Supplementary Material: Controlling rotation and migration of rings in a simple shear flow through geometric modifications

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1. Analytical integration of fluid disturbance due to particle along the azimuthal direction

The boundary integral equation given in equation (4 a) can be solved to obtain the force per unit area acting on the particle surface using a 2-D surface mesh. As mentioned in section 3, the azimuthal variation of the force per unit area for our problem, given by equation (8), can be used to reduce the dimensionality of the problem. Here, we give a detailed methodology of achieving this for a ring in a simple shear flow, such that **p** lies in the flow gradient plane. This could be easily extended for an axisymmetric particle with arbitrary orientation in a general linear flow field. The boundary integral equation (4 a) can be written as:

$$\mathbf{u}.\mathbf{n} = \mathbf{u}^{\infty}.\mathbf{n} + \frac{1}{8\pi\mu}\int ds' \int_{\phi}^{\phi+2\pi} d\phi'\, x' \left[f_n(\mathbf{r}')\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} + \frac{\left(x\cos(\phi) - x'\cos(\phi')\right)^2}{|\mathbf{r}-\mathbf{r}'|^3}\right) + f_b(\mathbf{r}')\left(\frac{\left(x\cos(\phi)-x'\cos(\phi')\right)\left(x\sin(\phi)-x'\sin(\phi')\right)}{|\mathbf{r}-\mathbf{r}'|^3}\right) + f_p(\mathbf{r}')\left(\frac{\left(x\cos(\phi)-x'\cos(\phi')\right)(y-y')}{|\mathbf{r}-\mathbf{r}'|^3}\right)\right], \quad \text{(A 1)}$$

$$\mathbf{u}.\mathbf{b} = \mathbf{u}^{\infty}.\mathbf{b} + \frac{1}{8\pi\mu}\int ds' \int_{\phi}^{\phi+2\pi} d\phi'\, x' \left[f_n(\mathbf{r}')\left(\frac{\left(x\cos(\phi)-x'\cos(\phi')\right)\left(x\sin(\phi)-x'\sin(\phi')\right)}{|\mathbf{r}-\mathbf{r}'|^3}\right) + f_b(\mathbf{r}')\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} + \frac{\left(x\sin(\phi)-x'\sin(\phi')\right)^2}{|\mathbf{r}-\mathbf{r}'|^3}\right) + f_p(\mathbf{r}')\left(\frac{\left(x\sin(\phi)-x'\sin(\phi')\right)(y-y')}{|\mathbf{r}-\mathbf{r}'|^3}\right)\right], \quad \text{(A 2)}$$

$$\mathbf{u}.\mathbf{p} = \mathbf{u}^{\infty}.\mathbf{p} + \frac{1}{8\pi\mu}\int ds' \int_{\phi}^{\phi+2\pi} d\phi'\, x' \left[f_n(\mathbf{r}')\left(\frac{\left(x\cos(\phi)-x'\cos(\phi')\right)(y-y')}{|\mathbf{r}-\mathbf{r}'|^3}\right) + f_b(\mathbf{r}')\left(\frac{\left(x\sin(\phi)-x'\sin(\phi')\right)(y-y')}{|\mathbf{r}-\mathbf{r}'|^3}\right) + f_p(\mathbf{r}')\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} + \frac{(y-y')^2}{|\mathbf{r}-\mathbf{r}'|^3}\right)\right], \quad \text{(A 3)}$$

where the subscripts n, b, p represent the components of a vector along **n**, **b** and **p** respectively. x and y are the position along $\mathbf{e_x}$ and $\boldsymbol{e_y}$ respectively and ϕ is the angular position. The primed variables (ds', x' and **r'**) are dummy variables. The integrals in (A 1) – (A 3) can be analytically integrated along the azimuthal direction by substituting $\phi'' = \phi' - \phi$ in equations to (A 1) – (A 3). Upon this substitution $|\mathbf{r} - \mathbf{r}'|$ is given by

$$r'' = |\mathbf{r} - \mathbf{r}'| = [A^2 - B^2 \cos(\phi'')]^{0.5}, \qquad \text{(A 4)}$$

