T_i k/Hd^2$	$W_i d/\kappa$
			$Ra_H = 10^4$		
512×512×64	16:16	0.144	1.436	0.323	19.7
1024×1024×64	32:32	0.124	1.689	0.322	20.1
			$Ra_H = 10^8$		
384×384×128	6:6	3.44–4.00	0.0128–0.0156	0.0389–0.0395	1092–1260
768×768×128	12:12	3.19–3.56	0.0146–0.0163	0.0389–0.0391	1185–1242

TABLE 2. Impact of the domain aspect ratio on the characteristics of internally-heated convection with free slip horizontal boundaries (symbols as in table 1).

For time-dependent regimes, we also observed that the number of downwellings decreases slightly with increasing aspect ratio. With a larger population than in steady-state regimes, the variation is more subdued, amounting to less than 10% when the aspect ratio is doubled. Values for temperature and velocity are not affected significantly by the change of lateral dimensions.

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