where $A^2 = x^2 + x'^2 + y^2$, $B^2 = 2xx'$. On changing the dummy variable to ϕ'' and substituting (A 4) and equation (8) into (A 1) – (A 3), the numerators of the integrands in (A 1) – (A 3) will be either a constant, powers of $\cos(\phi'')$ or powers of $\sin(\phi'')$, while the denominator will be either r'' or r''^3. Terms with odd powers of $\sin(\phi'')$ will integrate to zero, while any even powers of $\sin(\phi'')$ can be transformed into an equivalent term in $\cos(\phi'')$. A constant term and any power of $\cos(\phi'')$ can be transformed into an elliptic integral using elementary calculus as shown in Singh et al. (2013). This procedure can be understood from the transformation given by

$$\int_0^{2\pi} d\phi'' \frac{\cos^a(\phi'')}{(A^2 - B^2\cos(\phi''))^{\frac{m}{2}}} = \frac{4(-1)^a}{(C^2)^{m/2}} \int_0^{2\pi} d\phi'' \frac{\cos^a(2\phi'')}{(1 - D^2\sin^2(\phi''))^{\frac{m}{2}}}, \qquad \text{(A 5)}$$

where $C^2 = A^2 + B^2$, $D^2 = 2B^2/C^2 < 1$, a and m are integers. The left-hand side of (A 5) represents a general term expected in (A 1) – (A 3). The right-hand side of (A 5) can be given in terms of complete elliptic integrals of the first (K) and second (E) kind which are given by

$$K(D) = \int_0^{2\pi} \frac{d\phi''}{(1 - D^2\sin^2(\phi''))^{0.5}}, \qquad \text{(A 6)}$$

$$E(D) = \int_0^2 d\phi''(1 - D^2\sin^2(\phi''))^{0.5}. \qquad \text{(A7)}$$

The integrals in (A5) are obtained from Singh et al. (2013) for a = {0, 1, 2} and m = {1, 3} and are denoted as $S_1, S_2, \ldots, S_8$ following the same notation used in their paper. Two additional integrals are required for a = 3 and m = {1, 3} for our case which are given by S_9 and S_{10}. All the integrals involved in (A 1) – (A 3) are given by

$$S_1 = \int_0^{2\pi} d\phi" \frac{1}{(A^2 - B^2\cos(\phi"))^{0.5}} = \frac{4}{C} K \ , \qquad \text{(A 8 a)}$$

$$S_2 = \int_0^{2\pi} d\phi" \frac{\cos(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = -\frac{4}{C}\left(\frac{D^2-2}{D^2} K + \frac{2}{D^2} E\right), \qquad \text{(A 8 b)}$$

$$S_3 = \int_0^{2\pi} d\phi" \frac{1}{(A^2 - B^2\cos(\phi"))^{1.5}} = \frac{4}{C^3} \frac{E}{1-D^2} \ , \qquad \text{(A 8 c)}$$

$$S_4 = \int_0^{2\pi} d\phi" \frac{\cos(\phi")}{(A^2 - B^2\cos(\phi"))^{1.5}} = \frac{4}{C^3 D^2}\left(-2K + \frac{D^2-2}{(D^2-1)} E\right), \qquad \text{(A 8 d)}$$

$$S_5 = \int_0^{2\pi} d\phi" \frac{\cos^2(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = \frac{4}{C^3 D^4}\left(4(D^2-2)K + \frac{D^4-8D^2+8}{1-D^2} E\right), \qquad \text{(A 8 e)}$$

$$S_6 = \int_0^{2\pi} d\phi" \frac{\cos^2(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = \frac{4}{3CD^4}\left((3D^4 - 8D^2 + 8)K + 4(D^2-2)E\right), \qquad \text{(A 8 f)}$$

$$S_7 = \int_0^{2\pi} d\phi" \frac{\cos^3(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = \frac{-4}{3C^3 D^6}\left(2(9D^4 - 32D^2 + 32)K + \frac{(3D^6-38D^4+96D^2-64)E}{1-D^2}\right), \qquad \text{(A 8 g)}$$

$$S_8 = \int_0^{2\pi} d\phi" \frac{\cos^4(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = \frac{4}{10C^3 D^8}\left(16(D^2-2)(5D^4 - 16D^2 + 16)K + \frac{2(5D^8-96D^6+352\,D^4-512D^2+256)}{1-D^2} E\right), \qquad \text{(A 8 h)}$$

$$S_9 = \int_0^{2\pi} d\phi" \frac{\cos^3(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = -\frac{4}{15C\,D^6}\left((15D^6 - 62D^4 + 96D^2 - 64)K + 2(17D^4 - 32D^2 + 32)E\right), \qquad \text{(A 8 i)}$$

$$S_{10} = \int_0^{2\pi} d\phi" \frac{\cos^5(\phi")}{(A^2 - B^2\cos(\phi"))^{0.5}} = -\frac{4}{105C\,D^{10}}\left((1050D^8 - 7072D^6 + 19360D^4 - 24576D^2 + 12288)K + \frac{(-105D^{10}+2594D^8-13296D^6+29344D^4-30720D^2+12288)}{D^2-1} E\right). \qquad \text{(A 8 j)}$$

Applying the no slip boundary condition on the particle surface equations (A 1) – (A 3) are simplified to obtain a set of linear equations, which can be rewritten in the form:

$$-\mathbf{u}^{\infty}.\mathbf{n} = -(\mathbf{U} + \boldsymbol{\omega}_{\mathbf{p}} \times \mathbf{r}).\mathbf{n} + \sum_{j=1}^{N}\sum_{k=1}^{N_s} ds'_k x'_k\big[\big(f_0^j I_{nn0} + f_1^j I_{nn1} + f_2^j I_{nn2} + f_3^j I_{nn3}\big) + \big(f_4^j I_{nb1} + f_5^j I_{nb2} + f_6^j I_{nb3}\big) + \big(f_7^j I_{np0} + f_8^j I_{np1} + f_9^j I_{np2}\big)\big], \tag{A 9a}$$

$$-\mathbf{u}^{\infty}.\mathbf{b} = -(\mathbf{U} + \boldsymbol{\omega}_{\mathbf{p}} \times \mathbf{r}).\mathbf{b} + \sum_{j=1}^{N}\sum_{k=1}^{N_s} ds'x'\big[\big(f_0^j I_{bn0} + f_1^j I_{bn1} + f_2^j I_{bn2} + f_3^j I_{bn3}\big) + \big(f_4^j I_{bb1} + f_5^j I_{bb2} + f_6^j I_{bb3}\big) + \big(f_7^j I_{bp0} + f_8^j I_{bp1} + f_9^j I_{bp2}\big)\big], \tag{A 9b}$$

$$-\mathbf{u}^{\infty}.\mathbf{p} = -(\mathbf{U} + \boldsymbol{\omega}_{\mathbf{p}} \times \mathbf{r}).\mathbf{p} + \sum_{j=1}^{N}\sum_{k=1}^{N_s} ds'x'\big[\big(f_0^j I_{pn0} + f_1^j I_{pn1} + f_2^j I_{pn2} + f_3^j I_{pn3}\big) + \big(f_4^j I_{pb1} + f_5^j I_{pb2} + f_6^j I_{pb3}\big) + \big(f_7^j I_{pp0} + f_8^j I_{pp1} + f_9^j I_{pp2}\big)\big]. \tag{A 9c}$$

Here N is the number of elements in the primary mesh, N_s is the number of elements in the secondary mesh and ds'_k is the size of the secondary mesh, as shown in figure A.1 (c). The summation in equation (A 9) is performed using the secondary mesh points, which are subdivisions to the primary mesh, to get an accurate estimate of the Green's function. This secondary mesh is used to for the numerical integral in equation (A 9). Expressions to obtain $x', y'$and ds'_k are also shown in figure A. 1 (c). Furthermore, we assume that the values of $[f_0, f_1, \dots f_9]$ remain constant over secondary mesh as shown in figure A.1 (c) which works well as long as sufficient number of primary mesh points exist. The additional terms on the right-hand side of (A 9), denoted by I, can be obtained by simple but lengthy algebra from (A 1) – (A 3) after appropriate substitution. These additional terms are obtained by performing analytical integration over ϕ" and are given by

$$I_{nn0} = S_1 + x^2\cos^2(\phi)\,S_3 - 2xx'\cos^2(\phi)\,S_4 + x'^2(\cos(2\phi)\,S_5 + \sin^2(\phi)\,S_3), \tag{A 10 a}$$

$$I_{nn1} = \cos(\phi)\,S_2 + \cos(\phi)(x^2\cos^2(\phi)\,S_4 - 2xx'\cos^2(\phi)\,S_5 + x'^2(\cos(2\phi)\,S_7 + \sin^2(\phi)\,S_4) - \sin(\phi)\sin(2\phi)\big(xx'(S_3 - S_5) - x'^2(S_4 - S_7)\big), \tag{A 10 b}$$

$$I_{nn2} = \cos(2\phi)\,(2S_6 - S_1) + \cos(2\phi)\Big(x^2\cos^2(\phi)\ (2S_5 - S_3) - 2xx'\cos^2(\phi)\,(2S_7 - S_4) + x'^2\big(\cos(2\phi)\,(2S_8 - S_5) + \sin^2(\phi)\ (2S_5 - S_3)\big)\Big) - 2\sin^2(2\phi)\big(xx'(S_4 - S_7) - x'^2(S_5 - S_8)\big), \tag{A10 c}$$

$$I_{nn3} = \cos(3\phi)\,(4S_9 - 3S_2) + \cos(3\phi)\left(x^2\cos^2(\phi)\,(4S_7 - 3S_4) - 2xx'\cos^2(\phi)\,(4S_8 - 3S_5) + x'^2\left(\cos(2\phi)\,(4S_{10} - 3S_7) + \sin^2(\phi)\,(4S_7 - 3S_4)\right)\right) - \sin(3\phi)\sin(2\phi)\left(xx'\sin(2\phi)\,(5S_5 - S_3 - 4S_8) - x'^2(5S_7 - S_4 - 4S_{10})\right), \quad \text{(A 10 d)}$$

$$I_{nb1} = 0.5\sin(\phi)\sin(2\phi)\left(x^2S_4 - 2xx'S_5 + x'^2(2S_7 - S_4)\right) + \cos(\phi)\cos(2\phi)\left(-xx'(S_3 - S_5) + x'^2(S_4 - S_7)\right), \quad \text{(A 10 e)}$$

$$I_{nb2} = 0.5\sin^2(2\phi)\left(x^2(2S_5 - S_3) - 2xx'(2S_7 - S_4) + x'^2(4S_8 - 4S_5 + S_3)\right) + 2\cos^2(2\phi)\left(-xx'(S_4 - S_7) + x'^2(S_5 - S_8)\right), \quad \text{(A 10 f)}$$

$$I_{nb3} = 0.5\sin(3\phi)\sin(2\phi)\left(x^2(4S_7 - 3S_4) - 2xx'(4S_8 - 3S_5) + x'^2(8S_{10} - 10S_7 + 3S_4)\right) + \cos(3\phi)\cos(2\phi)\left(-xx'(-4S_8 + 5S_5 - S_3) + x'^2(-4S_{10} + 5S_7 - S_4)\right), \quad \text{(A 10 g)}$$

$$I_{np0} = (y - y')\cos(\phi)(xS_3 - x'S_4), \quad \text{(A 10 h)}$$

$$I_{np1} = (y - y')\left(\cos^2(\phi)\,(xS_4 - x'S_5) - \sin^2(\phi)\,x'(S_3 - S_5)\right), \quad \text{(A 10 i)}$$

$$I_{np2} = (y - y')\left(\cos(2\phi)\cos(\phi)\left(x(2S_5 - S_3) - x'(2S_7 - S_4)\right) - 2\sin(2\phi)\sin(\phi)\,x'(S_4 - S_7)\right), \quad \text{(A 10 j)}$$

$$I_{bn0} = 0.5\sin(2\phi)\left(x^2S_3 - 2xx'S_4 + x'^2(2S_5 - S_3)\right), \quad \text{(A 11 a)}$$

$$I_{bn1} = 0.5\sin(2\phi)\cos(\phi)\,(x^2S_4 - 2xx'S_5 + x'^2(2S_7 - S_4)) - \cos(2\phi)\sin(\phi)\left(x'^2(-S_7 + S_4) - xx'(-S_5 + S_3)\right), \quad \text{(A 11 b)}$$

$$I_{bn2} = 0.5\sin(2\phi)\cos(2\phi)\,(x^2(2S_5 - S_3) - 2xx'(2S_7 - S_4) + x'^2(4S_8 - 4S_5 + S_3)) - 2\cos(2\phi)\sin(2\phi)\left(x'^2(-S_8 + S_5) - xx'(-S_7 + S_4)\right), \quad \text{(A 11 c)}$$

$$I_{bn3} = 0.5\sin(2\phi)\cos(3\phi)\,(x^2(4S_7-3S_4)-2xx'(4S_8-3S_5)+x'^2(8S_{10}-10S_7+3S_4)) - 2\cos(2\phi)\sin(3\phi)\left(x'^2(-4S_{10}+5S_7-S_4)-xx'(-4S_8+5S_5-S_3)\right), \qquad \text{(A 11 d)}$$

$$I_{bb1} = \sin(\phi)\,S_2+\sin(\phi)\,(x^2\sin^2(\phi)\,S_4-2xx'\sin^2(\phi)\,S_5+x'^2(\cos^2(\phi)\,S_4 - \cos(2\phi)\,S_7))+\sin(2\phi)\cos(\phi)\,(-xx'(S_3-S_5)+x'^2(S_4-S_7)), \qquad \text{(A 11 e)}$$

$$I_{bb2} = \sin(2\phi)\,(2S_6-S_1)+\sin(2\phi)\left(x^2\sin^2(\phi)\,(2S_5-S_3)-2xx'\sin^2(\phi)\,(2S_7-S_4) + x'^2\left(-\cos(2\phi)(2S_8-S_5)+\cos^2(\phi)\,(2S_5-S_4)\right)\right)+2\sin(2\phi)\cos(2\phi)\,(-xx'(S_4-S_7) + x'^2(S_5-S_8)), \qquad \text{(A 11 f)}$$

$$I_{bb3} = \sin(3\phi)\,(4S_9-3S_2)+\sin(3\phi)\left(x^2\sin^2(\phi)\,(4S_7-3S_5)-2xx'\sin^2(\phi)\,(4S_8-3S_5) + x'^2\left(\sin^2(\phi)\,(4S_{10}-3S_7)+\cos^2(\phi)\,(-4S_{10}+7S_7-3S_4)\right)\right) + \sin(2\phi)\cos(3\phi)\,(-xx'(-4S_8+5S_5-S_3)+x'^2(-4S_{10}+5S_7-S_4)), \qquad \text{(A 11 g)}$$

$$I_{bp0} = (y-y')\sin(\phi)\,(xS_3-x'S_4)\,, \qquad \text{(A 11 h)}$$

$$I_{bp1} = (y-y')\left(\cos(\phi)\sin(\phi)\,(xS_4-x'S_5)+\sin(\phi)\cos(\phi)\,x'(S_3-S_5)\right), \qquad \text{(A 11 i)}$$

$$I_{bp2} = (y-y')\left(\cos(2\phi)\sin(\phi)\left(x(2S_5-S_3)-x'(2S_7-S_4)\right)+2\sin(2\phi)\cos(\phi)\,(S_4 - S_7)\right), \qquad \text{(A 11 j)}$$

$$I_{pn0} = (y-y')\cos(\phi)\,(xS_3-x'S_4)\,, \qquad \text{(A 12 a)}$$

$$I_{pn1} = (y-y')\left(\cos^2(\phi)\,(xS_4-x'S_5)-\sin^2(\phi)\,x'(S_3-S_5)\right), \qquad \text{(A 12 b)}$$

$$I_{pn2} = (y-y')\left(\cos(2\phi)\,(x\cos(\phi)(2S_5-S_3)-x'\cos(\phi)(2S_7-S_4)) - 2\sin(2\phi)\sin(\phi)\,x'(S_4-S_7)\right), \qquad \text{(A 12 c)}$$

$$I_{pn3} = (y-y')\left(\cos(3\phi)\left(x\cos(\phi)(4S_7-3S_4)-x'\cos(\phi)\,(4S_8-3S_5)\right) - \sin(3\phi)\sin(\phi)\,x'(-4S_8+5S_5-S_3)\right), \tag{A 12 d}$$

$$I_{pb1} = (y-y')\left(\sin^2(\phi)\,(xS_4-x'S_5)-\cos^2(\phi)\,(S_3-S_5)\right), \tag{A 12 e}$$

$$I_{pb2} = (y-y')\left(\sin(2\phi)\sin(\phi)\left(x(2S_5-S_3)-x'(2S_7-S_4)\right)-2\cos(2\phi)\cos(\phi)\,x'(S_4-S_7)\right), \tag{A 12 f}$$

$$I_{pb3} = (y-y')\left(\sin(3\phi)\sin(\phi)\,\left(x(4S_7-3S_4)-x'(4S_8-3S_5)\right) - \cos(3\phi)\cos(\phi)\,x'(-4S_8+5S_5-S_3)\right), \tag{A 12 g}$$

$$I_{pp0} = S_1+(y-y')^2S_3, \tag{A 12 h}$$

$$I_{pp1} = S_2\cos(\phi)+(y-y')^2\,S_4\cos(\phi), \tag{A 12 i}$$

$$I_{pp2} = (2S_6-S_1)\cos(2\phi)+(y-y')^2(2S_5-S_3)\cos(2\phi). \tag{A 12 j}$$

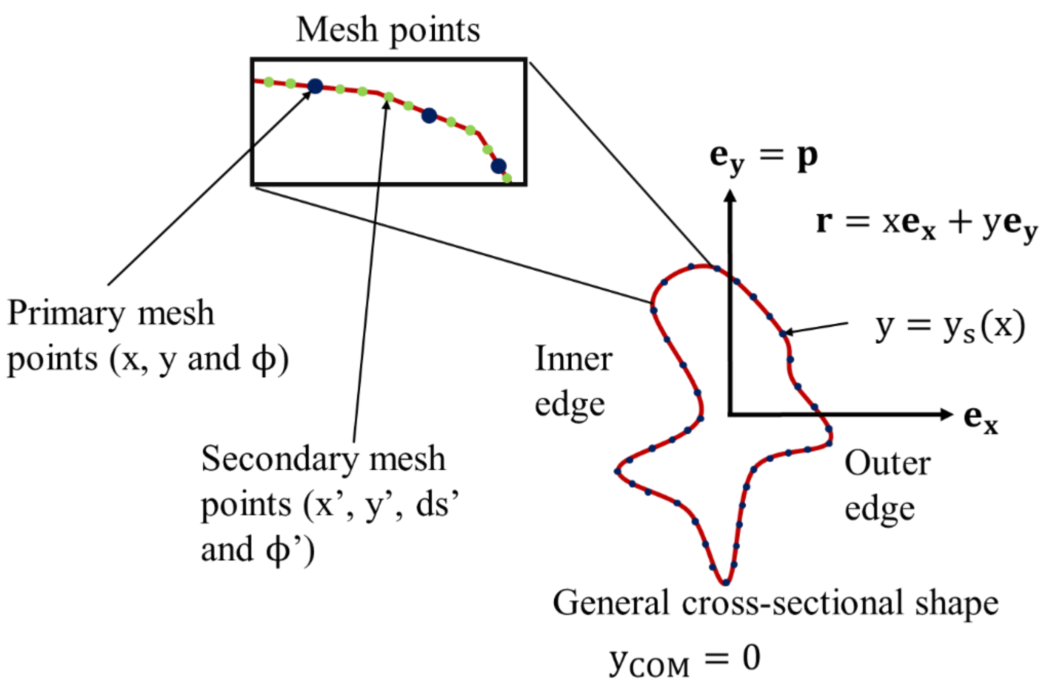
Using expressions in equations (A 10) – (A 12), the inner summation in equation (A 9) can be evaluated to give linear equations for $[f_0, f_1, \dots, f_9]$ at each mesh point. One gets 3 linear equations for each mesh point using equation (A 9), giving a total of 3N equations for a given value of ϕ. 10N equations can be obtained by evaluating equation (A 9) at four different values of ϕ. We also need to enforce force and torque free condition on the particle using equations (9) and (10) to obtain the linear and angular velocities of the particle for the mobility problem.

In the limit $D \rightarrow 1$, $S_1, S_2, \dots, S_{10}$, as per equation (A 8), become singular, which arises when **r**' approaches **r**. This singularity is logarithmic and integrable. We handled this singularity by analytically evaluating the elliptic integrals using the following asymptotic expansion of the elliptic integrals (Lee and Leal, 1982):

$$K = \ln\left(\frac{4}{D'}\right) + \frac{1}{2}\left(\ln\left(\frac{4}{D'}\right) - 1\right) D'^2 + \frac{9}{64}\left(\ln\left(\frac{4}{D'}\right) - \frac{7}{6}\right) D'^4, \tag{A 13 a}$$

$$E = 1 + \frac{1}{2}\left(\ln\left(\frac{4}{D'}\right) - \frac{1}{2}\right) D'^2 + \frac{3}{16}\left(\ln\left(\frac{4}{D'}\right) - \frac{13}{12}\right) D'^4, \tag{A 13 b}$$

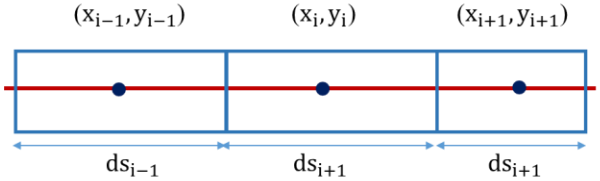
where $D' = (1 - D^2)^{0.5}$. Equations (A 13) are used when $D' \leq 10^{-3}$, else K and E are evaluated using numerically.



(a)

Primary mesh

N = number of elements in the primary mesh



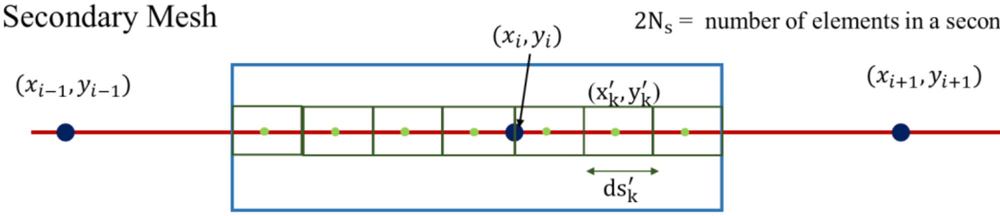
$$ds_i = \frac{1}{2}\left(\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} + \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}\right)$$

We assume that the values of $[f_0, f_1, \dots, f_9]$ are constant throughout a given cell.

(b)

Secondary Mesh

$2N_s$ = number of elements in a secondary mesh



$$\mathbf{X}'_k = \frac{(N_s + 1 - k)\frac{(\mathbf{X}_{i-1} + \mathbf{X}_i)}{2} + (k)\mathbf{X}_i}{N_s + 1}, \qquad 1 \le k \le N_s$$

$$\mathbf{X}'_k = \frac{(N_s + 1 - k)\mathbf{X}_i + (k)\frac{(\mathbf{X}_{i+1} + \mathbf{X}_i)}{2}}{N_s + 1}, N_s + 1 \le k \le 2N_s$$

Here $\mathbf{X_i} = (x_i, y_i)$ is a primary mesh point, while $\mathbf{X}'_{\boldsymbol{k}} = (x'_k, y'_k)$ is secondary mesh point. ds'_k is the size of the k^{th} secondary mesh point defined as

$$ds'_k = \frac{1}{2}\left(\sqrt{(x'_k - x'_{k-1})^2 + (y'_k - y'_{k-1})^2} + \sqrt{(x'_k - x'_{k+1})^2 + (y'_k - y'_{k+1})^2}\right)$$

This discretization ensures that the primary and secondary mesh point never coincide. The value of $[f_0, f_1, \dots, f_9]$ at each secondary mesh point of the i^{th} primary cell is equal to the value of $[f_0, f_1, \dots, f_9]$ for this primary mesh point .

(c)

Figure A.1. Schematic of the two meshes used in the calculation of the numerical integration of equation (4 a). The coefficients $[f_0, f_1, \dots f_9]$ are only evaluated at the primary mesh points and these values are assumed constant over the primary mesh cell. The summation in equation (A 9) is performed using the secondary mesh points, which are subdivisions to the primary mesh, to get an accurate estimate of the Green's function. This secondary mesh is used to for the numerical integral in equation (A 9).

2. References

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